COMMERCIAL ARITHMETIC

COMMERCIAL ARITHMETIC

BY

FRANK L. GRANT, M.A., F.R.A.S.

MATHEMATICAL MASTER, UNIVERSITY COLLEGE SCHOOL, LONDON

AND

ALEXANDER M HILL, M.A.

MASTER MATHEMATICAL DEPARTMENT, HIGH SCHOOL OF GLASGOW
MASTER, MATHEMATICAL DEPARTMENT, HIGH
EVENING SCHOOL, GLASGOW

WITH ANSWERS

NEW IMPRESSION

LONGMANS, GREEN AND CO.

39 PATERNOSTER ROW, LONDON

FOURTH AVENUE & 30TH STREET, NEW YORK

BOMBAY, CALCUTTA, AND MADRAS

1921

PREFACE TO THE THIRD EDITION

In the third edition an attempt has been made to bring the book into line with the modern methods of teaching Elementary Mathematics. This will be seen in the methods adopted in multiplication and division of decimals, in the rules used in contracted work, and in the inclusion and cancelling of concrete quantities in the solution of questions. The reason for every rule is fully stated so that the student will be able to follow all the working.

As the result of an expressed desire the section on Annuities has been considerably extended, and now covers the syllabus of the intermediate examinations for Chartered Accountants.

A considerable number of examples have been added, and the Examination Papers, which now include copies of papers in Commercial Arithmetic set by most of the Examining Bodies, have been brought up to date.

It is hoped that all errors have been corrected; but the authors will be glad to have their attention called to any that remain, and will gratefully acknowledge any suggestions as to improvements in the next edition.

F. L. G.
A. M. H.

January 1913.

PREFACE TO THE FIRST AND SECOND EDITIONS

THIS Text-book on Arithmetic has been prepared to meet the requirements of students completing a course of Commercial Education. The chief object in view has been to adopt those methods which will be of use in Mercantile Transactions. For that reason, only those portions of Elementary Arithmetic in which special methods are advocated have been considered. These include Subtraction by the Austrian method, Division by the Italian method, &c. In approximations, the logical system of working examples to a certain number of significant figures instead of places of decimals has been adopted, and attention is paid to the correctness of the last figure by making allowance when required. The extended use of approximations is explained by a short reference to the "method of prediction."

A special feature of the book is the continual use of decimalisation and the construction and use of tables of decimal values. The Metric System, the Imperial Standard British weights and measures, mean solar time, and the coinages (British and foreign), have all been used in this connection, and special tables given for the conversion from one system to another.

In Interest, Profit and Loss, Stocks and Shares, &c., full details have been given to enable students to work out problems on these subjects in an intelligent manner.

A short section on Algebra has been introduced in order that students may make use of the principles of simple equations in working problems. The methods adopted in the extraction of the square and cube roots are built up from the algebraic formulæ for the square and cube of a binomial expression, this being considered the most logical and satisfactory method. Throughout the section on mensuration, algebraic methods have been used, and the necessary formulæ given. Tables of four place logarithms have been added, and their use explained in connection with

involution, evolution, and compound interest; in connection with this, an explanation of the use of indices has been given.

The following extracts from papers which have appeared during the preparation of the book show that the principles, to which we have adhered, are gradually being adopted by the educational authorities throughout the country:—

"In Arithmetic, marks will be deducted on those answers in which bad and antiquated methods are used; for example, if the modern contracted method in division is not followed; if decimal workings are not properly contracted, if remainders are given in fractions instead of in decimals; if logarithms are not used where their use would save time. A knowledge of the elementary principles of mensuration is required. No questions will be asked in 'partnership,' 'tare and tret,' or other obsolete rules. . . ." *

"That in examinations particular methods of solution or demonstration should not as a rule be demanded, e.g., the use of algebra should not be prohibited in answering questions in arithmetic or geometry." †

Every effort has been made to select practical questions on the various rules. In most cases the answers are given correct to the nearest complete value (in money, to the nearest penny), and no unpractical fractions are used. While many of the questions are original, a number have been selected from papers set by the Scotch Education Department, the Oxford and Cambridge Local Examinations Board, the Institutes of Bankers and Chartered Accountants, and others.

Copies of the most recent papers set by several of these examining bodies have been added after the Miscellaneous Examples.

A table of contents and an index have been supplied.

F. L. G.
A. M. H.

THE HIGH SCHOOL OF GLASGOW.
June 1902.

* Prospectus of the City and Guilds Technical College.
† The Edinburgh Mathematical Society.

CONTENTS

COMMERCIAL ARITHMETIC

I.—ADDITION.

1. The following devices will generally prove helpful to speed and accuracy:—

To add 9; add 10 and deduct 1.
Thus, 43+9=53−1=52; 97+9=107−1=106.
To add 8; add 10 and deduct 2.
Thus, 56+8=66−2=64.

2. In adding, a lookout should be kept for combinations of figures in the columns, which make up 8, 9, 10, and 11. Thus, a 4 followed by a 5 should be added as a 9; a 6 followed by a 4, or an 8 by a 2, should be added as a 10, and so on.

3. In adding shillings, add first the unit figures, setting down unit figure of the answer; then add the tens column, divide the result by 2, setting down any remainder in the tens column of the answer, and carry the quotient to the pounds.

II.—SUBTRACTION (AUSTRIAN METHOD).

4. This method of subtraction consists in finding what requires to be added to the subtrahend figure to make the minuend.

Thus, to subtract 9 from 16, we say 9+7=16. Answer is 7.

Example.—From 67435 take 43927.

	Method—	7+8=15 (1 to carry)
67435	(2+1=3)	3+0= 3
43927		9+5=14 (1 to carry)
	(3+1=4)	4+3= 7
23508		4+2= 6

Examples 1.

Add the columns and rows, and find the grand total. This addition may be varied by taking the rows in sets of five and adding columns and rows accordingly.

	A	B	C	D	E	TOTALS
1	641,235	941,338	416,324	826,126	82,426	
2	313,721	216,727	311,021	7,318	1,395,342	
3	28,156	928,156	328,726	34,203	424,027	
4	834,785	4,789	773,417	712,937	2,331,784	
5	190,876	198,876	36,395	2,731	15,417	
6	828,739	759	8,237	326,208	317,183	
7	245,624	385,624	627,319	78,485	831	
8	313,977	343,976	2,962	2,939	4,215	
9	82,493	93	784,885	41[illegible],735	1,772	
10	737,926	837,926	869,139	173,572	5,208	
11	158,591	168,571	47,735	18,371	66,531	
12	827,197	227,197	142,578	412,172	138,314	
13	238,946	938,946	173,779	827,215	485,136	
14	442,839	2,839	12,172	8,5[illegible]	236	
15	190,164	490,766	82,715	272,572	4,126	
16	324,635	24,635	283,234	93,718	31,765	
17	71,296	76,298	272,547	316,825	8,545	
18	428,003	328,903	37,918	813	1,735,178	
19	18,795	18,795	31,925	145,178	21,955	
20	630,682	230,682	331,637	136,412	412,426	
Totals,						

Examples 2.

Add the columns and rows, and find the grand total. The addition may be varied by taking the rows in sets of five and adding columns and rows accordingly.

	A			B			C			D			E			F			Totals		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
1	826	13	9	126	14	7	35	6	11	328	18	4	726	10	5	615	8	10			
2	73	4	7	18	13	8	615	14	4	773	6	7	417	3	6	440	16	7			
3	342	14	[illegible]	93	16	8	89	14	10	363	14	11	95	16	8	89	14	0			
4	712	16	4	937	0	10	417	10	5	82	14	8	37	10	11	467	0	5			
5	27	2	5	317	3	8	65	16	10	627	2	5	319	17	8	865	18	10			
6	183	0	8	71	6	6	56	7	8	173	0	8	779	9	5	352	4	8			
7	412	3	4	172	13	10	317	3	10	12	4	7	172	13	10	17	3	11			
8	827	15	[illegible]	215	10	8	95	14	9	827	15	6	15	0	8	796	10	10			
9	85	5	10	34	8	11	124	6	8	283	8	11	234	18	11	134	16	8			
10	272	13	10	572	12	9	39	12	11	272	13	10	577	15	9	99	2	9			
11	326	8	11	208	9	9	778	12	6	29	3	11	62	9	10	70	16	11			
12	784	9	7	85	14	4	45	13	7	784	9	7	885	14	4	45	3	7			
13	29	9	8	39	12	11	85	14	9	869	19	6	139	2	6	925	14	4			
14	417	10	5	735	12	5	178	12	6	477	16	5	35	12	5	78	12	6			
15	173	0	9	572	14	9	784	9	7	142	10	10	578	16	9	734	19	9			
16	316	12	6	825	19	4	24	16	6	319	2	6	25	19	8	123	17	7			
17	31	3	8	3	17	11	777	13	9	931	8	3	63	17	10	678	3	9			
18	145	11	10	178	0	9	929	9	8	45	1	10	174	10	9	29	19	5			
19	136	4	6	412	3	7	42	16	11	236	14	9	12	18	4	442	16	11			
20	712	16	7	29	8	7	173	4	7	722	18	7	429	8	7	179	8	3			
Totals,																					

III.—SIMPLE MULTIPLICATION.

5. An example in simple multiplication is usually worked beginning with the right-hand figure of the multiplier and multiplying by the several figures in succession, *taking care to begin each line below the figure in use*, as in the following example :—

Multiply 68379 *by* 4837.

```
   68379 (multiplicand)
    4837 (multiplier)
  ------
  478653
 205137
547032
273516
---------
330749223 (product)
```

A beginner is, in such cases, very apt to overlook the fact that the lines of the working are obtained by multiplying the multiplicand by 7, 30, 800, and 4000, the 0's having been omitted. It is for this reason that there is the necessity for beginning each line immediately under the figure in use. The complete working out would be

68379
4837
478653
2051370
54703200
273516000
330749223

6. Examples may be worked out, beginning with the left-hand figure. In doing so, we take the figures in the order of their importance. In some cases it is advisable to use this method. The above example would then become

68379
4837
273516000
54703200
2051370
478653
330749223

7. It would be advantageous and more consistent if multiplication were always worked out by this method. The student should work out several examples in this way, the 0's being omitted if desired. It will be found useful when questions in contracted multiplication are considered (§ 51).

Example—

Multiply 789254 *by* 30780.

789254
30780
2367762
5524778
63140320
24293238120

NOTE.—It will be seen that all the figures of the working are placed in columns corresponding to their *place value*. Thus, in the adjoining example: 4 units multiplied by 3 ten-thousands gives 12 ten-thousands (the 2 is placed in the ten-thousand column); 4 units by 7 hundreds gives 28 hundreds (the 8 is placed in the hundreds column); 4 units by 8 tens gives 32 tens (the 2 is placed in the tens column).

8. Short Methods of Multiplication :—

In working out examples of multiplication a lookout should be kept for any relations that exist between the figures of the multiplier, and use made of them. With this in view, it is not necessary to perform the multiplication in any particular order of the figures of the multiplier, but the necessary 0's must be put down or care must be taken to place the first figure of each line correctly (§ 5). See Examples (ii.) and (iii.) below.

Examples—

(i.) *Multiply* 83924 *by* 1263.
Note: 6 = 3 × 2; 12 = 6 × 2.

```
  83924
   1263
 251772 . . . . (a)
 503544 . . . . (a × 2 = b)
1007088 . . . . (b × 2)
105996012
```

(ii.) *Multiply* 73624 *by* 416.
Note: 16 = 4 × 4.

```
   73624
     416
29449600
 1177984
30627584
```

(iii.) *Multiply* 936528 *by* 16648.
Note: 16 = 8 × 2; 64 = 16 × 4.

```
     936528
      16648
    7492224 . . . . (a)
  14984448  . . . . (a × 2 = b)
   59937792 . . . . (b × 4)
15591318144
```

(iv.) *Multiply* 85392 *by* 63472.
Note: 4 = 2 × 2; 63 = 7 × 9.

```
     85392
     63472
    170784 . . . . (a)
   597744  . . . . (b)
   341568  . . . . (a × 2)
 5379696   . . . . (b × 9)
5420001024
```

Examples 3.

Multiply, beginning with the left-hand figure (see §§ 6 and 7)—

1. 1485 by 357.
2. 5837 by 495.
3. 2599 by 608.
4. 60726 by 5007.
5. 74603 by 8537.
6. 839154 by 4671.
7. 936528 by 5964.
8. 286397 by 73005.
9. 547042 by 16259.
10. 385960 by 38060.
11. 759416 by 74237.
12. 965783 by 59807.

Multiply in as few lines as possible—

13. 53197 by 426.
14. 96285 by 832.
15. 39078 by 357.
16. 58762 by 972.
17. 854279 by 497.
18. 197045 by 819.
19. 946851 by 2463.
20. 857379 by 3963.
21. 564786 by 61854.
22. 679038 by 28567.
23. 862947 by 56082.
24. 6793865 by 63972.

IV.—SIMPLE DIVISION (ITALIAN METHOD)

9. An example in simple division is usually worked out thus :—

Example—*Divide* 3749529 *by* 849.

```
(Divisor) (Dividend) (Quotient)      This is a contraction of—
   849 ) 3749529 ( 4416              849 ) 3749529  / 4000
         3396                              3396000 |   400
         ----                              -------  |    10
          3535                              353529  |     6
          3396                              339600  |  ----
          ----                              ------  \  4416
           13929                             13929
            8490                              8490
           -----                             -----
            5439                              5439
            5094                              5094
            ----                              ----
             345 (remainder)                   345
```

10. When the principles involved in division are understood, the working may be greatly shortened by using the **Italian Method**. This method is used in a simple form in short division. Instead of writing down the products as obtained, the multiplications and subtractions are combined and only the differences are written down. These differences should be obtained by using the **Austrian Method of Subtraction** (§ 4). The above example worked out by the Italian method will show the great advantage of that method.

```
            4416
849 ) 3749529
       3535
        1392
         5439
          345
```

The working out is performed as follows :—

The quotient 4 having been obtained in the usual way, we have

4 × 9 = 36, + 3 = 39 (3 to carry)
4 × 4 = 16, + 3 = 19, + 5 = 24 (2 to carry)
4 × 8 = 32, + 2 = 34, + 3 = 37
and take down 5.

We then obtain the second 4 and proceed as before.

$4 \times 9 = 36, +9 = 45$ (4 to carry)
$4 \times 4 = 16, +4 = 20, +3 = 23$ (2 to carry)
$4 \times 8 = 32, +2 = 34 +1 = 35$
then take down 2.

The next quotient is 1, and we have

$9 + 3 = 12$ (1 to carry)
$4 + 1 = 5, +4 = 9$
$8 + 5 = 13$
then take down 9.

The last quotient is 6,

$6 \times 9 = 54, +5 = 59$ (5 to carry)
$6 \times 4 = 24, +5 = 29, +4 = 33$ (3 to carry)
$6 \times 8 = 48, +3 = 51, +3 = 54$

11. Position of the Quotient :—This method of division should be used in all cases, the quotient being written *above* the dividend, beginning above the right-hand figure of those used in the first operation. There must be one figure above each of the remaining figures of the dividend This is analogous to what is done in short division where the figures of the quotient are put in position below the dividend instead of above. It also provides us with a check on the number of figures in the quotient, which will be found specially useful in division of decimals.

It will be noticed that this method gives the place value of each figure of the quotient at once. Thus, in the above example, the first 4 is placed above the thousands figure (9) in the dividend.

Examples 4.

Divide—

1. 4709595 by 74.
2. 4625751 by 37.
3. 65530283 by 384.
4. 293515073 by 113.
5. 210681204 by 463.
6. 370573034 by 395.
7. 573005754 by 281.
8. 980347395 by 609.
9. 6023415937 by 873.
10. 50868192995 by 318.
11. 5686492978 by 837.
12. 258088709 by 495.
13. 41894514 by 2468.
14. 94873096 by 4937.
15. 83460924 by 5416.
16. 290381404 by 6803.
17. 383890248 by 7635.
18. 615370427 by 4008.
19. 492683752 by 9764.
20. 793478325 by 6837.
21. 62053581019 by 12345.
22. 36136780697 by 34006.
23. 1694624497 by 8345.
24. 91816074975 by 93274.

V.—DIVISION BY FACTORS.

12. When the divisor can be broken up into two or more factors it will often be found helpful, both to speed and accuracy, to use these factors and divide by short division. The smallest factor should be taken as the first divisor.

13. The great difficulty with beginners, in connection with division by factors, is the finding of the remainder. Once the principle of such division is understood, that should cease to be a difficulty.

Consider the following examples:—

(i.) *Divide* 6834592 *by* 42. (ii.) *Divide* 6834592 *by* 294.

The factors of 42 are 6 and 7; of 294 are 6, 7, and 7.

The division is performed thus:—

```
(i.)  42 { 6 | 6834592
         { 7 | 1139098 .. 4 }
             |  162728 .. 2 } 16

(ii.)      { 42 { 6 | 6834592
           {    { 7 | 1139098 .. 4 }
      294  {        |  162728 .. 2 } 16 }
           {      7 |   23246 .. 6      } 268.
```

(i.) Dividing by 6 we obtain 1139098. This is the number of times that 6 is contained in 6834592. Suppose the 6834592 to represent a number of pins; then the 1139098 is the number of sets each containing 6 pins, and the 4 which is over represents 4 single pins left. Similarly, when we divide the 1139098 by 7 we obtain 162728 bundles, each consisting of 7 sets of 6 pins, and we have 2 of these sets left over. Each of the bundles evidently contains 42 pins, so that from 6834592 pins we can make up 162728 bundles of 42 each. The pins not used in making up these bundles—2 sets of 6 each and 4 single pins—form the remainder. The total remainder is therefore $2 \times 6 + 4 = 12 + 4 = 16$.

(ii.) In dividing by 294 the first portion is worked out as before, and we have to divide 162728 by 7. On dividing we obtain 23246 large bundles, each made up of 7 bundles of 42 pins, and we have 6 of such bundles remaining. Each of the large bundles evidently contains $7 \times 42 = 294$ pins, so that from 6834592 pins we can make up 23246 bundles of 294 each. The pins not used in making up these bundles, *i.e.* the remainder, consist of 6 bundles of 42 each and 16 single pins. The total remainder is therefore $6 \times 42 + 16 = 252 + 16 = 268$.

14. The Rule for obtaining the Remainder is:—

Multiply each new remainder by the former complete divisor and add to this product the previous complete remainder.

Examples 5.

Divide, using factors—

1. 6839247 by 28.
2. 346368 by 35.
3. 524081 by 54.
4. 3961754 by 42.
5. 6238977 by 56.
6. 496463 by 63.
7. 7635296 by 48.
8. 8237645 by 72.
9. 2469387 by 121.
10. 7839285 by 99.
11. 2738016 by 144.
12. 638459 by 105.
13. 235437 by 336.
14. 269275 by 315.
15. 683974 by 462.
16. 456913 by 504.
17. 789406 by 729.
18. 3925008 by 1728.

15. The Italian Method of Division may, of course, be extended to all operations in which division is used. It shortens the working in Compound Division (§§ 105–6), Division of Decimals (§§ 33–4), Approximations in Division (§§ 56–9), and Greatest Common Measure. An example of the last of these, worked out by both methods, is given below. It shows the great advantage of the Italian Method.

VI.—GREATEST COMMON MEASURE.

16. The Greatest Common Measure is the greatest number that will divide, without remainder, two or more given numbers.

Examples.—(i.) *Find the G.C.M. of* 30599 *and* 271469.

Ordinary Method.

1	30599	271469	8
	26677	244792	
1	3922	26677	6
	3145	23532	
2	777	3145	4
	74	3108	
1	37	37	
	37		

Italian Method.

30599	271469
3922	26677
777	3145
37	37

G.C.M. = 37.

Note.—The answers to the divisions need not be written in, as they have no bearing on the final result; and if left in, they are apt to be mistaken for the answer to the G.C.M. This was especially the case in the old method of continued long division.

(ii.) *Find the G.C.M. of* 61103, 67338, *and* 89741.

61103	67338
4988	6235
89741	1247
2451	43
1204	
344	

G.C.M. = 43.

Examples 6.

Find the G.C.M. of—

1. 792 and 2552.
2. 1404 and 1264.
3. 4301 and 3553.
4. 12772 and 4257.
5. 11088 and 13104.
6. 47495 and 138355.
7. 63529 and 61013.
8. 42423 and 44671.
9. 48563 and 24293.
10. 1178760 and 1996995.
11. 3474, 13317, and 60023.
12. 6888, 15468, and 554470.

VII.—FACTORS AND PRIME NUMBERS.

17. If a number can be divided, without remainder, by no number except itself and unity, it is said to be a prime number. When it can be divided by any other number, the latter is said to be a factor or measure of the given number.

2, 3, 5, 7, 11, 13, 17, 19, &c., are prime numbers.

4, 6, 8, 9, 10, 12, 14, 16, 18, &c., are not prime numbers. They are sometimes called *composite* numbers.

A number is said to be a multiple of each of its factors.

18 Every composite number can be resolved into the product of two or more prime numbers. Thus $210 = 2 \times 3 \times 5 \times 7$. In such a case, 2, 3, 5, and 7, are called the prime factors of 210, since each is a prime number and a factor of 210.

19. A number can be resolved into its prime factors by trying each of the prime numbers beginning with 2 and dividing out the prime factors obtained till the quotient is a prime number or unity. In making these trial divisions, it is unnecessary to pro-

ceed further than the number which gives a quotient less than itself. No rule can be given as to the identification of prime numbers, which are unlimited in number, but a knowledge of the following methods of testing for the simple factors will be found useful in finding the prime factors of a number.

20. Since all numbers except those ending in 1, 3, 7, or 9, are divisible by 2 or 5, prime numbers can end only with the figures 1, 3, 7, or 9. It must be noted, however, that though a number ends with one of these four figures it is not necessarily prime, *e.g.*

$$3789 = 3 \times 3 \times 421$$
$$9873 = 3 \times 3 \times 1097.$$

21. Tests for Exact Divisibility :—

If the last digit is even, the number is divisible by . .	2
If the number formed by the last two digits is divisible by 4, the number is divisible by	2^2
If the number formed by the last three digits is divisible by 8, the number is divisible by	2^3
If the last digit is 0 or 5, the number is divisible by . .	5
If the sum of the digits is divisible by 3, the number is divisible by	3
If the sum of the digits is divisible by 9, the number is divisible by	3^2
If the sums of the alternate digits are equal, or if the differences of the sums of the alternate digits is divisible by 11, the number is divisible by	11

For other numbers it is generally more troublesome to apply the rules than to test by the numbers themselves.

22. To find the Prime Factors of a Number :—

In finding the prime factors of any given number use should be made of the above tests, and each prime factor, when found, divided out as often as it occurs. 4, 8, and 9 should be written as 2^2, 2^3, and 3^2 respectively in the prime factors. When the above numbers have all been divided out, and the number remaining is not unity, then other prime numbers beginning with 7, 13, 17, &c., must be tested for. It is unnecessary to proceed testing for prime factors when the quotient of the last trial divisor becomes less than the divisor itself. The number then being tested is a prime number.

Example—*Find the Prime Factors of 37944.*

Method—37944 is even, and therefore divides by a power of 2; testing the last three figures "944," we find that they divide by 8, ∴ the number divides by 8, giving 4743. 4743 is odd, and therefore contains no further power of 2; on adding the digits we get 18, and the number is therefore divisible by 9, giving 527. The sum of the digits is now 14, so that there is no further power of 3. The test for 11 gives 12 - 2 = 10, so that 11 is not a factor. We now test in succession for 7, 13, 17, and find 17 is a factor giving 31. The lowest trial divisor for 31 would be 17, which gives 1 on division, so that it is unnecessary to test further.

The example should be written out as follows:—

$$37944 = 2^3 \times 4743 \qquad (8 = 2^3)$$
$$= 2^3 \times 3^2 \times 527 \qquad (9 = 3^2)$$
$$= 2^3 \times 3^2 \times 17 \times 31$$

Prime Factors of $37944 = 2^3 \times 3^2 \times 17 \times 31$.

23. To find the G.C.M. and L.C.M. by the method of Prime Factors:—

The Least Common Multiple is the least number that can be divided without remainder by two or more given numbers. A knowledge of the methods used in factoring is often very useful in finding the G.C.M. and L.C.M. of numbers, and in simplifying expressions, by cancelling (§ 24).

Example—*Find the G.C.M. and the L.C.M. of* 12915 *and* 52360 *by the method of Prime Factors.*

Using the tests given in paragraph 21, and taking out the 5 first, we get

$$12915 = 5 \times 2583 \quad \text{and} \quad 52360 = 5 \times 10472$$
$$= 5 \times 3^2 \times 287 \quad ,, \quad = 5 \times 2^3 \times 1309$$
$$= 5 \times 3^2 \times 7 \times 41 \quad ,, \quad = 5 \times 2^3 \times 11 \times 119$$
$$= 5 \times 2^3 \times 11 \times 7 \times 17$$

For G.C.M. multiply up all the common factors;

$$\therefore \text{G.C.M.} = 5 \times 7 = 35,$$

and for L.C.M. take every factor, the common factors being taken only the maximum number of times they occur in any one;

$$\therefore \text{L.C.M.} = 2^3 \times 3^2 \times 5 \times 7 \times 11 \times 17 \times 41 = 19320840.$$

Examples 7.

Find the Prime Factors of—

1. 2016	5. 5808	9. 26624.
2. 1485	6. 11220	10. 39123
3. 3720	7. 16544	11. 48125
4. 6237	8. 24219	12. 62208

Find the L.C.M. and G.C.M. by the method of Prime Factors of—

13. 352 and 432. 17. 3157 and 3321. 21. 3843 and 10736.
14. 504 and 648. 18. 2205 and 4620. 22. 1584 and 15876.
15. 1155 and 1848. 19. 3663 and 8288. 23. 3456, 4212, and 5427.
16. 5355 and 7480. 20. 3445 and 4664. 24. 3168, 4536, and 5376.

Note: For L.C.M. and G.C.M. of Fractions, see §§ 39-41.

VIII.—CANCELLING.

24. A knowledge of the methods used in factoring and in finding the G.C.M. of numbers is of special use in simplifying expressions like the following—

Example—*Simplify* $\frac{171}{910} \times \frac{329}{459} \times \frac{425}{376}$.

$$\frac{\overset{19}{\cancel{171}}}{\underset{\underset{26}{\cancel{182}}}{\cancel{910}}} \times \frac{\overset{\cancel{47}}{\cancel{329}}}{\underset{\underset{3}{\cancel{51}}}{\cancel{459}}} \times \frac{\overset{\overset{5}{\cancel{85}}}{\cancel{425}}}{\underset{8}{\cancel{376}}} = \frac{19 \times 5}{26 \times 3 \times 8} = \frac{95}{624}.$$

Method—Examining these figures we see that 910 and 425 each cancel by 5, giving 182 and 85. The sum of the digits 171 = 9; and of 459 = 18; therefore each divides by 9, giving 19 and 51. The sum of the digits of 51 = 6; therefore 51 divides by 3, the other factor being 17; while 85 divided by 5 gives also 17 as its other factor; hence, dividing 51 and 85 by 17 we get 3 and 5. Dividing 182 by 2 we get 91, and on testing we find the factors of 182 to be 2, 7, and 13. Testing 329 we find its factors are 7, and 47, so we cancel 182 and 329 by 7. 376 divides by 8, and its factors are 8 and 47, therefore cancel 376 and 47 by 47. There now remains $\frac{19 \times 5}{26 \times 3 \times 8} = \frac{95}{624}$.

25. *Note:* In many cases, especially in questions dealing with percentages, it is often advantageous not to cancel.

In an example such as $\frac{64 \times 105}{50 \times 100}$ it is much better to cancel by 5 only, leaving $\frac{64 \times 21}{10 \times 100} = \frac{1344}{1000} = 1{\cdot}344$ (see § 165). If the other factors were cancelled out we should get $\frac{8 \times 21}{5 \times 25} = \frac{168}{125} = 1\frac{43}{125} = 1{\cdot}344$. The latter method involves much more work in the concluding division.

Again, take $\frac{631 \times 5 \times 3 \times 35}{365 \times 100}$ where one is very apt to cancel out the fives. A better method is to multiply both numerator and denominator by 2, giving when multiplied up $\frac{662550}{73000}$ from which we can obtain the answer by a short method as will be explained in § 218.

26. The student will learn by practice when it is advisable not to cancel. As a rule, do not cancel 10's except when the whole 10 cancels out, and where a 5 occurs which will not cancel, raise it to a 10 by multiplication. In other cases, cancel as much as possible.

Examples 8.

Simplify—

1. $\dfrac{252 \times 325}{585 \times 114}$.
2. $\dfrac{576 \times 196}{18 \times 64}$.
3. $\dfrac{13 \times 27 \times 25 \times 8}{63 \times 20 \times 11 \times 65}$.
4. $\dfrac{100 \times 1400}{56 \times 6}$.
5. $\dfrac{1444 \times 133}{57 \times 1805}$.
6. $\dfrac{252 \times 82}{8 \times 63}$.
7. $\dfrac{205 \times 189}{287}$.
8. $\dfrac{3456 \times 6543}{727}$.
9. $\dfrac{115 \times 145145}{92 \times 1160}$.
10. $\dfrac{85 \times 115115}{68 \times 1610}$.
11. $\dfrac{490 \times 204 \times 8}{105 \times 14 \times 136}$.
12. $\dfrac{364 \times 13 \times 9}{100 \times 4 \times 2}$.
13. $\dfrac{125 \times 8 \times 9}{300 \times 12 \times 2}$.
14. $\dfrac{75 \times 168 \times 120}{400 \times 105}$.
15. $\dfrac{60 \times 15 \times 2261}{420 \times 255}$.
16. $\dfrac{3 \times 66 \times 23 \times 34}{11 \times 17 \times 20 \times 69}$.
17. $\dfrac{8 \times 144 \times 35}{156 \times 12 \times 100}$.
18. $\dfrac{445 \times 15 \times 52}{100 \times 89}$.
19. $\dfrac{1472 \times 385}{200 \times 1728}$.
20. $\dfrac{39 \times 56 \times 115 \times 341}{713 \times 55 \times 91 \times 24}$.
21. $\dfrac{234 \times 870}{348 \times 582}$.
22. $\dfrac{45 \times 5760}{7008 \times 9}$.
23. $\dfrac{121 \times 425 \times 234}{1750 \times 187 \times 528}$.
24. $\dfrac{36 \times 600 \times 2555}{720 \times 2409 \times 846}$.
25. $\dfrac{111 \times 45 \times 325}{600 \times 296}$.
26. $\dfrac{702 \times 1001}{3 \times 819 \times 218}$.
27. $\dfrac{1739 \times 100}{296 \times 5}$.
28. $\dfrac{1082}{91} \times \dfrac{6}{541} \times \dfrac{737}{91} \times \dfrac{143}{4290}$.
29. $\dfrac{21780 \times 2 \times 57}{12 \times 2240 \times 171}$.
30. $\dfrac{3}{4} \times \dfrac{6023}{1} \times \dfrac{10}{4 \times 7291} \times \dfrac{184}{285}$.

26a. The principle involved in cancelling numbers should be extended to concrete quantities. Thus, just as 5 is made to divide into or cancel the number 20, giving 4 as a result, a concrete quantity in feet will cancel into one in yards (each foot being contained in each yard 3 times). The cancelling may be considered as taking place between the names

Examples—

$$\text{(i.)}\quad \frac{\overset{2}{\cancel{16}}\ \overset{3}{\cancel{\text{yards}}} \times \overset{5}{\cancel{35}}}{\underset{7}{\cancel{56}}\ \cancel{\text{feet}}} = 30.$$

$$\text{(ii.)}\quad \frac{6\ \text{men} \times \cancel{14}\ \cancel{\text{days}} \times \cancel{8}\ \overset{60}{\cancel{\text{hours}}} \times \overset{10}{\overset{20}{\cancel{£45}}}}{\cancel{60}\ \cancel{\text{days}} \times \underset{4}{\cancel{48}}\ \cancel{\text{min}} \times \underset{2}{\cancel{28}}\ \cancel{\text{shillings}}} = 450\ \text{men}.$$

In example (i.) all the names cancel out, hence the answer is the abstract number 30, whereas in (ii.) the name *men* remains untouched, and so forms part of the answer, 450 men.

$$\text{(iii.)}\quad \frac{\cancel{15}\ \overset{8}{\overset{\cancel{160}}{\cancel{\text{acres}}}} \times £5}{\underset{2}{\cancel{30}}\ \cancel{\text{sq. poles}} \times \cancel{10}\ \text{cwts.}} = \frac{£40}{\text{cwt.}}$$

In this example, when a name remains in both numerator and denominator the answer reads £40 per cwt.

$$\text{(iv.)}\quad 35\ \text{yards} \times 18\ \text{inches} \times \frac{10\ \text{shillings}}{\text{sq. yard}}.$$

In such cases sq. yards means yard × yard, and is cancelled as if it were written so.

$$\text{Thus, } 35\ \cancel{\text{yards}} \times \cancel{18}\ \cancel{\text{inches}} \times \frac{\overset{5}{\cancel{10}}\ \text{shillings}}{\cancel{\text{yard}} \times \underset{2}{\underset{\cancel{36}}{\cancel{\text{yard}}}}} = 175\ \text{shillings}.$$

It will not be necessary to write these examples in the extended form; sq. yards divided by yards give yards.

It will be found that the inclusion of the names is a safeguard against error. In questions involving cancelling an absurdity in the answer obtained will draw attention to some mistake in the working, thus $\frac{4 \text{ men} \times £16 \times £40}{30 \text{ men} \times 24 \text{ hours}}$; the answer obtained is 40 square £s per hour: an obvious absurdity. Had the names been omitted 40 might have been given as the answer, and a suitable name affixed.

Examples 8 (*continued*). For tables see pages 38–42.

Simplify—

31. $\frac{8 \text{ yards} \times 28 \text{ shillings}}{14 \text{ feet}}$.

32. $\frac{48 \text{ ounces} \times £5}{10 \text{ lbs.}}$.

33. $\frac{1 \text{ hour} \times 42 \text{ men} \times 180 \text{ pages}}{40 \text{ minutes} \times 30 \text{ men}}$

34. $24 \text{ sq. yards} \times \frac{5 \text{ shillings}}{\text{sq. foot}}$.

35. $4 \text{ gallons} \times \frac{6\text{d.}}{\text{gill}}$.

36. $\frac{£100 \times 3\frac{1}{2}\text{d.}}{1 \text{ shilling}}$.

37. $\frac{30 \text{ miles} \times 3 \text{ cwts.} \times 15 \text{ shillings}}{45 \text{ lbs.} \times 2 \text{ hours} \times £3}$

38. $\frac{25 \text{ yards} \times 14 \text{ feet}}{5 \text{ inches} \times 2 \text{ inches}} \times \frac{4 \text{ pence}}{100}$.

39. $340 \text{ miles} \times \frac{1 \text{ hour}}{40 \text{ miles}} \times \frac{7\text{s. } 6\text{d.}}{15 \text{ min.}}$.

40. $\frac{4 \text{ lbs} \times 30 \text{ sq. feet} \times 45 \text{ seconds} \times 15 \text{ tons}}{2 \text{ hours} \times 8 \text{ cwts.} \times 7\frac{1}{2} \text{ yards} \times 3 \text{ gallons} \times 4 \text{ inches}}$

IX.—VULGAR AND DECIMAL FRACTIONS.

27. Except in the case of the simpler vulgar fractions, such as $\frac{1}{2}$, $\frac{1}{4}$, &c., the student is advised to work with decimals, and, where possible, make use of the contracted methods explained in Section X. The chief work in connection with vulgar fractions will be:—their conversion into decimals and the conversion of mixed numbers into improper fractions and *vice versâ*.

28. There are three terms in common use which are applied to vulgar fractions. These are:—

Simple fraction, when the numerator is less than the denominator, *e.g.* $\frac{5}{8}$.

Improper fraction, when the numerator is greater than the denominator, *e.g.* $\frac{8}{5}$.

Mixed number, when we have both a whole number and a fraction, *e.g.* $1\frac{3}{5}$.

29. A mixed number can always be converted into an improper fraction by multiplying the whole number by the denominator and adding the numerator to the product. This gives the numerator of the improper fraction, the denominator remaining unchanged, thus: $1\frac{3}{5} = \frac{1 \times 5 + 3}{5} = \frac{8}{5}$.

Again, since $\frac{8}{5}$ means 8 divided by 5, if we perform the division we shall bring $\frac{8}{5}$ back from its form as an improper fraction to $1\frac{3}{5}$, its form as a mixed number.

30. **Decimal Fractions** are a series of fractions in which the denominators are powers of 10. Instead of the denominator being written below the numerator, it is designated by a division being made between the whole and the fractional part of the number. The number of figures after this division gives the power of 10 of the denominator. This division is indicated in various ways; in this country by a point put between the figures more than half way up, thus 16·38; (in France by a comma, thus 16,38; in Germany and the United States of America by a dot, thus 16.38). 16·38 represents therefore

$$16\frac{38}{10^2} \text{ or } 16\frac{38}{100} \text{ or } \frac{1638}{100}; \; 0·037 \text{ represents } \frac{37}{10^3} \text{ or } \frac{37}{1000}$$

31. **Addition and Subtraction** of decimals are worked exactly as addition and subtraction of whole numbers, the same care being taken to place the figures in the correct columns, and the decimal point is placed to indicate where the fractional portion begins.

Examples—

(i.) *Add* 63·839, 2·8563, ·372, 396·4, 8465.

```
  63·839
   2·8563
    ·372
 396·4
8465
8928·4673
```

(ii.) *From* 369·25 *take* 39·5437.

```
369·25 *
 39·5437
329·7063
```

* Gaps may, in imagination, be filled up with 0's.

32. **Multiplication.** — In multiplication of decimals the results of each multiplication should be put at once in its own *place value* column. Thus, in the example below when multiplying by the 8 begin in the ten-thousandths place, because 7 thousandths multiplied by 8 tenths gives 56 ten-thousandths The figures are thus all arranged according to value, the decimal points being in column as in addition and subtraction.

Example—*Multiply* 3 937 *by* ·8345.

```
3·937
 ·8345
3·1496
 ·11811
 ·015748
 ·0019685
3·2854265
```

The decimal points and the 0's are usually omitted in the working and the solution set down thus:—

```
3·937
 ·8345
3 1496
 11811
  15748
   19685
3·2854265
```

A check on the correctness of the position of the figures can also be obtained by adding together the decimal places of the two figures, *e.g.* 8 in 1st decimal place and 7 in 3rd decimal places gives 6 in 4th, or converting to vulgar fractions this becomes

$$\frac{3937}{1000}\times\frac{8345}{10000}=\frac{32854265}{10000000}=3\cdot2854265.$$

$$10^3 \quad 10^4 \quad 10^{3+4} = 10^7$$

The latter method shows the reason for adding the numbers of decimal figures.

Examples 9.

Add together—

1. 16·329, 147·42, 0·21013, 0·85693, 0·0078.
2. 3·2722, 0·00007, 16·3, 827·02, 10·3493.
3. 0·8217, 14·063, 0·2369, 0·01171, 77·15, 1829
4. 24·206, 183·85, 22·73, 0·0305, 13·3004, 771·5.
5. 29·494, 27·31, 0·3045, 837, 69·95, 0·02366.
6. From 183·6347 take 18·36347
7. From 0·0034695 take 0·0017824.
8. From 133·63925 take 0·837245
9. From 46·3269 take 39·8285.
10. From 639·82 take 499·34567.

Find the value of—

11. 8·637 × 3·208 ; 74·8 × 0·025.
12. 530·8 × 14·73 ; 14·44 × 0·0133.
13. 64·73 × 3·457 ; 79680 × 0·4005.
14. 98 × 0·0076 ; 230·98 × 56·702.
15. 752·006 × 8300·02 ; 3500 × 0·0035.
16. 5·7 × 180·5 ; 0·0057 × 1·805.
17. 0·02475 × 0·64 ; 0·002475 × 640.
18. 0·0079 × 0·079 ; 7·9 × 7900.
19. 54·6 × 0·00063 ; 0·0546 × 63.
20. 0·00375 × 9609 ; 0·000375 × 9·6
21. 0·3437 × 0·638.
22. 54·73 × 0·62.
23. 6·384 × 19·75.
24. 39·74 × 6·838.
25. 4·8375 × 0·00064.
26. 4·2 × 0·024 × 0·02.
27. 56·3 × 0·36 × 0·005.
28. 3·9 × 0·39 × 0·0039.
29. 860 × 6·5 × 0·038.
30. 3·4 × 34 × 0·034 × 3400.

Note.—The second answers of 16 to 20 should be obtained from the first without working.

33. Division:—

We have already seen that a decimal fraction such as 6·834 may be expressed in vulgar fraction form by removing the point and adding as a denominator the power of ten, denoted by the number of decimal figures; in this case $\frac{6834}{10^3}$ or $\frac{6834}{1000}$. If this fraction be multiplied by 10^3 or 1000, the result is a whole number.

34. A question in division, whether decimal or otherwise, may always be expressed in a fractional form, the divisor being the denominator. As a fraction is not altered in value if we multiply both numerator and denominator by the same number, it will be found advantageous in division of decimals to multiply

both divisor and dividend by the lowest power of ten, which will make the divisor a whole number. Then, adopting the method of §§ 10 and 14, and writing the answer as directed there, the decimal point of the quotient will be, in short division below, in long division above, the decimal point of the dividend.

Examples—

(i.) *Divide* 64·318 *by* 4.

4) 64·318
16·0795

(ii.) *Divide* 73·6191 *by* 300 (short division).

300) 73·6191
·245397

(iii.) *Divide* 9·219 *by* ·0012.

·0012) 9·219
or 12) 92190
7682·5

(iv.) *Divide* 538·16 *by* 3·2.
= 5381·6 *by* 32
(use factors)

32 { 4 | 5381·6
   { 8 | 1345·4
168·175

(v.) *Divide* 43·60092 *by* 6·834.

This may be expressed as $\frac{43 \cdot 60092}{6 \cdot 834}$

Multiplying numerator and denominator by 10³* we get $\frac{43600 \cdot 92}{6834}$

6·38
6834) 43600·92
25969
54672

(vi.) *Divide* 0·138145 *by* 39·47.

Proceeding as above and multiplying by 10² gives 13·8145 by 3947.

0·0035
3947) 13·8145
19735

(vii.) *Divide* 345 *by* 0·0037.

Multiplying by 10⁴ we get 3450000 by 37.

93243·243 †
37) 3450000·
120
90
160
120
90
160
120

* For multiplication of numbers by powers of ten, see "Mental Arithmetic," § 9.

† See § 35.

34a. It will be found advantageous, especially in connection with approximations, to adopt in division of decimals a method similar to that with integers (see Division, § 21; Multiplication, § 32). The first figure in the answer is written in its value place

at once. Thus in the above examples we obtain the place of the first figure of the answer as follows:—

(i.) 43 units divided by 6 units give 7 (6) units.
(ii.) 13 hundredths divided by 3 tens give 4 (3) thousandths.
(iii.) 34 tens divided by 3 thousandths give 11 (9) ten-thousands.

Reference to § 33 will show how these results are obtained, the working would then be written down—

```
(v.)              6·38
      6·834 ) 43·60092
                25969
                 54672

                0·0035
(vi.)  39·47 ) 0·138145
                 19735

(vii.)            93243·243
         0·0037 ) 345
                    120
                     90
                     160
                      120
                       90
                       160
                        120
```

Examples 10.

Divide, using short division where possible—

1. 0·2107206 by 4·206.
2. 362·6 by 0·00259.
3. 40·68 by 0·0018.
4. 7·3776 by 0·024.
5. 9·04878 by 386·7.
6. 0·002 by 16.
7. 36·5248 by 0·0125.
8. 6·4521 by 0·375.
9. 4·3046721 by 0·0729.
10. 37·07724 by 0·08.
11. 26·8975 by 0·03625.
12. 420·6969 by 1·23.
13. 0·749265 by 0·02585.
14. 7·56 by 0·00108.
15. 0·0015 by 0·003125.
16. 1·173 by 0·575.
17. 389464 by 0·356.
18. 40804 by 0·0202.
19. 80223 by 0·2057; 802·23 by 2·057.
20. 11214 by 0·00534; 112·14 by 0·534.
21. 173889 by 0·0417; 0·173889 by 417.
22. 179·89 by 71956; 17989 by 0·71956.
23. 329·875 by 4375; 0·0329875 by 437500.
24. 274·104 by 846; 2741040 by 0·000846.
25. Multiply 0·02475 by 64 and divide the result by 0·000125.
26. Multiply 3·456 by 0·006543 and divide the product by 0·0727.
27. Multiply 16·38 by 0·00647 and divide the product by 8·411.
28. Multiply 0·0037 by 0·645 and divide the result by 0·0043.
29. Multiply 3825 by 68·4 and divide the result by 0·0076.
30. Multiply 4738 by 67·5 and divide the result by 0·00927.

Note.—The second answers of 19 to 24 should be obtained from the first without working.

35. When in a decimal fraction a digit or set of digits recurs, the fraction is called a **Repeating, Recurring, or Circulating Decimal**, *e.g.* in the example (iii.) § 34, the decimal portion of the quotient, if continued, would be ·243243243243 . . . . Such results are abbreviated by placing a dot above the digit that recurs, if there is only one, or a dot above the first and last digits of a recurring set. Thus $\cdot\dot{6}$ is written for ·666 . . . . . . and as in (iii.) § 34, $\cdot\dot{2}4\dot{3}$ for ·243243 . . . . When all the digits of the fractional part recur, the fraction is called a **Pure Recurring Decimal**; when one or more digits do not recur, *e.g.* $0{\cdot}6\dot{8}\dot{3}$, $36{\cdot}35\dot{4}2\dot{7}$, it is called a **Mixed Recurring Decimal.**

36. Conversion of Recurring Decimals to Vulgar Fractions:— Pure recurring decimals are converted into vulgar fractions by making the repeating digit or digits the numerator, and putting for denominator as many nines as there are repeating digits, thus:—

$$\cdot\dot{6} \text{ becomes } \tfrac{6}{9} = \tfrac{2}{3}.$$
$$\cdot\dot{2}4\dot{3} \quad \text{,,} \quad \tfrac{243}{999} = \tfrac{9}{37}.$$

37. When the fraction is a mixed recurring decimal, or when we wish to convert a whole number as well as fraction into an improper fraction, neglecting the decimal point, we first subtract from the entire number a number made up of all the digits which do not repeat, and take the result as numerator. The denominator is made up by taking as many 9's as there are digits in the recurring portion, followed by as many 0's as there are digits in the *non-recurring portion of the fraction.*

Example—

(i.) *Reduce* $0{\cdot}6\dot{8}\dot{3}$ *to a vulgar fraction.*

683 − 6 = 677 = numerator.
990 = denominator.

$$\therefore 0{\cdot}6\dot{8}\dot{3} = \frac{677}{990}.$$

(ii.) *Express* $36{\cdot}35\dot{4}2\dot{7}$ *as a vulgar fraction.*

(*a*) as a mixed number.

35427 − 35 = 35392 = numerator.
99900 = denominator.

$$\therefore 36{\cdot}35\dot{4}2\dot{7} = 36\frac{35392}{99900}.$$

(*b*) as an improper fraction.

3635427 − 3635 = 3631792 = numerator.
99900 = denominator.

$$\therefore 36{\cdot}35\dot{4}2\dot{7} = \frac{3631792}{99900}.$$

Examples 11.

1. Express $0·\dot{8}$, $0·\dot{1}\dot{6}$, $0·\dot{3}\dot{4}$, $0·\dot{6}8\dot{3}$ as vulgar fractions, in lowest terms.
2. „ $0·\dot{7}8\dot{2}$, $0·\dot{9}38\dot{4}$, $0·5\dot{3}8\dot{2}$ as vulgar fractions, „ „
3. „ $6·\dot{3}$, $18·\dot{6}$, $435·\dot{3}\dot{7}$ as mixed numbers, „ „
4. „ $36·\dot{4}$, $8·\dot{5}2\dot{3}$, $96·\dot{3}7\dot{6}$ as improper fractions, „ „
5. „ $0·38\dot{2}4\dot{3}$, $0·54\dot{7}\dot{1}$, $0·3\dot{7}27\dot{9}$ as vulgar fractions, „ „
6. „ $16·3\dot{8}\dot{2}$, $453·9\dot{7}\dot{6}$, $835·\dot{3}92\dot{7}$ as mixed numbers, „ „
7. „ $34·5\dot{4}\dot{0}$, $19·38\dot{2}\dot{7}$, $6·73\dot{8}3\dot{4}$ as improper fractions, „ „

38. Conversion of Vulgar into Decimal Fractions :—

We have seen above how a decimal fraction is converted into a vulgar fraction. To convert a vulgar fraction into a decimal, the **Rule** is :—

Divide the numerator by the denominator, affixing 0's as required. If the division is performed as directed in §§ 10, 12, 34, there should be no difficulty regarding the position of the decimal point.

Examples—

(i.) *Convert* $\frac{3}{8}$ *into a decimal fraction.*

$$8\ )\ 3·000$$
$$·375$$

$\therefore \frac{3}{8} = 0·375$

Note.—When the divisor is under 13, or can be broken up into 2 or 3 factors, use short division, § 12; supply 0's till the division terminates or has been carried as far as desired.

(ii.) *Convert* $\frac{375}{512}$ *into a decimal fraction.*

```
            ·732421875
512 ) 375·00
       16 60
        1 240
          2160
           1120
             960
            4480
             3840
              2560
```

$\therefore \frac{375}{512} = 0·732421875.$

Examples 12.

Convert into vulgar fractions—

1. 13·28.	4. 8·832.	7. 4·2506.	10. 0·148.
2. 0·375.	5. 17·6.	8. 8·3.	11. 4·808.
3. 0·07.	6. 0·005.	9. 36·004.	12. 0·0006.

Convert into decimal fractions—

13. $\frac{5}{16}$.	16. $\frac{207}{1472}$.	19. $4\frac{56}{625}$.	22. $\frac{339}{16017}$.
14. $4\frac{8}{25}$.	17. $\frac{481}{625}$.	20. $1\frac{442}{768}$.	23. $\frac{77}{64}$.
15. $16\frac{7}{32}$.	18. $18\frac{17}{125}$.	21. $\frac{19}{16000}$.	24. $\frac{361}{3475}$.

Examples 12a.

1. Find the sum of $5\frac{1}{8}$, $\frac{2}{3}$ of $3\frac{1}{2}$, and $\frac{7}{8}$.

2. Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{3}{16}$.

3. Simplify $3\frac{7}{18}+\frac{7}{36}+26\frac{5}{7}+5\frac{29}{70}+20\frac{1}{21}$.

4. Which is the greater $\frac{128}{15}$ or $8\frac{3}{5}$, and by how much?

5. From $15\frac{11}{12}$ take the sum of $2\frac{3}{8}$, $3\frac{5}{12}$, $5\frac{3}{16}$.

6. If $\frac{11}{18}$ are cut from a pole, and then $\frac{7}{16}$ of the remainder, what fraction of the pole is left?

7. From $\frac{3}{8}$ of $1\frac{1}{2}$ take $\frac{1}{6}$ of $\frac{3}{5}$.

8. A shipowner possessed $\frac{3}{4}$ of a ship, sold $\frac{5}{8}$ of his share to B, who again sold $\frac{3}{10}$ of his share to C; what part of the ship did C buy?

9. What fraction added to the sum of $1\frac{2}{3}$, $4\frac{5}{6}$, and $7\frac{3}{8}$ will make a total of 15?

10. Three men owned a ship. The first owned $\frac{5}{9}$ of it, and the second $\frac{1}{6}$. What fraction did the third man own?

11. How much greater than $\frac{19}{21}$ is the fraction got by increasing this numerator and denominator by 7?

12. What must be added to $\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}$ to make unity?

13. What number subtracted from $41\frac{1}{4}$ leaves $19\frac{1}{2}$?

14. Find the least fraction which added to the sum of $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{29}{12}$ shall make the result an integer.

15. A boy cut away $\frac{1}{3}$ of a stick and found that $\frac{1}{4}$ of the remainder measured $4\frac{1}{2}$ inches. How long was the stick?

16. Which is the greater $\frac{1}{5}$ of 4 or $\frac{1}{4}$ of 5, and by how much?

17. If I pay away $\frac{1}{3}$ of my money, then $\frac{1}{5}$ of what remains, and then $\frac{1}{4}$ of what still remains, what fraction of the whole will be left?

18. A man has $\frac{3}{5}$ of an estate, he gives his son $\frac{1}{2}$ of his share, what portion of the estate has he then left?

19. A man travels $\frac{2}{7}$ of his journey by road, $\frac{4}{9}$ by sea, and then 320 miles by rail. Find the total length of his journey.

20. One-ninth of a number added to one-seventh of the same number makes 96; find the number.

21. A man left half his property to his wife, one-third of the remainder to his eldest son, and the rest to be divided equally among five younger children, each of whom got £600. How much did the wife get?

22. A person pays away $\frac{1}{3}$ of his money, then $\frac{1}{4}$ of the remainder, and then $\frac{1}{5}$ of what now remains. What fraction of his original money has he left?

23. Divide the sum of the fractions $\frac{2}{7}$ and $\frac{4}{15}$ by the product of $\frac{9}{11}$ and $\frac{13}{14}$.

24. Divide the product of $2\frac{4}{15}$ and $2\frac{5}{8}$ by the difference of $2\frac{3}{8}$ and $2\frac{1}{4}$.

25. What number multiplied by $2\frac{2}{3}$ of $\frac{4}{11}$ produces $3\frac{1}{2}$ of $\frac{4}{5}$?

26. Find the product of the sum and difference of $5\frac{1}{3}$ and $5\frac{1}{2}$.

27. Express the fractions $\frac{5}{16}$, $\frac{10}{31}$, $\frac{2}{11}$ with the same numerator, and then arrange them in ascending order of magnitude.

28. What fraction multiplied by $3\frac{1}{2}$ of $\frac{4}{5}$ produces $2\frac{2}{3}$ of $\frac{4}{13}$?

29. Three persons having £$1\frac{1}{13}$, £$2\frac{3}{8}$, £$7\frac{1}{32}$ subscribe respectively $\frac{5}{13}$, $\frac{1}{6}$, $\frac{2}{15}$ of their money. What is their total subscription?

30. Simplify (i.) $(2\frac{1}{3}+1\frac{4}{7})\div(1\frac{1}{21}-\frac{2}{7})$
(ii.) $2\frac{1}{3}+1\frac{4}{7}\div1\frac{1}{21}-\frac{2}{7}$.

31. Divide the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ by the sum of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

32. From a piece of rope 25 feet long, as many pieces as possible $2\frac{5}{6}$ feet long are cut. How many pieces are obtained, and what fraction of the original length remains?

33. From the product of $7\frac{5}{8}$ and $8\frac{3}{4}$ take their difference.

34. Multiply together the sum, difference, and product of $\frac{3}{4}$ and $\frac{5}{8}$.

35. Into how many parts has $\frac{3}{7}$ of an estate been divided if each part is $\frac{1}{28}$ of the whole estate?

36. Simplify $\frac{4}{7}$ of $5\frac{1}{4}+6\frac{3}{4}$ of $\frac{8}{9}-\frac{5}{11}$ of $1\frac{1}{10}$.

37. What quantity multiplied by $\frac{7}{8}$ will make $\frac{9}{16}$?

38. A man left £2400 to be divided among his four sons. The eldest has $\frac{5}{16}$; the second $\frac{3}{5}$ of $\frac{7}{9}$ of $\frac{15}{28}$; the third $\frac{1}{4}$ of $\frac{7}{8}$. How much money had the youngest?

39. Divide unity by $\dfrac{\frac{2}{3}+\frac{3}{4}}{\frac{2}{5}+\frac{3}{4}}$.

40. Find the value of $(\frac{2}{3}+\frac{1}{5})-(\frac{2}{5}+\frac{1}{7})+(\frac{2}{7}+\frac{1}{8})$.

41. Simplify $9\frac{1}{2}-5\frac{1}{10}+6\frac{8}{25}$.

42. Simplify $\dfrac{\frac{2}{3}+\frac{4}{5}}{\frac{3}{5}+\frac{1}{3}}\div\dfrac{\frac{3}{4}-\frac{2}{5}}{\frac{2}{5}+\frac{1}{4}}$.

43. Find the value of $\frac{2}{5}$ of £1 + $\frac{4}{9}$ of a guinea + $5\frac{2}{3}$ of a crown − $2\frac{3}{8}$ of a florin.

44. What fraction multiplied by $\frac{2}{5}$ of $1\frac{1}{3}$ of $1\frac{1}{2}$ will produce 1?

45. Simplify $\dfrac{1\frac{4}{7}-\frac{2}{3}}{2\frac{1}{2}\div1\frac{3}{4}}\times(1-\frac{4}{15})+\frac{5}{6}$ of $\frac{4}{5}-\frac{1}{6}$.

46. A person paid away $\frac{2}{3}$ of the contents of his purse, and was robbed of $\frac{4}{7}$ of the remainder. If he had still £3, 15*s.* left, how much had he at first?

47. Find the value of $\frac{9}{16}$ of £5, 17*s.* − $\frac{3}{7}$ of £8, 2*s.* 9*d.* + $\frac{10}{13}$ of £1, 7*s.* 9*d.*

48. What is the value of $\dfrac{3\frac{3}{5}+\frac{2}{3}\times6\frac{1}{4}-1\frac{5}{6}}{3\frac{3}{5}-\frac{2}{3}\times6\frac{1}{4}+1\frac{5}{6}}$ of £5, 8*s.* to nearest penny.

49. Two persons share an estate equally. One sells $\frac{3}{8}$ of his share to the other. What share of the whole estate has each now got?

50. One-fifth of an estate was sold to A, $\frac{3}{8}$ to B, and $\frac{2}{3}$ of the remainder to C, who paid £714. Find the value of A's share.

39. G.C.M. and L.C.M. of Fractions :—

The Greatest Common Measure and Least Common Multiple of Decimal Fractions are obtained in the same way as those of whole numbers (§§ 15, 23). Special methods, however, require to be used for vulgar fractions.

40. G.C.M. of Vulgar Fractions :—

Since the G.C.M. is the Greatest Fraction which can divide each of the given fractions without a remainder, it is necessary to express the fractions with a common denominator—the least common denominator being the most suitable. Having done so, we have then to find the G.C.M. of the numerators of the obtained fractions. The G.C.M. of the fractions is this G.C.M. as numerator and the L.C.M. as denominator. The **Rule** for finding the G.C.M. of fractions is therefore :—

Take the G.C.M. of the numerators and the L.C.M. of the denominators, and the fraction so obtained is the G.C.M. of the fractions given.

Example—*Find the G.C.M. of* $\frac{4}{15}$, $\frac{2}{3}$, $\frac{4}{9}$.

G.C.M. of numerators = 2
L.C.M. of denominators = 45

$\therefore$ **G.C.M.** $= \frac{2}{45}$

41. L.C.M. of Vulgar Fractions :—

The L.C.M. is the Least Fraction or number that can be divided without a remainder by each of the fractions. The denominator of the fraction must therefore be such, that it is a factor of each of the denominators given, *i.e.* we require the G.C.M. of the denominators. The numerator must be such that it can be divided by each of the numerators, *i.e.* we require the L.C.M. of the numerators. We then obtain the **Rule** for finding the L.C.M. of vulgar fractions :—

Take the L.C.M. of the numerators and the G.C.M. of the denominators, and the fraction so obtained is the L.C.M. of the fractions.

Example—*Find the L.C.M. of* $\frac{8}{15}$, $\frac{2}{3}$, $\frac{4}{9}$.

L.C.M. of numerators = 8
G.C.M. of denominators = 3

$\therefore$ **L.C.M.** $= \frac{8}{3}$ or $2\frac{2}{3}$.

To find the G.C.M. and L.C.M. of mixed numbers reduce to improper fractions, and proceed as above.

Examples 13.

Find the G.C.M. and L.C.M. of—

1. $\frac{1}{3}$, $\frac{2}{9}$, $\frac{4}{15}$.	5. $\frac{5}{117}$, $\frac{50}{91}$, $\frac{75}{143}$.	9. $\frac{1}{7}$, $\frac{2}{21}$, $\frac{5}{18}$.
2. $\frac{3}{8}$, $\frac{[illegible]}{16}$, $\frac{3}{24}$.	6. $\frac{2}{85}$, $\frac{8}{51}$, $\frac{4}{153}$.	10. $\frac{3}{8}$, $\frac{5}{12}$, $\frac{10}{21}$.
3. $\frac{12}{25}$, $\frac{8}{35}$, $\frac{4}{5}$.	7. $\frac{4}{11}$, $\frac{16}{[illegible]3}$, $\frac{24}{165}$.	11. $\frac{4}{5}$, $2\frac{1}{2}$, $\frac{7}{10}$.
4. $\frac{7}{9}$, $\frac{28}{135}$, $\frac{[illegible]6}{81}$.	8. $\frac{39}{57}$, $\frac{13}{171}$, $\frac{117}{133}$.	12. $3\frac{1}{6}$, $5\frac{3}{7}$, $4\frac{1}{14}$.

X.—APPROXIMATIONS.

42. In many calculations connected with business and with practical work of every kind the results are not required, and, in most cases, are not correct, beyond a certain number of figures. It is, therefore, necessary to adopt methods where, with the least amount of calculation, the result will be correct to the required number of figures.

43. Although, in the majority of cases, approximations are required in connection with decimals, it is not necessary they should be confined to them. Professor Perry, in his course of Lectures on Practical Mathematics, calls attention to the abuse of figures in this connection. "In a leading newspaper a few days ago I saw the indicated horse-power of a marine engine quoted as 3562·74 horse-power. Well, it is very probable that this measurement is in error at least 5 per cent. That is, the person who made the measurements and calculations is not sure whether the answer might not be 3700 or 3400, and he pretends that the last figure has a meaning. I am sorry to say that many misleading figures of this kind are published in the best books written on the steam-engine. . . . When I was at school the mean distance from the earth to the sun was stated as 95,142,357 miles. I wonder why furlongs and inches were not mentioned. The best knowledge we now have of this distance is that it is not greater than 93 nor less than $92\frac{1}{2}$ millions of miles."

44. In order to state the degree of accuracy required in a calculation, we introduce the idea of **Significant Figures**. The number of significant figures is the number of figures which are considered correct; 4058, 4058000, ·004058, ·4058, 23·00, and 2300 may be taken as examples of numbers correct to four significant figures.

45. Making Allowances. If the first five figures of the number 6834593 are known to be correct, but there is some doubt about the 93, then instead of 6834593 we may write 6834600, which is the number correct to five significant figures. It would be incorrect to write 6834500, because, a value of the last two figures being 93, it is quite evident that the number is nearer 6834600 than 6834500. In making allowances we therefore add one to the last of the significant figures if the first of the remaining figures is 5 or above 5.

Example—0·683746	*becomes*	0·68375	*when correct to*	5	*significant figures.*
"	"	0·6837	" "	4	"	"
"	"	0·684	" "	3	"	"
"	"	0·68	" "	2	"	"

46. Approximations in Addition and Subtraction:—

The method to be adopted in such cases will be best understood by considering an example and working it out entire; and then, making use of the principle of § 45, find the answer correct to the given number of significant figures.

Example—(i.) *Add together correct to four significant figures* 63·8379, 435·832, 0·38642, 8·7695, 73·3071, 4·6325.

Worked out complete we have—

```
 63·8379
435·832
  0·38642
  8·7695
 73·3071
  4·6325
586·76542
```

47. Taking only the first four figures and making allowances, § 45, we have—

```
 63·8
435·8
  0·4
  8·8
 73·3
  4·6
586·7
```

48. The first answer, however, is nearer 586·8 than 586·7, so it will generally be found advisable to work to a figure more than the required correctness. Then we have—

```
 63·84
435·83
  0·39
  8·77
 73·31
  4·63
586·77
586·8
```

49. From the above, it will be seen that when an answer is required to a certain number of significant figures it is better to work for one more than the required number of figures, using the principle of § 45, and then striking off the extra figure from the result, thus—

(ii.) *Add together, correct to five significant figures,* 839,247; 683,423; 392,473; 1,689,257; 2,005,638; 736,927; 835,924.

8392	5
6834	2
3924	7
16892	6
20056	4
7369	3
8359	2
71829	
7182900	

Although in this answer the required significant figures are 71829, the sum is not 71829 but 7182900. Care must be taken in such examples to complete the number by adding the necessary 0's.

This can be done by adding the 0's required in the first line, or by writing the decimal point in its place.

50. If the answer is to be correct to a certain number of decimal figures, the principle is the same; instead of counting from the extreme left-hand figure, we count from the first decimal place.

Example 14.

Add together—

1. 3·35, 30·8, 242·06, 18·385, 1·2954 correct to 4 significant figures
2. 48·048, 101·645, 5·49, 13·913, 85·693 correct to 4 significant figures.
3. 6172, 22630, 17283, 11404, 234375 correct to 5 significant figures
4. 106·5918, 0·4206969, 0·0000128, 1·173, 2400·6 correct to 6 significant figures.
5. 1250480, 11252, 5972971, 83867, 698038 correct to 3 significant figures.
6. 16·0790, 3571·3253, 15·2865, 6·8821, 0·0896, 24·3073 correct to 4 significant figures.
7. 8·2541, 46·3518, 0·3106, 1·4507, 12·73587 correct to 4 significant figures.
8. 0·12514, 0·0477, 4·1228, 0·0073, 1·5629, 0·74554 correct to 3 significant figures.
9. 53·17519, 0·81929, 0·0197, 1·69596, 43·75358 correct to 4 significant figures.
10. $189\cdot\dot{6}$, $67\cdot\dot{2}\dot{7}$, $1\cdot\dot{6}9$, $7\cdot\dot{9}1\dot{3}$, $34\cdot7\dot{6}\dot{7}$, $90\cdot\dot{5}1\dot{7}$, $9\cdot\dot{6}4\dot{7}$ correct to 6 significant figures.
11. 732·4218, 8415, 0·897210, 79·6875, 8·192 correct to 2 decimal places.
12. 29·44335, 0·14721675, 206·10347, 26·49976 correct to 4 decimal places.
13. $312\cdot3\dot{8}\dot{7}$, $16\cdot8\dot{5}\dot{7}$, $4\cdot\dot{4}1\dot{6}$, $0\cdot7\dot{3}\dot{6}$, $20\cdot\dot{8}\dot{7}$, 6·397748 correct to 5 decimal places.

14. 90·685518, 9·7163055, 103·640592, 10·30517, 16·193847 correct to 4 decimal places.

15. 0·463827, 0·0392456, 0·9387275, 0·96751376, 5·6834269, 0·9873746 correct to 5 decimal places.

Examples 15.

Find the difference between—

1.	78·364	and	3·8375	correct to	3 significant figures.
2.	3947·65	"	345·8327	"	4 " "
3.	64·3925	"	8·32569	"	3 " "
4.	3456 834	"	83755	"	4 " "
5.	69·5	"	9·83724	"	4 " "
6.	38279·635	"	82 3725	"	2 decimal places.
7.	44·$\dot{6}$	"	3·734	"	4 significant figures.
8.	68·$\dot{3}\dot{7}$	"	8·$\dot{2}4\dot{3}$	"	5 " "
9.	3456·$\dot{3}$	"	6·3$\dot{8}1\dot{6}$	"	4 " "
10.	23·3$\dot{9}2\dot{5}$	"	8·37$\dot{3}\dot{6}$	"	5 " "
11.	6·83924	"	0·93265	"	3 decimal places.
12.	39·7$\dot{2}5\dot{6}$	"	18·$\dot{3}24\dot{7}$	"	5 " "

51. Approximation in Multiplication:—

We shall again take an example and treat it in a similar way as the example in addition.

Example—(i.) *Multiply* 68·347 *by* 0·45369 *correct to* 5 *significant figures.*

This example worked out complete by the method of § 6 would be—

This gives as the result, correct to 5 significant figures, **31·008** (§ 45).

68·347	
·45369	
27 3388	
3 4173	5
2050	41
410	082
61	5123
31·0083	**5043**

52. Adopting the principle of §§ 45–48, and obtaining only the figures which are necessary, *i.e.* those on the left of the line in the above working, we have—

68·347	
0·45369	
27 338	8
3 417	4
205	0
41	0
6	1
31·008	

53. In performing multiplication by the contracted method, it is necessary to pay attention to the following **Rules**:—

1. Take as many figures in the multiplicand as you wish correct figures in the answer. If the product of the first significant figures in the multiplicand and multiplier is less than ten, an extra figure must be taken.

2. If there is not a sufficient number of figures in the multiplicand add 0's to make up the number. If there are more than required, strike off the surplus figures. Count from the left.

3. After multiplying by any figure of the multiplier, strike out the right-hand figure of the remaining multiplicand before multiplying by the next.

4. Always begin with the left-hand figure of the multiplier, and multiply the figure last struck out to find a carrying figure, applying the principle of § 45.

5. The decimal point is obtained as directed in § 32. In the above example 7 thousandths multiplied by 4 tenths gives 28 ten-thousandths.

54. When the answer is required to a certain number of decimal places begin the multiplication so that the first figure of the working has a value one place smaller than the places required. In example (iii.) below, the 7 tens must multiply a figure in the hundred-thousandths place to give a result in the ten-thousandths place which is a value one place smaller than the place required (thousandths). In an example such as 123·4567 by 0·006824 to three decimal places, the first figure of the multiplier (6 thousandths) would be multiplied into a figure in the tenths place in order to give a result in the ten-thousandths place. The remaining figures 567 would be struck off and the first product would be 6 × 4, with a carrying figure of 3 added, and the 7 obtained would be placed in the ten-thousandths place.

55. The following examples have been selected to show the application of these rules:—

Examples—

(i.) *Multiply* 683954 *by* 7364 *correct to* 4 *significant figures.*

Rules 1 and 2 give ·683954 × 7364.

683954		Rule 3.
·7364		
4787	7*	Rule 4 and § 32.
205	2	
41	0	
2	7	
5037	000000	§ 45.

$= 5037 \times 10^6$.

(The complete answer is 5036637256.)

7 not 6, because 35 counts as 40 (§ 45).

(ii.) *Multiply* 16·88 *by* 0·05478 *correct to 5 significant figures.*

Rules 1 and 2 give 16·3800 × 0·05478.

```
 16·3800  . . .  Rule 3.
 0·05478
 ------
  81900|0
   6552|0
   1146|6
    131|0
 ------
 ·89730| . . .  Rule 5.
```

(iii.) *Multiply* 6·83954 *by* 73·64 *to 3 decimal places.*

In this example the first multiplier is 7 tens; in order to get ten-thousandths (4 decimal figures) we must multiply hundred-thousandths by tens. The hundred-thousandth figure in the multiplicand is the 4 (§ 54).

```
   6·83954  . . .  Rule 3.
   73·64
  -------
 478 767|8
  20 518|6
   4 103|7
     273|6
  -------
 503·664|
```

Examples 16

Multiply—

1.	6443824	by	39534	correct to	5 significant figures.		
2.	83705	by	40837	”	5	”	”
3.	736924	by	6837	”	5	”	”
4.	8247	by	839245	”	5	”	”
5.	726309	by	54387	”	4	”	”
6.	837263	by	80706	”	4	”	”
7.	354326	by	30807	”	4	”	”
8.	726952	by	63825	”	4	”	”
9.	6[illegible]3076	by	47879	”	nearest ten millions.		
10.	524327	by	63825	”	nearest million.		
11.	738560	by	83724	”	nearest ten millions.		
12.	345·37	by	373·54	”	nearest ten.		
13.	789·42	by	47·638	”	5 significant figures.		
14.	34·837	by	49·805	”	5	”	”
15.	638·345	by	8·327	”	5	”	”
16.	68·325	by	6·785	”	4	”	”
17.	70·836	by	0·06347	”	4	”	”
18.	83675	by	0·00487	”	4	”	”
19.	689·456	by	0·875	”	4	”	”
20.	34567·8	by	0·6382	”	nearest unit.		
21.	247·384	by	0·00357	”	4 significant figures.		
22.	685·607	by	0·00456	”	4	”	”
23.	0·00347	by	0·08567	”	3	”	”
24.	0·0049	by	0·087634	”	4	”	”

Multiply—

25.	6·3825	by	4·9324	correct to	3	decimal places.
26.	84·724	by	3·825	,,	3	,,
27.	63·837	by	45·83	,,	3	,,
28.	94·637	by	3·8025	,,	3	,,
29.	74·8925	by	63·924	,,	3	,,
30.	45·8327	by	0·0037	,,	4	,,
31.	769·567	by	0·0894	,,	4	,,
32.	3·2725	by	0·080507	,,	4	,,
33.	0·00327	by	0·00895	,,	5	,,
34.	0·83247	by	0·089327	,,	5	,,
35.	7·63249	by	0·93256	,,	5	,,
36.	0·3276925	by	0·836274	,,	5	,,
37.	4375	by	1·34525	,,	3	,,
38.	13·428725	by	0·7392	,,	2	,,
39.	827·952	by	18·43	,,	1	,,
40.	6934	by	3·14159	,,	units place.	
41.	1·26532	by	0·79032	,,	,,	
42.	44·5192	by	0·022462	,,	,,	
43.	719386	by	28763	,,	nearest million.	
44.	23·7$\dot{1}\dot{8}$	by	5·84	,,	3	decimal places.
45.	194·6$\dot{1}$6$\dot{9}$	by	94·6$\dot{5}\dot{8}$	,,	2	,,
46.	753·$\dot{9}$1$\dot{2}$	by	0·84$\dot{3}\dot{7}$	,,	4	,,

56. Approximation in Division :—

We shall illustrate this by working out an example complete, and then finding the figures unnecessary for the approximation.

Example—*Divide* 6841775000 *by* 73452 *correct to* 4 *significant figures.*

This example worked out complete to 4 significant figures of the quotient by the method of § 10 gives

```
                   93146 = 93150
73452 ) 684177|5000
         23109|5          (correct
          1073|90          to 4 sig-
           339|380         nificant
            45|5720        figures).
             1|5008
```

To obtain the answer correct to 4 significant figures with the least possible working, we omit all the figures on the right of the line and make use of the principles of §§ 45 and 48.

```
                   93146 = 93150
73452 ) 6841785000
          23110
           1074
            339
             45
```

57. In performing division by the contracted method it is necessary to pay attention to the following **Rules**:—

1. Find the position of the first figure in the quotient (see § 34a).

2. Take one more figure in the divisor (adding 0's if necessary) than is required in the quotient. In the case of significant figures this is obtained direct from the example, in decimal places from No. 1.

3. Find the number of figures of the dividend required for the first multiplication and strike off from the dividend the surplus figures (or add 0's if fewer figures), making the necessary corrections (§ 45).

4. Continue the division, but always strike off one figure from the right of the divisor before obtaining a new figure in the quotient.

5. In multiplying by each figure of the quotient, first multiply the figure of the dividend last struck off to obtain a carrying figure, applying the principle of § 45.

58. When the answer is required to a **certain number of decimal figures**, find the position of the first figure of the answer (see § 34a). From this, ascertain the number of figures required to make the answer correct to the required number of decimals, and proceed as above.

59. The following examples have been selected to show the application of these rules:—

Examples—

(i.) *Divide* 12345·6789 *by* 68·3 *to 4 significant figures.*

This becomes 12345·6789 by 68·300. Rule 2.

180·75 = 180·8.

Rule 4. 68·300) 12345·7789 Rule 3.

```
 5515 7
 ------
   51 7
   ----
    3 9
    ---
      5
```

(ii.) *Divide* 378 *by* 647538 *to 3 significant figures.*

This becomes 378 by 647538. Rule 2.

Rule 4. 0·0005837 = 0·000584

647538) 378·00 . . Rule 3.

```
 54 23
 -----
  2 43
  ----
    49
    --
     4
```

(iii.) *Divide* 6·374 *by* 4·97 *to* 3 *decimal figures.*

(34a) gives 1 unit as first figure in quotient, and the answer has therefore to be correct to 4 significant figures. We have 6·374 ÷ 4·9700. Rule 2.

1·2825 = 1·283.

Rule 4. 4·9700) 6·3740 . Rule 3.

```
4·9700 ) 6·3740
         1 4040
           4100
            124
             25
```

(iv.) *Divide* 30·8073 *by* 638·27 *to* 5 *decimal places.*

(§ 34a). This gives (5) four-hundredths as the first figure in quotient. The answer has therefore to be correct to 4 significant figures. We have 30·8073 ÷ 638·27. Rule 2.

0·048267 = 0·04827.

Rule 4. 638·27) 30·8073 . Rule 3.

```
638·27 ) 30·8073
          5 2765
            1703
             427
              44
```

Examples 17.

Divide—

1.	683427	by	47	correct to 3 significant figures.
2.	543894	by	·327	" 3 " "
3.	8924736	by	499	" 4 " "
4.	83456729	by	6834	" 4 " "
5.	493672549	by	7386	" 4 " "
6.	83765438	by	4937	" 4 " "
7.	392568375	by	9386	" nearest ten.
8.	7956493	by	347	" " hundred.
9.	738569247	by	2976	" " "
10.	589347836	by	6925	" " ten.
11.	69·345	by	3·7	" 3 significant figures
12.	437·68	by	14·763	" 3 " "
13.	824·725	by	82·9	" 4 " "
14.	76·8375	by	0·68725	" 4 " "
15.	384·7254	by	0·0478	" 4 " "
16.	935·683	by	16·8374	" 5 " "
17.	39·73854	by	837·63567	" 5 " "
18.	493·68327	by	5678·63976	" 5 " "
19.	83·3	by	68·516	" 5 " "
20.	495·737	by	0·07654	" nearest tenth.
21.	42·3876	by	6·8372	" " tenthousandth.
22.	4956·732	by	18·6924	" " hundredth.

Divide—

23.	438·76	by	63·27	correct to	3	decimal places.
24.	926·475	by	8·653	„	3	„ „
25.	24·6837	by	18·624	„	3	„ „
26.	934·9256	by	3·7637	„	4	„ „
27.	85·7637	by	29·6834	„	4	„ „
28.	256·63	by	3·141592	„	4	„ „
29.	93·6382	by	8·63854	„	5	„ „
30.	367·7546	by	86·635	„	5	„ „
31.	1	by	3·14159	„	4	„ „
32	6·83759	by	493·72	„	3	„ „
33.	0·0843765	by	0·0038426	„	2	„ „
34.	0·00001	by	3141·59265	„	10	„ „

35. Find the reciprocal of 1·27628156 correct to 7 decimal places.
36. „ „ 1·62889463 „ 7 „
37. „ „ 1·10872 „ 5 „
38. „ „ 4·515063 „ 5 „
39. Find a decimal within $\frac{1}{100000}$ of $\frac{5}{36}$.
40. Find to nearest ten-thousandth 4·3÷5·7.

60. Method of Prediction :—

The student will often find it necessary to obtain an answer correct to a certain number of decimals, when the working-out of the problem may involve both multiplication and division. Such questions are to be found in Simple Interest (§§ 214–215) especially. As the final answer is to be correct to a certain number of figures, it is necessary that we should know to how many figures the intermediate steps must be obtained. In all such cases it is advisable to retain the extra figure required by § 48, in all but the final result.

61. In questions producing an answer in money it is often necessary to find the result correct to 3 or 4 decimal places. The following rules have been drawn up to suit such a question involving a multiplication followed by a division. They may be easily extended to more involved questions. (§ 64.)

62. In order to determine the degree of correctness of the multiplication, 1st. Find the place value of the first figure of the multiplication. This is obtained by multiplying the first

figures in multiplicand and multiplier (excluding 0's). Care must be taken when this product is less than 10, and in all cases the carrying figure should be allowed for (§ 53).

2nd. From this value and the value of the first figure of the divisor find the value of the first figure of the quotient. Make allowances when there is any doubt of the first figure being greater than the first figure of the product.

3rd. Calculate the number of significant figures required in quotient. One more than this is number of significant figures required in product.

63. Examples—

(i.) 638·45 × 4·3972 ÷ 83·6394 *to 3 places of decimals.*

1. First figure of product in thousands place (6 hundreds by 4 units give 2 thousands—24 hundreds).

2. First figure of quotient in ten place (24 hundreds by 8 tens give 3 tens).

3. Number of significant figure in quotient is five (tens place with 3 decimal gives 5)—work for 6

```
 638·450                          33·5654 = 33·565
   4·3972              83·6394 ) 2807·392
 --------                         298 210
2553 800                           47 292
 191 535                            5 472
  57 460                              454
   4 469                               36
     128                                3
 --------
2307·392
```

(ii.) 36·392 × 0·03745 ÷ 83·46 *to 5 places of decimals.*

1. First figure of product—units place.
2. First figure of quotient—hnndredths place.
3. Therefore quotient to contain 4 significant figures—work for 5.

```
 36·392                    0·016329 = 0·01633
  0·03745          83·460 ) 1·36288
 --------                    52828
1 09176                       2752
  25474                        248
   1456                         81
    182
 --------
 1·36288
```

64. This principle may be extended to any number of multiplications and divisions. When the answer is required to significant figures, the number of figures required in each product and quotient can easily be found by the rules given in the notes of § 62. In all continued work it is advisable to perform the multiplications before the divisions.

Examples 18.

Find—

1.	68·374	× 83·526	÷ 38·546	correct to	3	decimal	places.
2.	94·6874	× 2·839	÷ 8·6347	„	3	„	„
3.	385·3926	× 13·68	÷ 39·827	„	3	„	„
4.	18·7364	× 7·825	÷ 4·6386	„	3	„	„
5.	256·39	× 94·637	÷ 0·9247	„	3	„	„
6.	58·738	× 8·739	÷ 8·638	„	3	„	„
7.	345·6824	× 5·68	÷ 7·7089	„	3	„	„
8.	856·6834	× 6·825	÷ 33·694	„	5	„	„
9.	73·6925	× 34·83	÷ 4·83725	„	5	„	„
10.	387·39276	× 76·7	÷ 67·60908	„	5	„	„
11.	44·82639	× 85·9	÷ 5·683	„	5	„	„
12.	397·63256	× 39·76	÷ 89·3456	„	5	„	„
13.	73·62897	× 43·685	÷ 367·6347	„	5	„	„
14.	486·6352	× 376·68	÷ 695·82476	„	5	„	„

XI.—TABLES OF BRITISH WEIGHTS AND MEASURES.

Avoirdupois Weight.

16 drams (drs.)	=	1 ounce (oz.)
16 oz.	=	1 pound (lb.)
14 lbs.	=	1 stone (st.)
2 sts. } 28 lbs. }	=	1 quarter (qr.)
4 qrs.	=	1 hundredweight (cwt.)
20 cwt.	=	1 ton

112 lbs.	=	1 cwt.
2240 lbs.	=	1 ton

66. **Linear Measure.**

12 inches (in.) . = . . 1 foot (ft.)
3 ft. . . = . . 1 yard (yd.)
22 yards = 1 chain (ch.) or { 5½ yds. = 1 rod or pole (pl.)
10 chs. = 1 furlong (fur.) { 40 pls. = 1 furlong (fur.)
8 furs. . . = . . 1 mile (ml.)

220 yds. . . = . . 1 furlong
1760 yds. }
5280 ft. } . . 1 mile
63,360 in. }
100 links . = . . 1 chain

67. **Square Measure.**

144 sq. inches . . = . 1 sq. foot (sq. ft.)
9 sq. feet . . . = . 1 sq. yard (sq. yd.)
484 sq. yds. = 1 sq. chain or { 30¼ sq. yds. = 1 sq. perch or pole (sq. pl.)
10 sq. chains = 1 acre (ac.) { 40 sq. pls. = 1 rood (rd.)
{ 4 roods = 1 acre (ac.)
640 acs. . . . = . . 1 sq. mile

4840 sq. yds. . . = . . 1 acre.

68. **Cubic Measure.**

1728 cub. in. . . . = . . 1 cub. ft.
27 cub. ft. . . = . . 1 cub. yard

69. **Liquid Measure.**

4 gills . . . = . . 1 pint (pt.)
2 pts. . . . = . . 1 quart (qt.)
4 qts. . . . = . . 1 gallon (gal.)
36 galls. . . . = . . 1 barrel

The imperial gallon = 277·274 cubic inches and holds 10 lbs. (avoir.) of pure water.

70. **Corn or Dry Measure.**

4 gills . . . = . . 1 pint (pt.)
2 pts. . . . = . . 1 quart (qt.)
4 qts. . . . = . . 1 gallon (gall.)
2 galls. . . . = . . 1 peck (pk.)
4 pks. . . . = . . 1 bushel (bush.)
8 bushels . . = . . 1 quarter (qr.)

71. Troy Weight.

24 grains (grs.)	=	1 pennyweight (dwt.)
20 dwts.	=	1 ounce (oz.)
12 oz.	=	1 pound (lb. T.)
5760 grains	=	1 lb. (T.)
7000 grains	=	1 lb. (Avoir.)

72. Apothecaries Weight.*

20 grains	=	1 scruple (scr. or ℈)
3 scrs.	=	1 dram (dr. or ʒ)
8 drs.	=	1 ounce (oz. or ℥)
12 oz.	=	1 lb. (Troy)

73. Apothecaries Fluid Measure.*

60 minims (m.)	=	1 drachm (ʒ)
8 drachms	=	1 ounce (oz. or ℥)
20 oz.	=	1 pint (pt.)
8 pts.	=	1 imp. gallon.

* Usually employed in Photography.

74. Measure of Time.

60 seconds (″)	=	1 minute (min.)
60 mins. (′)	=	1 hour (hr.)
24 hrs.	=	1 day
7 days	=	1 week
4 weeks	=	1 month
52 weeks 12 calendar mths. 365 (or 366) days	=	1 year
100 years	=	1 century

Note.—As the number of weeks in a month or year is not exact and the number of days in a month is not constant, these units should not be used unless when specially required.

75. The Calendar.

The Calendar in use is the Gregorian Calendar, and dates from the year 1582, when Pope Gregory caused the necessary changes to be made in the calculation of the calendar. Before that time, in accordance with the Julian Calendar, every fourth year was made a Leap-year. If this were correct, then the length of the year would be $365\frac{1}{4}$ days; but the length of the year is rather less than that (365·242 days). The error amounts to rather more than

three days in each 400 years. To make allowance for this error, out of every 400 years three which would otherwise be taken as Leap-years are counted as ordinary years. The years selected are those at the end of the first three of each set of four centuries. The student is reminded that, as the year before 1 A.D. is called 1 B.C., *i.e.* there is no year 0, the first century of the Christian Era ended with the 100th year.

76. To find whether a year is a Leap-year or not:—

If the year is not the last year of a century and the number is divisible by four without a remainder, the year is a Leap-year; when the century is divisible by four the last year is a Leap-year; thus—1896, 1912, 1600, 2000, are Leap-years, but 1906, 1700, 1900, 1902, are not Leap-years.

77. The calendar month is an arbitrary division of time, and the number of days varies from 28 to 31. These are usually learned by means of some mnemonic aid such as the following—

Thirty days hath September,
April, June, and November,
All the rest have thirty-one
Excepting February alone,
Which hath but twenty-eight days clear,
And twenty-nine in each Leap-year.

78. **Angular Measure.**

60 seconds (″)	=	1 minute (min.)
60 minutes (′)	=	1 degree (deg.)
90 degrees (°)	=	1 right angle (rt. ang. or ⌊rt.)

79. **Measurement of Temperature.**

The scales in common use are the **Centigrade** and **Fahrenheit**.

In the Centigrade scale, the freezing point of water is 0°, and the boiling point 100°, the interval being divided into 100 equal divisions, each 1° Centigrade. In the Fahrenheit scale, the freezing point is 32°, the boiling point 212°, the interval being divided into 180 equal divisions, each 1° Fahrenheit.

In Russia and some parts of Germany the **Réaumur** scale is used. In it the freezing and boiling points are 0° and 80° respectively, the interval being divided into 80 equal divisions, each 1° R.

The formulæ for conversion are $\frac{C}{100}=\frac{F-32}{180}=\frac{R}{80}$; where C. F. and R. are the readings on a Centigrade, Fahrenheit, and Réaumur Thermometer respectively.

Example—*Find the corresponding readings on the Centigrade and Réaumur scales of 59° F.*

$$\frac{F-32}{180}=\frac{C}{100} \text{ or } \frac{R}{80}.*$$

$$\frac{59-32}{180}=\frac{27}{180}=\frac{C}{100} \quad \therefore C=\frac{100\times 3}{20}=15° \text{ C.}$$

$$\frac{59-32}{180}=\frac{27}{180}=\frac{R}{80} \quad \therefore R=\frac{80\times 3}{20}=12° \text{ R.}$$

* Algebra (Section xvii.).

80. **Paper Measure.**

24 sheets	=	1 quire
20 quires	=	1 ream
10 reams	=	1 bale

81. Alphabetical list of terms in use not included in above tables :—

1 chaldron (coke)	=	12 sacks (measure of capacity)
1 cubit	=	18 inches (measure of length)
1 degree	=	$69\frac{1}{6}$ miles (measure of length)
1 fathom	=	6 feet (measure of length)
1 hand	=	4 inches (measure of length)
1 hogshead (hhd.)	=	54 gallons (liquid measure)
1 knot	=	1 nautical mile per hour (measure of speed)
1 nautical mile	=	$6086\frac{1}{2}$ feet (measure of length)
1 league	=	3 miles (measure of length)
1 sack (of coke)	=	3 bushels (measure of capacity)
1 span	=	9 inches (measure of length)
1 ton of shipping	=	42 cubic feet (cubic measure)
1 truss (of hay)	=	56 lbs. (avoir.) (measure of weight)
1 tun (of oil)	=	210 gallons (liquid measure)

A **carat** is used as a proportional measure of one twenty-fourth in stating the fineness of gold. It is also used as a measure of weight for diamonds, &c., and equals about $3\frac{1}{5}$ grains.

XII.—REDUCTION.

82. It is not customary to express sums of money, weights, measures, &c., always in terms of one unit, but by means of names representing multiples and measures of the unit, to express the amount desired. Hence the need of tables as shown on pp. 38–42.

83. From the relations shown to exist between the various amounts or quantities mentioned in the tables, we have a ready means of expressing an amount, written under many heads, in terms of any particular one; or of expressing an amount, written under one head, in terms of several. Such processes are termed **Reduction.** Since the various amounts indicated by each of the names in the separate tables are multiples or measures of one another, we see that to reduce an item expressed in terms of one name to those of another, we must use either multiplication or division.

Examples—

(i.) *Express £5, 4s. 8d. in terms of pence, or Reduce £5, 4s. 8d. to pence.*

Method—£5 × 20 + 4 = 104s.
104s. × 12 + 8 = 1256d.
Every £1 = 20s. ∴ £5, 4s. = 5 times 20 + 4 = 104s.
Similarly with the shillings.

```
 £5   4s.  8d.
 20
 104 shillings.
  12
1256 pence
```

(ii.) *Reduce 114593 ounces to tons, &c.*

Method—Since 16 oz. = 1 lb.; every 16 oz. in 114593 oz. will make 1 lb. Hence divide by 16.
Similar reasoning should be applied to all the necessary reductions.

114593 oz. ÷ 16 = 7162 lbs. 1 oz.
7162 lbs. ÷ 28 = 255 qrs. 22 lbs.
255 qrs. ÷ 4 = 63 cwts. 3 qrs.
63 cwts. ÷ 20 = 3 tons 3 cwts.
Total = 3 tons 3 cwts. 3 qrs. 22 lbs. 1 oz.

```
16 { 4 | 114593 oz.
   { 4 |  28648 " 1 }
28 { 4 |   7162 " 0 } 1 oz. (§ 14).
   { 7 |   1790 " 2 }
     4 |    255 " 3 } 22 lbs.
    20 |     63 " 3 qrs.
       3 tons 3 cwts. 3 qrs. 22 lbs. 1 oz.
```

84. No trouble need arise in working Reduction questions on any table. Decide whether the process involves multiplication or division, and proceed as above, using the appropriate numbers. When reducing a quantity expressed in terms of a higher unit to those of a lower, multiply; when from a lower to a higher unit, divide.

Examples 19.

1. Reduce £7, 10s. 4d. to pence.
2. „ £56, 18s. 3½d. to half-pence.
3. „ £2, 17s. 11¾d. to farthings.
4. „ £58, 19s. 0¼d. to farthings.
5. „ 754 shillings to pence.
6. „ 59376 pence to £'s, &c.

7. Reduce 152738 farthings to shillings, &c.
8. " 92187 pence to half-sovereigns, &c.
9. " 6 tons 13 cwts. 3 qrs. 15 lbs. to lbs.
10. " 15 cwts. 110 lbs. 13 oz. 5 drs. to drs.
11. " 6 cwts. 5 st. 6 lbs. 10 oz. to ounces.
12. " 849215 ounces to tons, cwts. qrs. lbs. oz.
13. " 724519 drs. to cwts. lbs. oz. drs.
14. Multiply £5, 10*s.* 6*d.* by 365. Reduce the answer to halfpence, and supposing each halfpenny weighs $\frac{1}{6}$ oz., find the whole weight in lbs. avoirdupois.
15. Reduce 5 mls. 3 furs. 15 pls. to yards.
16. " 18 furs. 4 chs. 18 yds. 2 ft. to inches.
17. " 6 mls. 1325 yds. to feet.
18. " 628417 ins. to mls. furs. chs. yds. ft.
19. " 562380 links to mls. furs. chs. links.
20. " 3285 yds. to mls. furs. pls. yds.
21. " 420 acs. 3 rds. 15 sq. pls. to square poles.
22. " 10 acs. 1294 sq. yds. to square inches.
23. " 925816 sq. ins. to sq. yards, &c.
24. " 415913 sq. yds. to acs. rds. pls.
25. " 75129158 sq. ft. to sq. mls. acs. sq. yds.
26. " 52 cub. yds. 13 cub. ft. 1345 cub. ins. to cubic inches.
27. " 724196 cub. ins. to cub. yds. ft. ins.
28. " 16 qrs. 3 bush. 2 pks. 1 gall. to gills.
29. " 4 barrels 18 galls. 3 qts. 1 pt. to pints.
30. " 423569 pts. to barrels, galls. qts. pts.
31. " 8 lbs. 7 oz. 12 dwts. to grains (Troy).
32. " 563459 grs. to lbs. oz. dwts. grs. (Troy).
33. " 84276 grs. to lbs. (Avoir.).
34. " 7 ℥ 5 ʒ 2 ℈ to grains.
35. " 7241 grs. to ℥ ʒ ℈.
36. " 3 pts. 10 fluid oz. to minims.
37. " 6842 fluid drs. to imperial gallons.
38. " 5 yrs. 184 days 10 hrs. to minutes.
39. " 25 days 14 hrs. 16 mins. to seconds.
40. " 842167 mins. to yrs. days, hrs. mins.
41. " 381726 hrs. to weeks.
42. How many minutes in the month of September? February? July?
43. At 3 A.M. on 5th July a boy is 362954 mins. old. When was he born?
44. Reduce 8 right angles 15° 30′ 24″ to seconds.
45. " 53816″ to right angles, degrees, mins. secs.
46. " 2 bales, 5 reams, 10 quires to sheets.
47. " 3846 sheets to reams, quires, sheets.
48. " 58347 fathoms to yards.
49. " 85 ins. to hands, inches.
50. " 3892 galls. to hhds.

85. In certain questions it will be found advantageous to work with the tables as written below.

Mls.	Furs.	Pls.	Yds.	Ft.
1 =	8			
	1 =	40		
		2 =	11	
			1 =	3

or

Ft.	Yds.	Pls.	Furs.	Mls.
3 =	1			
	11 =	2		
		40 =	1	
			8 =	1

(i.) *Hence to reduce 64 miles to feet, proceed thus*—

$$64 \text{ mls.} = 64 \times \tfrac{8}{1} \text{ furs.} = 64 \times \tfrac{8}{1} \times \tfrac{40}{1} \text{ pls.}$$

$$= 64 \times \tfrac{8}{1} \times \tfrac{40}{1} \times \tfrac{11}{2} \text{ yds.} = 64 \times \tfrac{8}{1} \times \tfrac{40}{1} \times \tfrac{11}{2} \times \tfrac{3}{1} \text{ ft.}$$

This is usually written at once as—

$$64 \text{ mls.} = 64 \times \tfrac{8}{1} \times \tfrac{40}{1} \times \tfrac{11}{2} \times \tfrac{3}{1} \text{ ft.}$$

The 1's may be omitted if desired.

(ii.) *Again, reduce 847629 feet to miles.*

As above $847629 \text{ ft.} = 847629 \times \tfrac{1}{3} \times \tfrac{2}{11} \times \tfrac{1}{40} \times \tfrac{1}{8} \text{ mls.}$

(iii.) *Reduce 645 half-crowns to florins.*

TABLE.

Half-crs.	Sixpences.	Florins.
1 =	5	
	4 =	1

$\therefore$ 645 half-crowns $= 645 \times \tfrac{5}{1} \times \tfrac{1}{4}$ florins.

(iv.) *Reduce 648 lbs. (Avoir.) to lbs. (Troy).*

TABLE.

lbs. (Avoir.).	Grains.	lbs. (Troy).
1 =	7000	
	5760 =	1

$$\therefore 648 \text{ lbs. (Avoir.)} = 648 \times \frac{7000}{1} \times \frac{1}{5760} \text{ lbs. (Troy).}$$

86. It will be noticed that both kinds of reduction are worked alike, *i.e.* in both cases the table is written from left to right in the direction of the required reduction, and the reduction is performed by multiplying by the number on the right and dividing by the number on the left in each line of the table.

87. This method of reduction is the most suitable when the reduction occurs in the working out of another question, and where it is necessary to express a quantity, having a given name, in terms of another single quantity, *e.g.* feet as miles, not as miles, furs. pls., &c.; or miles as feet, not miles, furs. pls., &c., as feet.

38. The principle of reduction is usually associated with quantities of the same kind, but this is an unnecessary restriction. Thus, while we may reduce yards to feet, since we know that 3 feet are equivalent to 1 yard, so also, if cloth is being sold at 9*d.* per yard, we may take 9 pence as the equivalent of 1 yard and use it in a similar way. We may thus solve many questions by reduction, having first constructed the necessary table of equivalent values.

The so-called **Chain Rule** questions are of this type.

Examples—

(i.) *If 3 horses are worth 5 cows, and 4 cows worth 30 sheep, and 6 sheep worth 15 lambs, find the value of a horse if a lamb is worth 18s.*

Here the table, as constructed, would read—

Horses.	Cows	Sheep.	Lambs.	Shillings.
3 =	5			
	4 =	30		
		6 =	15	
			1 costs	18

Hence 1 horse costs $1\times\frac{5}{3}\times\frac{30}{4}\times\frac{15}{6}\times\frac{18}{1}=562\frac{1}{2}$ shillings $=$ £28 2*s.* 6*d.*

(ii.) *If 4 women earn as much as 9 boys, and 3 men as much as 8 boys, and 18 girls as much as 5 women, find the wages of a girl if a man gets 32s. per week.*

Girls	Women.	Boys.	Men.	Shillings.
18 =	5			
	4 =	9		
		8 =	3	
			1 gets	32

1 girl's wage $=1\times\frac{5}{18}\times\frac{9}{4}\times\frac{3}{8}\times\frac{32}{1}=$ 7*s.* 6*d.*

(iii.) *Express £14 per ton as pence per lb.*

Here we wish to express 1 lb. by its equivalent in pence. The table would read—

Lbs.	Ton.	£	Pence.
2240 =	1		
	1 costs	14	
		1 =	240

1 lb. costs $1\times\frac{1}{2240}\times\frac{14}{1}\times\frac{240}{1}=1\frac{1}{2}d.$

(iv.) *Express 40 miles per hour in feet per second.*

Seconds.	Minutes.	Hour	Miles	Yards	Feet.
60 =	1				
	60 =	1			
		1 is time of	40		
			1 =	1760	
				1 =	3

$\therefore$ 1 second is time of $1 \times \frac{1}{60} \times \frac{1}{60} \times \frac{40}{1} \times \frac{1760}{1} \times \frac{3}{1} = 58\frac{2}{3}$ feet.

Examples (iii.) and (iv.) may also be set down thus—

(iii.) $\frac{£14}{\text{ton}} = \frac{14 \times 20 \times 12 \text{ pence}}{1 \times 2240 \text{ lb.}} = \frac{1\frac{1}{2}d}{\text{lb}}$

$= 1\frac{1}{2}d.$ per lb (see §26a, Ex. (iii.)).

(iv.) $\frac{40 \text{ miles}}{\text{hour}} = \frac{40 \times 1760 \times 3 \text{ ft.}}{60 \times 60 \text{ seconds}} = \frac{58\frac{2}{3} \text{ ft}}{\text{second}}$

$= 58\frac{2}{3}$ feet per second.

For a further use of the above method of reduction, see Exchange, §§ 145–148.

Examples 20.

1. Reduce 59 miles to yards.
2. „ 84 tons to lbs.
3. „ 175936 in. to miles, correct to 2 decimals.
4. „ 8475 sq. yards to acres, correct to 2 decimals.
5. „ 528·19 cwts to ounces.
6. „ 4591830 seconds to days, correct to 2 decimals.
7. „ £85 to pence.
8. „ 74938 farthings to £'s, correct to 3 decimals.
9. „ 5284 florins to half-crowns, correct to 2 decimals.
10. „ 25 lbs (Troy) to ounce (Avoir.).
11. „ 835 links to poles
12. „ 947 yards to links.
13. „ 64 knots to miles per hour.
14. Express 50 miles per hour in yards per sec.
15. „ 10*s.* 6*d.* a yard as pence per inch.
16. „ 10 guineas per cwt. as shillings per lb.
17. „ 4½*d.* per gill, as shillings per gall.
18. „ 7*s.* 6*d.* per sq. yard as £'s per acre.
19. „ 3½*d.* per oz. as £'s per ton.
20. „ £500 a year as pence per hour.

21. If 5 ducks are worth 2 geese, 3 geese worth ten hens, 4 hens worth 27 chickens, find the value of 3 ducks if 5 chickens are sold for 7*s.* 6*d.*

22. 50 gooseberries are worth 4 apples, and 5 apples are worth 3 pears. Find how many gooseberries should be given in exchange for 60 pears.

23. If 2 knives be worth 5 tops, and 3 tops be worth 60 marbles, and marbles cost 1*d.* for 10, how much does a boy gain or lose who gives his knife for 6*d.*?

24. Find in the form of a vulgar fraction in its lowest term a multiplier to convert miles per hour to yards per sec. Use it to convert 45 miles per hour to yards per sec.

XIII.—DECIMALISATION OF MONEY, WEIGHTS, AND MEASURES.

89. The conversion of one unit into the decimal of a higher unit, and of the decimal of a high unit into its value in lower units, are special applications of the methods of §§ 32 and 33.

90. **Decimalisation of Money.**

To reduce a sum of money expressed in pounds, shillings, pence, and farthings, to the decimal of a pound.

Divide the farthings by 4 to express them in decimals of a penny.

Divide the total pence so found by 12 to express them in decimals of a shilling.

Divide the total shillings so found by 20 to express them in decimals of a pound.

Example—*Express £14, 17s. 5¼d. as the decimal of £1.*

Method— $\frac{1}{4}$ = ·25*d.* £14 17 5¼

Total pence 5·25 ÷ 12 = ·4375*s.* 5·25*d.*

Total shillings 17·4375*s.* ÷ 20 = ·871875£. 17·4375*s.*

∴ £14 17 5¼ = £14·871875.

The requisite divisions should all be done mentally.

91. To find the value of a sum of money expressed as the decimal of £1.

Multiply the decimal portion by 20 to find its value in shillings.

Multiply the decimal portion of the shillings so found by 12 to find its value in pence.

Multiply the decimal portion of the pence so found by 4 to find its value in farthings.

Example—*Find the value of £7·653125.*

Method—£·653125 × 20 = 13·062500*s.*

0·0625*s.* × 12 = 0·7500*d.*

0·7500*d.* × 4 = 3·00 farthings.

∴ £7·653125 = £7, 13*s.* 0¾*d.*

```
£7·653125
       20
----------
13·062500s.
       12
----------
 0·7500
      4
----------
 3·00
```

91a. An approximate decimalisation may be obtained mentally thus. Since 2s. = £·1, 6d. = £·025, and ¼ = £·0010416̇, we get the following rule:—

Set down ·1 for each complete florin, followed (as second and third decimals) by 25 for each remaining 6d., increased by 1· for each additional farthing, affixing (as fourth and fifth decimals) 4 times these additional farthings increased by 1 in 25.

Should only 3 decimals be desired, obtain them as above, increasing the third decimal by 1 when the additional farthings are 12 or more (or when the fourth decimal would be 5 or over).

Note.—1 in 25 means 1 when over 12 · 2 when over 37 . 3 when over 62 , 4 when over 87 (§ 45).

Examples—

(i.) 16s. 8½d. = 8 florins, 1 sixpence, 10 additional farthings
= ·8 ; (25 + 10 = 35) ; (10 × 4 + 2 = 42)
= ·83542 or ·835 to 3 decimals.

(ii.) 9s. 10¾d = 4 florins, 3 sixpences, 19 additional farthings
= ·4 ; (3 × 25 + 19 = 94) ; (19 × 4 + 3 = 79)
49479 or ·495 to 3 decimals.

Conversely, using the first three decimals (with allowance if 5 decimals are given), count the first decimal as florin, 6d. for each complete 25 in second and third decimals, and ¼d. each for what is left (less 1 if over 12).

Examples—

(i.) £·827 ; ·8 = 16s. , 27 = 25 + 2 = 6 + ½d Total, 16s. 6½d.

(ii.) £·99083 = £·991 ; ·9 = 18s. , 91 – 75 + (16 – 1) = 1s. 6d. + 3¾d
Total, 19s 9¾d

92. Methods similar to those in § 90 and § 91 are adopted in decimalising weights and measures of all kinds The weights and measures in common use in this and other countries will be found on pp. 38–42.

Examples—(i.) *Express* 16 *tons* 15 *cwts.* 1 *qr.* 3 *lbs.* 8 *oz. as the decimal of* 1 *ton.*

Method—	**8 oz. = ·5 lbs.**	**3·5 lbs**
		·875
Total lbs. 3·5 ÷ 28 (*i.e.* 4 × 7)	**= ·125 qrs.**	**1·125 qrs.**
Total qrs. 1·125 ÷ 4	**= ·28125 cwts.**	**15·28125 cwts.**
Total cwts. 15·28125 ÷ 20	**= ·7640625 tons**	**16·7640625 tons.**

∴ 16 tons 15 cwts. 1 qr. 3 lbs. 8 oz. = 16·7640625 tons.

When the divisors are too large for mental divisions, divide by factors, setting down the intermediate results as in line 2 of above reduction.

(ii.) *Find the value of* $3{\cdot}68\dot{3}$ *of a mile.*

Method—$0{\cdot}68\dot{3}$ mls. × 8 = $5{\cdot}4\dot{6}$ fur.

$0{\cdot}4\dot{6}$ fur. × 40 = $18{\cdot}\dot{6}$ pls.

$0{\cdot}\dot{6}$ pls. × $5\frac{1}{2}$ = $3{\cdot}\dot{6}$ yds.

$0{\cdot}\dot{6}$ yds. × 3 = $1{\cdot}\dot{9}$ ft. *i.e.* 2 ft.

```
3·68333 of a mile
      8
5·46666 . .
     40
18·66 . . .
    5·5
3·66 . . .
    3
1·99 = 2 ft.
```

∴ $3{\cdot}68\dot{3}$ of a mile = **3 mls 5 furs. 18 pls. 3 yds. 2 ft.**

or

```
3·68333 of a mile
      8
5·4666 . .
     10
4·666 . .
   22
1333
 1333
14·666 .
     3
1·99 . . . = 2 ft.
```

∴ $3{\cdot}68\dot{3}$ of a mile = **3 mls. 5 furs. 4 chs. 14 yds. 2 ft.**

93. In the following sets of examples on the tables all multiplications and divisions are to be worked in decimals, and by the contracted methods when the number of required figures is stated.

Examples 21.

1. Reduce 16*s.* $8\frac{1}{4}$*d.*; £1, 7*s.* $3\frac{1}{2}$*d.*; £3, 6*s.* $8\frac{3}{4}$*d.* to the decimal of £1.
2. " 13*s.* $9\frac{3}{4}$*d.*; £5, 8*s.* $6\frac{3}{4}$*d.*; £2, 9*s.* 3*d.* " " £1.
3. " 4*s.* $8\frac{1}{4}$*d.*; £2, 16*s.* $5\frac{1}{4}$*d.*; £8, 7*s.* $6\frac{1}{4}$*d.* " " £1.
4. Decimalise £3, 7*s.* $8\frac{1}{4}$*d.* and multiply by 4376, correct to 3 decimal places.
5. Decimalise £6, 13*s.* $5\frac{1}{2}$*d.* and multiply by 8765, correct to 3 decimal places.
6. Decimalise £4, 1*s.* $10\frac{3}{4}$*d.* and multiply by 8463, correct to 3 decimal places.
7. Decimalise £4463, 17*s.* $6\frac{1}{2}$*d.* and divide by 834. Answer correct to 3 decimal places.
8. Decimalise £7256, 13*s.* $5\frac{3}{4}$*d.* and divide by 6834. Answer correct to 5 decimal places.
9. Find the values of £·394; £·63754; £12·8327.
10. " " £13·837; £64·5837; £35·72634.

11\. Find the values of 37·634*s.*; 395·632*s.*; 645·7326*s.*
12\. " " 3926·63*d.*; 836·95*d.*; 473·8326*d.*
13\. " " 83 articles @ £12·75 each. Answer in £ *s. d.*
14\. " " 747 articles @ £42·835 each. Ans. in £ *s. d.*
15\. " " 3687 " @ £3·7394 " " "
16\. " " 8326 " @ 16·638*s.* " " "
17\. " " 4289 " @ 3·6835*s.* " " "
18\. " " 89325 " @ 4·685*d.* " " "
19\. " " 68474 " @ 8·639*d.* " " "
20\. " " £683·635 ÷ 368. Answer in £ *s. d.*
21\. " " £837·5634 ÷ 793. " "
22\. " " £6927·824 ÷ 8376 (3 places). Ans. in £ *s. d.*
23\. " " 837·6924*s.* ÷ 5246 (3 "). " "
24\. " " 8926·6345*s.* ÷ 6937 (3 "). " "

Examples 22.

Avoirdupois Weight. § 65.

1\. Express 5 cwts. 2 qrs. 10 lbs. 8 oz. as decimal of 1 ton.
2\. " 8 cwts. 1 qr. 1 st. as decimal of 1 ton.
3\. " 17 cwts. 93 lbs. as decimal of 1 ton.
4\. " 39 lbs. 10 oz. as decimal of 1 cwt.
5\. " 436 lbs. 5 ozs. 8 drs. as decimal of 1 cwt.
6\. " 7 ozs. 10 drs. as decimal of 1 lb.
7\. Find the gross weight of 685 bags each weighing 1 cwt. 1 qr. 1 lb.
8\. " " " 373 " " 1 cwt. 36 lbs.
9\. Find the value of 6·83475 of a ton.
10\. " " 0·86394 of a ton.
11\. " " 6·3452 of a cwt.
12\. " " 4·8376 of a cwt.
13\. " " 5·63275 of a lb.
14\. " " 568·3724 tons ÷ 864, correct to 5 significant figures, and reduce the answer to nearest lb.

Examples 23.

Linear Measure. § 66.

1\. Express 7 furs. 5 chs. 15 yds. as decimal of 1 mile.
2\. " 18 furs. 3 chs. 7 yds. " " 1 "
3\. " 5 furs. 16 pls. 3 yds. " " 1 "
4\. " 2 ft. 7 ins. as decimal of 1 yard.
5\. " 27384 ins. " " 1 mile.
6\. Find the value of 8·3764 of 1 mile.
7\. " " 13·63825 of 1 mile.
8\. " " 618·63526 of 1 yard.
9\. " " 325·6347 of 1 chain.
10\. " " 47·8269 of 1 furlong.

Examples 24.

SQUARE AND CUBIC MEASURES. §§ 67, 68.

1. Reduce 15 acs. 3 rds. 20 pls. to the decimal of 1 sq. mile.
2. „ 76 acs. 5 sq. chs. 360 sq. yds to the decimal of 1 sq. mile.
3. „ 3 rds. 24 pls. to the decimal of 1 acre.
4. „ 15 sq. chs. 375 sq. yds. 5 sq. ft. to the decimal of 1 acre.
5. „ 13 cub. ft. 432 cub. ins. to the decimal of 1 cub. yard.
6. Find the value of 3·6835 of a square mile.
7. „ „ 13·674 of an acre.
8. „ „ 8·911 of an acre in acres, square chains, square yards, &c.
9. Find the value of 2·65 of a cubic yard.
10. „ „ 3476·785 of a square yard.

Examples 25.

LIQUID AND CORN MEASURES. §§ 69, 70.

1. Reduce 15 galls. 3 qts. 3 gills to the decimal of 1 gallon.
2. „ 3 qts. 1 pt. 3 gills „ „ 1 „
3. „ 4 bush. 2 pks. 1 gall. „ „ 1 quarter.
4. „ 33 qrs. 5 bush. 3 pks. 3 qts. to the decimal of 1 quarter.
5. Find the value of 0·6375 of a gallon.
6. „ „ 3·624 of a quart.
7. „ „ 0·837 of a bushel.
8. „ „ 81·6345 of a quarter.
9. A vessel has an internal volume of 2287·5105 cubic inches. Find how many gallons it holds.
10. A vessel is completely filled with 73·3125 lbs. of water. What is its capacity expressed in liquid measure.

Examples 26.

TROY AND APOTHECARIES WEIGHTS, APOTHECARIES FLUID MEASURE. §§ 71–73.

1. Express 7 oz. 7 dwts. 12 grs. as the decimal of 1 pound (Troy).
2. „ 1 lb. (Troy) „ „ 1 „ (Avoir.).
3. „ 5 drs. 2 scrs. 12 grs. „ „ 1 ounce (Troy).
4. „ 5 pts. 16 oz. 4 drs. „ „ 1 gallon.
5. „ 5 oz. 5 drs. 15 minims „ „ 1 pint.
6. Find the value of 0·83475 of a pound (Troy).
7. „ „ 3·687 of a pound (Troy)—Apothecaries Weight.
8. „ „ 2·934 of a gallon—Apothecaries Fluid Measure.

Examples 27.

MEASURE OF TIME. §§ 74–77.

1. Express 313 days 6 hrs. as the decimal of 1 year.
2. „ 14 hrs. 32 min. 12 secs. as the decimal of 1 day.
3. „ 94 days 15 hrs. 15 min. „ „ 1 year.
4. „ 59 hrs. 15 mins. „ „ 1 week.
5. Find the value of 0·68754 of 1 year.
6. „ „ 14·9325 of 1 year.
7. „ „ 6·278 of 1 week.
8. „ „ 0·0368 of 1 day.
9. „ „ 12·347 of 1 hour.
10. Find the number of days from 12th January till 3rd November 1904, and express the result as the decimal of the year.

Examples 28.

ANGULAR MEASURE, MEASURE OF TEMPERATURE, PAPER MEASURE. §§ 78–80.

1. Express 34° 13′ 20″ as the decimal of 1 right angle.
2. „ 17° 3′ 42″·5 „ „ 1 „
3. Find the value of 0·68375 of a right angle.
4. „ „ 17°·839.
5. Express 4 reams 12 quires as the decimal of 1 bale.
6. „ 3 reams 6 quires 18 sheets as the decimal of 1 bale.
7. Find the value of 18·3456 reams.
8. Express 59° F., 107° F., 200° F. in the Centigrade Scale.
9. „ 4° C., 50° C., 80° C., in the Fahrenheit Scale.

Tables of Decimal Values.

94. In offices and warehouses where much use is made of any particular set of units, tables of decimal values of those are often prepared to facilitate the work. Below we give examples of two such tables.

95. *Table of Shillings and Pence expressed as decimals of £1.*

s.	d.	Decimal of £1.	s.	d.	Decimal of £1.	s.	d.	Decimal of £1.	s.	d.	Decimal of £1.	s.	d.	Decimal of £1.
	1	00417	4	1	20417	8	1	40417	12	1	60417	16	1	80417
	2	00834		2	20834		2	40834		2	60834		2	80834
	3	0125		3	2125		3	4125		3	6125		3	8125
	4	01667		4	21667		4	41667		4	61667		4	81667
	5	02084		5	22084		5	42084		5	62084		5	82084
	6	025		6	225		6	425		6	625		6	825
	7	02917		7	22917		7	42917		7	62917		7	82917
	8	03334		8	23334		8	43334		8	63334		8	83334
	9	0375		9	2375		9	4375		9	6375		9	8375
	10	04167		10	24167		10	44167		10	64167		10	84167
	11	04584		11	24584		11	44584		11	64584		11	84584
1	—	05	5	—	25	9	—	45	13	—	65	17	—	85
	1	05417		1	25417		1	45417		1	65417		1	85417
	2	05834		2	25834		2	45834		2	65834		2	85834
	3	0625		3	2625		3	4625		3	6625		3	8625
	4	06667		4	26667		4	46667		4	66667		4	86667
	5	07084		5	27084		5	47084		5	67084		5	87084
	6	075		6	275		6	475		6	675		6	875
	7	07917		7	27917		7	47917		7	67917		7	87917
	8	08334		8	28334		8	48334		8	68334		8	88334
	9	0875		9	2875		9	4875		9	6875		9	8875
	10	09167		10	29167		10	49167		10	69167		10	89167
	11	09584		11	29584		11	49584		11	69584		11	89584
2	—	1	6	—	3	10	—	5	14	—	7	18	—	9
	1	10417		1	30417		1	50417		1	70417		1	90417
	2	10834		2	30834		2	50834		2	70834		2	90834
	3	1125		3	3125		3	5125		3	7125		3	9125
	4	11667		4	31667		4	51667		4	71667		4	91667
	5	12084		5	32084		5	52084		5	72084		5	92084
	6	125		6	325		6	525		6	725		6	925
	7	12917		7	32917		7	52917		7	72917		7	92917
	8	13334		8	33334		8	53334		8	73334		8	93334
	9	1375		9	3375		9	5375		9	7375		9	9375
	10	14167		10	34167		10	54167		10	74167		10	94167
	11	14584		11	34584		11	54584		11	74584		11	94584
3	—	15	7	—	35	11	—	55	15	—	75	19	—	95
	1	15417		1	35417		1	55417		1	75417		1	95417
	2	15834		2	35834		2	55834		2	75834		2	95834
	3	1625		3	3625		3	5625		3	7625		3	9625
	4	16667		4	36667		4	56667		4	76667		4	96667
	5	17084		5	37084		5	57084		5	77084		5	97084
	6	175		6	375		6	575		6	775		6	975
	7	17917		7	37917		7	57917		7	77917		7	97917
	8	18334		8	38334		8	58334		8	78334		8	98334
	9	1875		9	3875		9	5875		9	7875		9	9875
	10	19167		10	39167		10	59167		10	79167		10	99167
	11	19584		11	39584		11	59584		11	79584		11	99584
4	—	2	8	—	4	12	—	6	16	—	8	20	—	—

Note.—**For every additional farthing add ·00104.**

96. *Table of Quarters and Lbs. expressed as decimals of 1 cwt.*

Qrs.	Lbs.	Decimal of Cwt.	Qrs.	Lbs.	Decimal of Cwt.	Qrs.	Lbs.	Decimal of Cwt.	Qrs.	Lbs.	Decimal of Cwt.
—	1	00893	1	1	25893	2	1	50893	3	1	75893
	2	01786		2	26786		2	51786		2	76786
	3	02679		3	27679		3	52679		3	77679
	4	03572		4	28572		4	53572		4	78572
	5	04465		5	29465		5	54465		5	79465
	6	05358		6	30358		6	55358		6	80358
	7	0625		7	3125		7	5625		7	8125
	8	07143		8	32143		8	57143		8	82143
	9	08036		9	33036		9	58036		9	83036
	10	08928		10	33928		10	58928		10	83928
	11	09822		11	34822		11	59822		11	84822
	12	10715		12	35715		12	60715		12	85715
	13	11608		13	36608		13	61608		13	86608
	14	125		14	375		14	625		14	875
	15	13393		15	38393		15	63393		15	88393
	16	14286		16	39286		16	64286		16	89286
	17	15179		17	40179		17	65179		17	90179
	18	16072		18	41072		18	66072		18	91072
	19	16965		19	41965		19	66965		19	91965
	20	17858		20	42858		20	67858		20	92858
	21	1875		21	4375		21	6875		21	9375
	22	19643		22	44643		22	69643		22	94643
	23	20536		23	45536		23	70536		23	95536
	24	21429		24	46429		24	71429		24	96429
	25	22322		25	47322		25	72322		25	97322
	26	23215		26	48215		26	73215		26	98215
	27	24108		27	49108		27	74108		27	99108
	28	25		28	5		28	75		28	—

Examples 29.

Miscellaneous.

1. Express 0·175 of a ton + 0·195 of a cwt. + 0·145 of a qr., in pounds and decimals of a pound.
2. Add together 0·2625 of £1, 0·0623 of 13*s.* 4*d.*, and 8·25 of 9*d.*
3. Express 5 cwts. 2 qrs. 21 lbs. as a decimal fraction of a ton, correct to six significant figures.
4. Reduce £3, 15*s.* 7½*d.* to the decimal of 2*s.*
5. Find the value of 0·875 of a bushel.
6. Find the value of 7·134225 tons + 8·903 cwts. + 10·2 qrs.
7. Express 41 yds. 9 in. as the decimal fraction of a furlong.
8. Express 0·0890625 of a day (24 hours) in minutes and seconds.
9. Express 5 cwts. 2 qrs. 7 lbs. as the decimal of 1 ton.
10. Express 1 lb. (Troy) as the decimal of 1 lb. (Avoir.).

11. A map is drawn on the scale of 1 inch to a mile. If the distance between two places on the map measures 1·65 inches, what is the real distance between the places?

12. A map is drawn on a scale of one mile to the inch. The distance between two villages is 10 miles 3 fur. 4 chains. What will be the distance between these villages on the map?

13. What decimal fraction of the year had elapsed at 12 noon on March 10th, 1900?

14. Baltic and American deals are sold in the English market by the St. Petersburg standard, the solid content of which is 165 cub. ft. Express 15 3 St. Petersburg standards in cub. yds., cub. ft., cub. in.

15. Express 35 chains 15 links as the decimal of 1 mile.

16. Cloth was formerly sold by the ell. If, under an old agreement, a person was to receive yearly 16 Scotch ells of cloth, what does he receive according to modern measurements? A Scotch ell = 37·06 in.

17. The old beer gallon had a capacity of 282 cub. in. Express this in gallons, pints, and quarts.

18. A Winchester bushel has a capacity of 2150·42 cub. in. Express the capacity as a decimal of an imperial bushel, and in gallons, quarts.

19. Express a cental (100 lbs.) as the decimal of 1 cwt.

20. Express a nautical mile as the decimal of a geographical mile.

21. A tonneaux equals 0·98421 of a ton. Find its value in cwts. qrs. lbs. &c.

22. The Japanese koku equals 4·99918 imp. bushels. Express this in gallons, quarts.

Examples 30.

Construct tables for the following :—

1. Ounces and drams as decimals of one pound (5 places).
2. Feet and inches as decimals of one yard (5 places).
3. Quarts, pints, and gills as decimals of one gallon (4 places).
4. Bushels and pecks as decimals of one quarter (3 places).
5. Ounces and pennyweights as decimals of one pound (Troy) (5 places).
6. Minutes and seconds as decimals of one hour (5 places).
7. Centigrade degrees (0, 5, 10, &c.) in values of Fahrenheit degrees (2 places).
8. Hundredweights, quarters, stones in decimals of one ton (5 places).
9. Furlongs and chains in decimals of one mile (4 places).
10. Gallons and quarts in decimals of one barrel (4 places).
11. Quires and double-sheets in decimals of one ream (5 places).
12. Ounces (Troy) in pounds (Avoir.) (4 places).

***Note.*—For method of using tables, and exercises on, see § 126.**

XIV.—COMPOUND RULES.

97. Compound Multiplication:—

Multiplication by all numbers up to and including 20 ought to be done mentally.

Example—*Multiply £42, 13s. 7¾d. by 9.*

$= £384, 2s. 9\frac{3}{4}d.$

Method—9 times $\frac{3}{4} = 6\frac{3}{4}$.

$9 \times 7 + 6 = 69d. = 5s. 9d.$
$9 \times 13 + 5 = 122s. = £6, 2s.$
$9 \times 2 + 6 = 24.$
$9 \times 4 + 2 = 38.$

98. In dealing with larger multipliers multiply the multiplier by the farthings, pence, and shillings, and also by the pounds if the amount is less than the multiplier or simpler to multiply by: *e.g.* if we had £68 × 493, we multiply 493 by 68; £2003 × 738, we multiply 738 by 2003. In every case select the simpler number as multiplier.

Example—*Multiply £43, 17s. 2½d. by 427.*

£43, 17s. 2½d.
427
£18728, 7s. 11½d.

Working:

4 | 854
213 – ½
854
12 | 1067
88 – 11
2989
427
20 | 7347
367 – 7
1281
1708
18728

Method—

2 times 427 = 854 farthings = 213½ pence.
2 „ 427 + 213d. = 1067 pence = 88 and 11d.
17 „ 427 + 88s. = 7347 shillings = £367 and 7s.
43 „ 427 + £367 = £18728.

99. Advantage should be taken of the decimal method (§ 90), especially where the money can be readily expressed in the decimal form.

Example—

Multiply £7, 9s. 7½d. by 436.
£7, 9s. 7½d. = £7·48125.

7·48125
436
2992 500
224 4875
44 88750
3261·82500 = £3261 16s. 6d.
20
16·500
12
6·0

100. When the decimal repeats and the multiplier is large, care should be taken to extend the decimal of the £ far enough to ensure the product being correct to 3 places of decimals. Allowance should always be made for carrying figures, as explained in § 45.

Example—

Multiply £18, 6*s.* 5*d.* *by* 247.

£18, 6*s.* 5*d.* = £18·32083̇.

```
 18·320833̇ (§ 54)
 347
 ─────────────
 5496 250 | 0
  832 833 | 3
  128 246 | 8
 ─────────────
£6357·329     = £6357, 6s. 7d.
     20
   ─────
    6·580
     12
   ─────
    6·96
```

101. Similar methods are used in multiplication of the units of other measures.

Examples—

(i.) *Multiply* 3 *tons* 5 *cwts.* 1 *qr.* 15 *lbs.* *by* 45.

tons	cwts.	qrs.	lbs.
3	5	1	15
			45
147	**2**	**1**	**3**

Method—

15 times 45 = 675 lbs. = 24 qrs. 3 lbs.
1 × 45 + 24 = 69 qrs. = 17 cwts. 1 qr.
5 times 45 + 17 = 242 cwts. = 12 tons 2 cwts.
3 × 45 + 12 = 147 tons.

Working:

```
28 | 675
     24 – 3 lbs.
     45
 4 | 69
     17 – 1 qr.
    225
2,0 | 24,2
     12 – 2 cwts.
    135
    147 tons
```

or, by decimalising

3 *tons* 5 *cwts.* 1 *qr.* 15 *lbs.* × 45 = 3·2692 *tons* × 45.

```
  3·2692
  45
 ────────
 130 768
  16 3460
 ────────
 147·114 = 147 tons 2 cwts. 1 qr. 3 lbs.
     20
   ─────
   2·280
     4
   ─────
   1·12
    28
   ─────
   3·36
```

(ii.) *Multiply* 16 *yds.* 1 *ft.* 9 *ins. by* 735.

Decimalising we get 16·583̇ yds × 735.

```
  16·5833333
   735
11680 333 | 3
  497 500 | 0
   82 916 | 6
12188·750 = 12188 yds. 2 ft. 3 in.
        3
     2·25
       12
     3·00
```

(iii.) *Multiply* 6 *lbs.* 8 *oz.* 15 *dwts. by* 98.

Decimalising we get 6·72916̇ lbs. × 98.

```
 6·729166
 98
605 624 | 9
 53 033 | 3
659·458 = 659 lbs. 5 oz. 10 dwts
   12
  5·496
   20
  9·920
```

Examples 31.

Find the values of—

1. £7, 16s. 3¾d. × 8.
2. £19, 11s. 10¼d. × 9.
3. 18s. 7¾d. × 7.
4. £57, 13s. 0¼d. × 6.
5. £3, 0s. 5d. × 11.
6. £5, 6s. 11¼d. × 18.
7. £453, 7s. 8½d. × 57.
8. £3075, 13s. 4¼d. × 72.
9. £57, 0s. 6¾d. × 341.
10. £519, 18s. 0½d. × 459.

Find, by decimals, the values of (correct to nearest penny)—

11. £35, 7s. 4½d. × 12.
12. £37, 18s. 2¼d. × 13.
13. £304, 16s. 2d. × 19.
14. £63, 7s. 4½d. × 15.
15. £379, 6s. 3½d. × 159.
16. £8005, 10s. 0¼d. × 576.
17. £68, 14s. 10¾d. × 327.

Find, by decimals, the values of (correct to nearest farthing)—

18. £4176, 1s. 1¾d × 1005.
19. £645, 19s. 11d. × 390.
20. £5103, 15s. 6¾d. × 2185.

Examples 32.

Find the values of—

1. 3 cwts. 2 qrs. 21 lbs. 14 oz. × 7.
2. 15 tons 6 cwts. 3 qrs. 18 lbs. × 19.
3. 40 yds. 2 ft. 11 ins. × 53.
4. 65 acs. 3 rds. 18 pls. × 425.
5. 10 mls. 3 furs. 5 chs. 16 yds. × 175.
6. 76 lbs. 8 oz. 14 dts. × 306.
7. 12 lbs. 6 ozs. 18 dwts. 12 grs. × 49.
8. 53 days, 6 hrs. 18 mins. 45 secs. × 781.
9. 16 yrs. 218 days, 10 hrs. 20 mins. × 503.
10. 108 qrs. 6 bush. 3 pks. 2 gills × 6090.

Find, by decimals, the values of—

11. 17 tons 3 cwts. 2 qrs. 14 lbs. × 75.
12. 5 qrs. 18 lbs. 2 ozs. 8 drs. × 300.
13. 16 yds. 2 ft. 9 ins. × 425.
14. 34 acs. 1 rd 30 pls. × 2800.
15. 105 mls. 7 furs. 2 chs. 17·6 yds. × 1350.
16. 29 lbs. 11 ozs. 9·6 drs. × 490.
17. 25 lbs. 7 ozs. 4 dwt. 16 grs. × 376.
18. 183 days 16 hrs 24 mins. 30 secs. × 524.
19. 35 yrs 29 days 4 hrs. 48 mins. × 130.
20. 16 qrs. 4 bush. 3 pks. 2 gills × 596
21. 57 tons, 3 cwts. 2 qrs. 18 lbs. 6 oz. × 154, correct to nearest pound.
22. 23 mls. 2 furs. 8 chs 16 yds. × 38, correct to nearest furlong.
23. 64 sq yds. 8 sq. ft. 108 sq. ins. × 625, correct to nearest square yard.
24. 5 lbs. 3 ozs. 6 dwts. 18 grs. × 327, correct to nearest ounce.

102. Compound Division:—

In dividing by any number up to 20, short division ought to be employed.

Example—

Divide £846, 13*s*. 4½*d* *by* 17.

17) £846, 13*s* 4½*d*.

£49, 16*s*. 0¾*d*. $\frac{15}{17}$.

Method—

Dividing the £'s we get 49 and 13 over.
Multiply 13 by 20 and add 13*s*. = 273*s*.
273 ÷ 17 = 16 and 1 over.
Multiply 1 by 12 and add 4*d*. = 16*d*.
16 ÷ 17 = 0 and 16 over.
Multiply 16 by 4 and add 2*f*. = 66*f*.
66 ÷ 17 = 3 and 15 over.

103. All numbers consisting of the figures 1 to 20 followed by one or more 0's can be dealt with in a similar way, care being taken to mark off the necessary figures (one for each 0 in the divisor) before dividing the pounds, the shillings, &c.

104. Factors may be used in Compound Division. The rule regarding the remainder (which in business is often omitted—especially fractions of the lowest unit) is the same as that explained in § 13.

Example—*Divide* £931, 14*s*. 2½*d*. *by* 72.

72 = 8 × 9; we divide by 8 first (§ 12) and then by 9, retaining or neglecting the remainder as desired.

72 { 8 | £931, 14*s*. 2½*d*.
     { 9 | £116, 9*s*. 3¼*d*. " 2
           | £12, 18*s*. 9½*d*. " 7 } $\frac{58}{72} = \frac{29}{36}$

105. Long division is usually worked out by the following method.

Examples—

(i.) *Divide* £11946, 11*s.* 11¾*d.* *by* 341.

```
        £35, 0s. 8 0/4 d. 223/341
       ----------------------
341 ) £11946, 11s. 11¾d.
       1716
        ----
         11
         20
        ----
        231
         12
       ----
       2783
         55
          4
        ---
        223
```

Method—

£11946 ÷ 341 = £35 and 11 over.

£11 × 20 + 11*s.* = 231 shillings.

231*s.* ÷ 341 = 0*s.* and 231 over.

231*s.* × 12 + 11 = 2783*d.*

2783*d.* ÷ 341 = 8*d.* and 55 over.

55*d.* × 4 + 3 = 223 farthings.

223 farthings ÷ 341 = 0 farthings and 223 over $= \frac{223}{341}$.

The principle of decimalisation (§ 90) is of great use in division.

(ii.) *Divide* £345, 13*s.* 10*d.* *by* 84.

This is equal to £345·69167 ÷ 84 (see table, page 47).
Dividing to 4 places we get £4·1154,
Which, on reduction, = £4, 2*s.* 3·6*d.*

106. As in multiplication, similar methods are used with the other tables.

Examples—

(i.) *Divide* 724 *yds.* 2 *ft.* 8 *ins.* *by* 57.

```
     12 yds. 2 ft. 1 47/57 in.
    --------------------------
57 ) 724 yds. 2 ft. 8 ins.
     154
      40
       3
     ---
     122
       8
      12
     ---
     104
      47
```

Method—

724 yds. ÷ 57 = 12 yds. and 40 over.

40 yds. × 3 + 2 ft. = 122 ft.

122 ft. ÷ 57 = 2 ft. and 8 over.

8 ft. × 12 + 8 ins. = 104 ins.

104 ins. ÷ 57 = 1 in. and 47 over $= \frac{47}{57}$.

(ii.) *Divide* 325 *days* 7 *hrs.* 25 *mins.* 30 *secs. by* 89.

Decimalising, this becomes 325·3094 ÷ 89
= 3·6552 days
= **3 days 15 hrs. 43 mins. 29 secs.**

Practice will enable the student to decide when to decimalise.

Examples 33.

1. Divide £649, 2*s.* 6*d.* by 8.
2. „ £5943, 13*s.* 10½*d.* by 36 (factors)
3. „ £8527, 18*s.* 11¾*d* by 392.
4. „ £62841, 1*s.* 0½*d.* by 5040.
5. „ £651, 7*s.* 11¼*d.* by 9.
6. „ £8476, 13*s.* 8¾*d* by 64 (factors).
7. „ £12957, 11*s.* 2¼*d.* by 487.
8. „ £81276, 13*s.* 10*d.* by 3104.
9. „ 15 tons 3 cwts. 2 qrs. 10 lbs. by 8.
10. „ 34 tons 18 cwts. 12 lbs. 14 oz. by 48 (factors).
11. „ 628 cwts. 109 lbs. 12 oz by 321.
12. „ 6259 mls. 1 fur. 8 chains 16 yds. by 425.
13. „ 36 mls. 1625 yds. 2 ft. 10 ins. by 84 (factors).
14. „ 45 acs. 1 rd. 35 pls. by 7.
15. „ 181 ac. 3276 sq. yds. 3 sq. ft. 116 sq. ins. by 72 (factors).
16. „ 325 ac . 90 sq. pls. 18 sq. yds. by 425.
17. „ 37 lbs. 6 oz. 12 dwts. 10 grs. by 5.
18. „ 481 qrs. 6 bush. 3 pks. 2 gills by 44 (factors).
19. „ 3281 days 16 hrs. 15 mins. by 385.
20. „ 12 yrs. 151 days 34 mins. by 27 (factors).
21. „ £850, 13*s.* 4½*d.* by 320 (decimalise).
22. „ £27, 18*s.* 10½*d.* by 45 (decimalise) and (factors).
23. „ £341, 16*s.* 2*d.* by 250 (decimalise) and (factors).
24. „ 84 tons 15 cwts. 2 qrs. 21 lbs. by 300 (decimalise)
25. „ 625 cwts. 3 qrs. 17 lbs. 8 oz by 375 (decimalise).
26. „ 284 mls. 3 furs. 20 pls. by 80 (decimalise).
27. „ 1371 yds. 2 ft. 9 ins. by 350 (decimalise) and (factors).
28. „ 427 acs. 3 rds. 16 pls. by 50 (decimalise).
29. „ 4321 sq. yds. 8 sq. ft. 72 sq. ins. by 330 (decimalise).
30. „ 385 acs. 6 sq. chs. 8456 sq. links by 560 (decimalise).
31. „ 59 lbs. 7 oz. 10 dwts. by 30 (decimalise).
32. „ 645 lbs. 3 oz. 6 dwts. 16 grs. by 450 (decimalise).
33. „ 275 days 1 hr. 30 mins. by 200 (decimalise).
34. „ 528 days 16 hrs. 48 mins. by 350 (decimalise).
35. „ 65 qrs. 3 bush. 3 pks. 1 gill by 80 (decimalise).
36. „ 25 right angles 15° 25′ 30″ by 400 (decimalise).

107. The following are examples in both multiplication and division. As a general rule, multiply first; but, when it is observed that the amount may be divided evenly by the given divisor, divide first. Experience will enable the student to decide.

Examples—

(i.) £37, 18s. $4\frac{1}{2}$d. × $\frac{3}{5}$.

	£	s.	d.
	37	18	$4\frac{1}{2}$
			3
5	113	15	$1\frac{1}{2}$
	£22	15	$0\frac{1}{4}\ \frac{1}{5}$

(ii.) £42, 12s. $7\frac{1}{2}$d. × $\frac{95}{243}$.

```
   42·63125
   95
 3836 8125
  213 1563
 4049·969
```

```
243 ) 4049·969    16·666
      1619
      1619
      1616
      1589
       131
```

```
16·666 = £16, 13s. 4d.
   20
13·320
    12
 3·84
```

Examples 34.

1. Find the value of $\frac{3}{8}$ of £2, 14s. 6d.
2. " " $\frac{57}{160}$ of £5, 10s. $4\frac{1}{2}$d.
3. " " $\frac{\text{£84, 10s. 3d.} \times 59}{100}$.
4. " " $\frac{\text{£36, 1s. 2d.} \times 38}{250}$.
5. " " $\frac{\text{£684, 7s. } 10\frac{1}{2}\text{d.} \times 49}{700}$.
6. " " $\frac{\text{£21, 18s. 9d.} \times 32}{500}$.
7. " " $\frac{\text{3 tons 12 cwts. 2 qrs.} \times 27}{125}$.
8. " " $\frac{7}{15}$ of 12 cwts. 2 qrs. 24 lbs. 8 oz.
9. " " $\frac{\text{350 lbs. 4 oz. 12 drs.} \times 15}{200}$.
10. " " $\frac{27}{35}$ of 32 mls. 2 furs. 16 pls.
11. " " $\frac{64 \times \text{25 acs. 3 rds. 10 pls.}}{1800}$.

12. Find the value of $\frac{375}{450}$ of 34 acs. 8 sq. chs.
13. " " 37 yds. 2 ft. 9 in. × $14\frac{2}{7}$.
14. " " $66\frac{2}{3}$ times £18, 10*s*, 9*d*.
15. " " $6\frac{1}{4}$ times 3 tons 5 cwts. 56 lbs.
16. " " $\frac{\text{35 times 31 days 4 hrs. 48 min}}{100}$
17. " " $\frac{9}{16}$ of 35 lbs. 8 oz. 10 dwt
18. " " $\frac{34\frac{1}{4}\text{ lbs.} \times 27}{300}$ (Avoir.)
19. " " £528, 17*s*. 6*d*. ÷ $6\frac{2}{3}$.
20. " " 75 qrs. 3 bush. 2 pks. ÷ $16\frac{2}{3}$.
21. Prove that $3\frac{2}{3}$ of 10*s*. $0\frac{3}{4}$*d*. + $2\frac{3}{5}$ of 14*s*. $10\frac{3}{4}$*d*. + $2\frac{1}{4}$ of 17*s*. $7\frac{3}{4}$*d*. + $1\frac{3}{5}$ of 14*s*. $0\frac{1}{4}$*d*. is equal to £7.

XV.—THE METRIC SYSTEM.

108. The **Metric System** of weights and measures is now used in most civilised countries. The great exceptions are the two English-speaking nations. In this country various Acts have been passed preparatory to its being adopted, but the Government strong enough to make the change has not yet been formed. The only important objection to the change from our arbitrary and cumbrous tables to the other systematic sets of weights and measures, is the chaos that *might* take place during the change. The present Government has made inquiries as to the method of change and period of transition found necessary by those countries which have adopted the system. The information thereby obtained shows clearly that the change could be made in less time, and with less disturbance to trade than is generally supposed.

109. The following are extracts from the reply sent by Sir F. Lascelles to the Marquess of Salisbury in February 1900 in reply to the above inquiries as regards Germany:—

"The difficulties were overcome with comparative success. . . . The purchasing public soon learned to appreciate the simplicity of the new system, and they accepted without serious complaint the inconvenience inevitably connected with the period of transition. . . . The Weights and Measures Regulations came into force on the 1st January 1872; that is to say, about three and a half years after its introduction had been announced. Permission to use the new measures was granted, however, as early as January 1870, in so far as the other party to a particular transaction concurred in their use. The interval thus granted

was sufficient to ensure the adoption of the new system in all its details; it also enabled the local bureaux to acquire the necessary apparatus, and to produce and certify the accuracy of the new measures in such quantities as to render their exclusive use in the various branches of industry an accomplished fact by the 1st January 1872. . . . This is all the more noteworthy, as previous to that date a very large number of different systems had been in use in Germany at the same time . . . It cannot be said that a serious desire exists in Germany at the present day to revert to the former state of things."

110. Acquaintance with the system and its possibilities will show that the adoption would be of great advantage to all classes of the community. All commercial operations would be enormously simplified; at least one-third of the time allotted in schools to elementary Arithmetic could be given to other subjects; and ordinary calculations could be performed in one-half the time required at present.

111. One of the greatest advantages of the metric system is that there is one definite unit taken for each set of measures, and all the others are powers of ten of this unit. To construct a table we require to know this unit, and then we can form the others by adding the following prefixes:—

myria-	=	10000 times.
kilo-	=	1000 times.
hecto-	=	100 times.
deca-	=	10 times.
	unit.	
deci-	=	$\frac{1}{10}$ of
centi-	=	$\frac{1}{100}$ of
milli-	=	$\frac{1}{1000}$ of

112. **Measure of Length.**

The fixed unit in this measure is the **metre**, which is a little longer than a yard.

10 millimetres (mm.)	=	1 centimetre (cm.) $\frac{1}{100}$ metre.
10 centimetres	=	1 decimetre (dm.) $\frac{1}{10}$ metre.
10 decimetres	=	1 metre.
10 metres	=	1 decametre (Dm.) 10 metres.
10 decametres	=	1 hectometre (Hm.) 100 metres.
10 hectometres	=	1 kilometre (Km.) 1000 metres
10 kilometres	=	1 myriametre (Mm.) 10000 metres.

A Micron = $\frac{1}{1000000}$ metre is used for extremely small measures.

1 metre	=	39·37079 inches.
1 yard	=	91·43835 centimetres.

113. Measure of Area.

In measurements of areas we may use the **Square Metre** and the squares of the other units. The table of square measure is then similar to that of length, but we have 100's in place of 10's.

The square decametre is called an **Are**, and then we have the following table. Those omitted are seldom used.

10 Centiares ($\frac{1}{100}$ are)	=	.	1 Deciare ($\frac{1}{10}$ are).
10 Deciares	=	.	1 **Are**.
10 **Ares**	=	.	1 Decare.
10 Decares	=	.	1 Hectare.
1 are	=	.	119·603 sq. yds.
1 sq. mile	=	.	258·98945 hectares.

114. Measure of Volume.

In this we can use the **Cubic Metre** and the cubes of the other units of the measure of length. The table then advances by powers of 1000.

E.g. 1000 cubic decimetres = 1 cubic metre. The cubic metre is called a **Stere**. The table is—

10 Decisteres	=	.	1 **Stere**.
10 **Steres**	=	.	1 Decastere.
1 stere	=	.	1·30802 cub. yard.
1 cub. yard	=	.	0·7645 steres.

115. Measure of Capacity.

A relation exists between the measures of length and capacity. A cubic decimetre is called a **litre**, and the litre is taken as the unit of capacity. The table is then constructed as in § 111.

10 millilitres (ml.)	=	1 centilitre (cl.) = $\frac{1}{100}$ litre.
10 centilitres . .	=	1 decilitre (dl.) = $\frac{1}{10}$ litre.
10 decilitres	=	1 **litre**.
10 **litres** . .	=	1 decalitre (Dl.) = 10 litres.
10 decalitres .	=	1 hectolitre (Hl.) = 100 litres.
10 hectolitres .	=	1 kilolitre (Kl.) = 1000 litres.
1 litre .	=	. **1·7608 pints.**
1 gallon	=	. **4·5435 litres.**

116. Measure of Weight.

A cubic centimetre of pure water at maximum density point (4° C.), and at normal pressure (760 mm.) weighs 1 **gramme.** The gramme is taken as the unit of weight.

10 milligrammes (mg.)	=	1 centigramme (cg.) = $\frac{1}{100}$ gramme.
10 centigrammes	=	1 decigramme (dg.) = $\frac{1}{10}$ gramme.
10 decigrammes	=	1 gramme.
10 grammes	=	1 decagramme (Dg.) = 10 grammes.
10 decagrammes	=	1 hectogramme (Hg.) = 100 grammes.
10 hectogrammes	=	1 kilogramme (Kg.) = 1000 grammes.

100 kilogrammes is called a quintal.
1000 kilogrammes is called a tonneau or metric ton.

Since 1 c.cm. = $\frac{1}{1000}$ of a litre and 1 gramme = $\frac{1}{1000}$ of a kilogramme, we get 1 litre of water weighs 1 kilogramme.

1 gramme	=	15·4323 grains.
1 kilogramme	=	2·20462 lbs. Avoir.
1 grain	=	0·0648 grammes.
1 lb. Avoir.	=	0·4536 kilogrammes.

117. Reduction:—

With metric tables reduction is performed by moving the decimal point to right or left as required. To understand this, write the "metric" table thus (use prefixes only)—

Kilo.	Hecto.	Deca.	(Metre.)	deci.	centi.	milli
48	5	6	7 ·	2	3	8

To express the above in terms of any one unit set the figures down and place a decimal point after the figure of the desired unit, thus—

48·567238 kilometres,
485·67238 hectometres,
48567·238 metres,
&c.

It will be observed that the *figures* remain the same in every case, the *decimal point* alone being moved.

The following is the method of reduction recommended:—

Examples—

(i.) *Reduce* 32·62574 *hectometres to centimetres.*

Rewrite the figures, moving the point to its new position, which will be found by (1) noting the direction in the above table—Hecto to centi (to the

right); (2) moving the point in this direction one figure at a time, naming the prefixes till you reach the one required; insert the point after this figure.

32·62574 hectometres
= 326257 4 centimetres.

(ii.) *Express* 684732·4 *decimetres as kilometres*
(*direction, left*) = **68·47324 kilometres.**

Since there are 100 square centimetres in a square decimetre and 100 cubic centimetres in a cubic decimetre (§§ 113, 114), it should be noted that in reduction where the name square occurs we follow the same plan as above, but move the decimal point two figures at a time, and three figures with the name cubic. With all other tables, including are, litre, &c., move the point only one figure at a time.

(iii.) *Express* 63925·485 *square metres as square kilometres*
(*direction, left, 2 figures at a time*) = **0 063925485 sq. kilometres.**

(iv.) *How many cubic decimetres are there in* 1 *cubic decametre?*
(*direction, right, 3 figures at a time*) 1 *cubic decametre*
= **1,000,000 cubic decimetres.**

As the square decametre is also called the are, the cubic decimetre the litre, and the cubic metre the stere, it is often necessary to perform a double reduction.

(v.) *Reduce* 64372·18 *sq. centimetres to deciares*
(*direction, left, 2 figures at a time*) = 0 0643721 *sq. decametres or ares*
(*direction, right, 1 figure at a time*) = **0·6437218 deciares.**

(vi.) *Express* 1 *hectolitre in cubic metres.*
(*right, 1 figure at a time*) 1 *hectolitre* = 100 *litres or cubic decimetres*
(*left, 3 figures at a time*) = **·1 cubic metre.**

Examples 35.

1. Express 843762·5 centimetres as decimetres; 12947·628 grammes as Kilogrammes; 821·7095 Hectolitres as decilitres.

2. Express 1294738·5 sq. cm. as sq. M.; 0·328629 sq. Dm. as sq. dm.; 6128473·8 c. cm. as c. M.; 5·837 c. dm. as c. mm.

3. Express 7·935826 Decares as deciares; 429·6175 centilitres as Hectolitres; 4325·9 steres as Decasteres.

4. Express 528407 square metres as Hectares; 4283205 cubic centimetres as litres; 325·92837 Decasteres as cubic decimetres.

5. Express 429385·294 cubic metres as Kilolitres; 384276 centilitres as cubic centimetres.

6. Find the weight in metric tons of 90,000 Kilolitres of water.

7. How many rails, each 10 metres long, are required in laying a single line of rails a distance of 20 kilometres?

8. How many pieces of cloth, each 200 square centimetres in area, can be cut from a roll containing 500 square metres?

9. How many allotments of 1·5 ares can be made from a piece of land 300 hectares in area?

10. How many jugs, each of 20 cubic centimetres capacity, can be filled from a can containing 10 litres?

11. A stack of wood, containing 150 blocks, measures 100 decasteres. Find the volume of each block of wood in steres.

12. A cubic metre of aluminium is to be cut up into 4000 small cubes. What will be the volume of each?

13. A rod of iron weighing 15 kilogrammes is cut into 40 equal pieces. What will be the weight of each piece in grammes?

14. A piece of platinum, 1 square centimetre in area, weighs 1 decigramme. How many such pieces can be cut from a plate which weighs 2 kilogrammes?

15. A cubic centimetre of mercury weighs 13·6 grammes. What will be the weight of mercury in a flask containing half a litre?

16. A train is travelling 96 kilometres per hour. What is the rate in centimetres per second?

118. In converting from the British System to the Metric System, it is advisable to express the British units in one unit, with decimals if necessary (Section XIII.), and then make use of values given for conversion.

Example—*Convert 16 tons 5 cwts. 36 lbs. to kilogrammes.*

16 tons 5 cwts. 36 lbs. = 325 cwts. 36 lbs.
= 36436 lbs.
1 lb. = 0·453593 kilogrammes
∴ 36436 lbs = (0·453593 × 36436) kilogrammes
= 16527·1 kilogrammes.

119. In converting from the Metric System to the British System, we convert to one unit and then find the value by the method of Section XIII.

Example—*Convert 6834·2 metres to miles and yards.*

1 metre = 1·093633 yards
6834·2 metres = (1·093633 × 6834·2) yards
= 7474·10 yards
= 4 miles 434·10 yards.

Examples 36.

1. The distance from London to Edinburgh is 400 miles. Express this distance in kilometres.

2. During the ten months ending April 1901 the United States of America imported 65,893,989 sq. yds. of unbleached cloth. Express this amount in square metres.

3. During the four months ending April 1901 Sweden exported 59,100 cubic metres of boards. Find the equivalent in cubic yards.

4. In May 1901 the United Kingdom exported the following quantities of jute yarn :—to Germany 2200 yds., to Spain and Canaries 387,700 yds., to United States of America 21,400 yds., to Brazil 2,454,500 yds., and to other countries 741,500 yds. Express these lengths in metres.

5. In May 1901 the United Kingdom exported the following quantities of wool :—to Germany 87,100 lbs., to Holland 12,300 lbs., to France 6800 lbs., to United States of America 1,830,900 lbs., to Canada 25,300 lbs, to other countries 23,200 lbs. Express these quantities in kilogrammes.

6. For the five months ending 31st May 1901 the United Kingdom imported the following quantities of wine :—Champagne 511,194 galls., Saumur 51,651 galls., Burgundy 5516 galls., Hock 20,359 galls., Moselle 18,816 galls. Express these quantities in hectolitres.

7. The following are the total quantities of coal and culm exported :—1896, 32,947,680 tons; 1897, 35,354,296 tons; 1898, 35,058,430 tons; 1899, 41,180,332 tons; 1900, 44,089,197 tons. Express these quantities in tonneaux.

8. The height of Mount Everest is 29,000 ft. Express this height in metres.

9. The railway journey from Copenhagen to Stockholm *viâ* Gothenburg is 701 kilos. Express this distance in miles and furlongs.

10. The total area of the German Empire is 539,812 sq. kilos. Find the area in square miles.

11. A train travels at the rate of 54 miles per hour. Express the rate in kilometres per hour.

12. Express the weight of a gramme in grains and decimals of a grain.

13. The area of a certain farm is 50 ac. 2 ro. What is the area expressed in hectares?

14. A person walks at the rate of 4 miles per hour. How long will he take to travel 40 kilos.?

15. At Glasgow, during August 1901, the mean atmospheric pressure, corrected for temperature and reduced to mean sea-level, was 29·943 ins. Express this pressure in centimetres, correct to three places of decimals.

16. For the same period the average velocity of the wind per hour in miles was 10·5. Express this velocity in kilometres, correct to two places of decimals.

17. A piece of wire measures 8·76 centimetres. Express this length in inches, correct to two places of decimals.

18. A tradesman buys 342·45 metres of cloth. Find the length in yards, correct to two places of decimals.

19. A measure is graduated to hold 40 fluid ounces. Express this amount in litres, correct to two places of decimals.

20. A labourer has to carry 12·816 quintals of plaster a distance of 8·34 metres. Express these quantities in lbs. and yards, correct to three places of decimals.

21. The area of Scotland is 19,777,490 acres. Express this area in hectares, correct to six significant figures.

22. During the five months ending May 1901, France imported 48,963,600 kilogrammes of machinery. Find the equivalent in tons, correct to six significant figures.

23. During the same period France imported 879,427 hectolitres of wines. Find the equivalent in gallons, correct to six significant figures.

24. The exports of coke and cinders from the United Kingdom for the years 1896 to 1900 were as follows:—1896, 676,811 tons; 1897, 978,327 tons; 1898, 769,742 tons; 1899, 867,295 tons; 1900, 985,365 tons. Express these quantities in tonneaux, correct to four significant figures.

120. Commercial Uses :—

In commerce the only units of the measure of length in common use are the centimetre, metre, and kilometre. The metre and centimetre are used where we would use yards, feet, and inches; the kilometre where we would use the mile. For scientific measurements, in every country, the centimetre, millimetre, and micron ($\frac{1}{1000}$ mm.) are used. The following are approximate values which may be used for rough calculations, and in checking results.

1 centimetre	is nearly equal to	$\frac{2}{5}$	of an inch.
1 metre	,, ,,	$\frac{12}{11}$	of a yard.
1 kilometre	,, ,,	$\frac{5}{8}$	of a mile.
1 pole	,, ,,	5	metres.
1 furlong	,, ,,	200	metres.

121. In measuring areas the hectare is used where we would use acres, and the square metre where we would use square yards.

1 hectare	is nearly equal to	$2\frac{1}{2}$ acres.
1 square metre	,, ,,	$1\frac{1}{5}$ square yards.

122. The cubic metre corresponds to the cubic yard and is almost $1\frac{3}{10}$ of a cubic yard. In measuring wood, however, the volume is expressed in steres.

In measuring liquids the cubic centimetre, litre, and hectolitre are the most commonly used; the litre and hectolitre being used in measuring grain and fruit also.

1 litre	is nearly equal to	$1\frac{3}{4}$ pints.
1 hectolitre	,, ,,	22 gallons.
1 cubic inch	,, ,,	$16\frac{1}{2}$ cubic centimetres.

123. The kilogramme is the commonest unit in the measure of weight; the quintal and tonneau are used for heavy weights, and the milligramme and centigramme in scientific calculations.

1 kilogramme	is nearly equal to		$2\frac{1}{5}$ lbs
1 quintal	„	„	2 cwts.
1 tonneau	„	„	1 ton.
1 gramme	„	„	$15\frac{1}{2}$ grains.

Examples 37.

(In this set the values given in §§ 120-123 are to be used.)

1. An approximate value of the distance from the earth to the sun is $92\frac{1}{2}$ million miles. What will be the corresponding approximate value expressed in kilometres?

2. A person buys 11 lbs. of tea at 2*s.* 6*d.* per lb. If the tea was sold in kilogrammes, what amount would he buy and what would be the price per kilogramme?

3. A farmer lays $2\frac{1}{2}$ tons of manure on a field of 3 acres extent. Express these quantities in quintals and hectares.

4. The distance from Paris to Madrid is 1450 kilometres. What is the distance in miles?

5. The distance from London to Paris is 288 miles. Express the distance in kilometres.

6. Motor machines are allowed to travel in the United Kingdom at the rate of 20 miles per hour. Express the rate in kilometres.

7. The reading of a barometer on a certain day is 74 centimetres. What is the reading in inches?

8. A person in France wishes to buy about two pints of milk. What quantity should he ask for?

9. A flag-pole is 58·5 metres high. What is the height in feet?

10. A sovereign contains about 113 grains of pure gold. Express the amount in grammes.

11. Sound travels at the rate of 1090 ft. per second. Express the velocity in centimetres per second.

12. Light travels at the rate of about 300,000 kilometres per second. Find the corresponding value in miles (three significant figures).

13. A barrel of apples contains 14 stone. What is the weight in kilogrammes?

14. A man buys a barrel of grapes containing 5·5 kilogrammes. How many lbs. does he buy?

15. A chest of tea contains 160 lbs. What is the weight of the tea in kilogrammes?

16. A person wishes to buy about 15 yds. of cloth. How many metres should he ask for?

124. For commercial purposes tables of equivalents are usually constructed similar to those below.

The following tables are taken from the "Encyclopædia Britannica," Ninth Edition.

Table for conversion of British and Metric Units.

Inches.	Millimetres.	Metres.	Feet.	Cubic Inches.	Cubic Centimetres.	Cubic Metres.	Cubic Feet.
1	25·399	1	3·2809	1	16·386	1	35 316
2	50·799	2	6 5618	2	32·772	2	70·633
3	76 199	3	9·8427	3	49·168	3	105 950
4	101 598	4	13·1236	4	65 545	4	141·266
5	126·998	5	16·4045	5	81 931	5	176 583
6	152 397	6	19·6854	6	98·317	6	211·899
7	177·797	7	22·9663	7	114·703	7	247·216
8	203 196	8	26 2472	8	131 089	8	282·533
9	228·596	9	29 5281	9	147·476	9	317·849
Pints.	**Litres.**	**Litres.**	**Gallons.**	**Grains**	**Grammes.**	**Kilos.**	**Pounds (Troy).**
1	0 56755	1	0·22024	1	0 064799	1	2 6792
2	1·13510	2	0·44049	2	0·129598	2	5·3584
3	1·70265	3	0·66073	3	0·194397	3	8 0377
4	2·27020	4	0·88098	4	0·259196	4	10·7169
5	2·83775	5	1·10122	5	0 323994	5	13·3961
6	3·40530	6	1 32146	6	0·388794	6	16·0754
7	3·97286	7	1·54171	7	0·453593	7	18·7546
8	4 54041	8	1·76195	8	0·518392	8	21·4338
9	5 10796	9	1·98220	9	0·583190	9	24·1130

125. Two of the most common tables are the conversion of tons, cwts. lbs. to kilogrammes, and of metres to miles and yards.

Table of conversion of Tons, Cwts. Lbs. to Kilogrammes.

Lbs.	Kilos.	Cwts.	Kilos.	Tons.	Kilos.	Kilos.	Lbs. (Avoir.).
1	0·45359	1	50·80238	1	1016·0475	1	2·2046
2	0·90719	2	101·60475	2	2032·0951	2	4·4092
3	1·36078	3	152·40713	3	3048·1426	3	6·6139
4	1·81437	4	203·20951	4	4064·1902	4	8·8185
5	2·26796	5	254·01188	5	5080·2377	5	11 0231
6	2·72156	6	304·81426	6	6096·2852	6	13·2277
7	3·17515	7	355·61664	7	7112·3328	7	15 4323
8	3·62874	8	406·41902	8	8128·3803	8	17·6370
9	4·08233	9	457·22140	9	9144·4279	9	19·8416

Table of conversion of Metres to Yards and Miles.

Metres.	Yards	Kilometres	Miles.
1	1·09363	1	0·62138
2	2·18727	2	1·24276
3	3·28090	3	1·86415
4	4·37453	4	2·48553
5	5·46817	5	3·10691
6	6·56179	6	3·72829
7	7·65543	7	4·34968
8	8·74907	8	4·97106
9	9·84269	9	5·59244

Examples 38.

Construct tables for the conversion of—

1. Yards, chains, furlongs, and miles, to kilometres.
2. Square yards, roods, and acres, to hectares.
3. Grains, pennyweights, ounces, and pounds (Troy), to grammes.
4. Pints, quarts, and gallons, to litres.
5. Quintals to cwts., tonneaux to tons.
6. Centimetres to inches, metres to chains.
7. Square metres to square yards, hectares to square miles.
8. Litres to pints, hectolitres to bushels.

126. HOW TO USE THE TABLES.

(Take the examples of §§ 118–119.)

Express—(i.) *16 tons 5 cwts. 36 lbs. in kilogrammes.*

This is worked out thus :—

10 tons (1 ton × 10)	=	10160·475 kilogrammes.
6 tons	=	6096·2852 „
5 cwts.	=	254·01188 „
30 lbs. (3 lbs. × 10)	=	13·6078 „
6 lbs.	=	2·72155 „
16 tons 5 cwts. 36 lbs.	=	16527·1 kilogrammes.

Express—(ii.) 6834·2 *metres in miles and yards.*

6000 metres	(6 metres × 1000)	=	6561·79 yards
800 „	(8 „ × 100)	=	874·907 „
30 „	(3 „ × 10)	=	32·8090 „
4 „		=	4·37453 „
2 „	(2 metres ÷ 10)	=	·218727 „
6834·2 metres			7474·10 yards = 4 mls. 434·10 yds.

Examples 39.

(Using tables of §§ 124–125.)

Convert—

1. 6·5 in. ; 45 in. ; 2 ft. 6 in. ; 3 yds. 5 in., into millimetres.
2. 4 yds. 1 ft. 8 in. ; 3 yds. 7 in. ; 6 yds. 2 ft. 7 in., into metres.
3. 8·5 metres ; 613·75 cms. ; 8467·5 mms., into feet.
4. 126 cub. in. ; 20 cub. ft. ; 3 cub. yds., into cub. centimetres.
5. ·45 cub. metres ; 68345 c. cms. ; 8·934 cub. metres, into cub. ft.
6. 63 pints, 18 gallons, 8 bushels, into litres.

Express—

7. 3·9 litres ; 46375 c. cms. ; 10·06 litres, in gallons.
8. 63 grains ; 7000 grains ; 5760 grains, in grammes.
9. 45 kilos; 39·27 kilos; 837 grammes; 453·6 grammes, in lbs. (Troy).
10. 3 tons 5 cwts. 12 lbs. in kilos.
11. 7 tons 12 cwts. 20 lbs. in kilos.
12. 18 tons 8 cwts. 7 lbs. in kilos.
13. 7345 metres in (1) yds. (2) miles.
14. 8936 metres in (1) yds. (2) miles, (3) miles and yds.
15. 67·385 kilometres in (1) miles, (2) yds.

Examples 40.

(Using tables of §§ 95–96, &c.)

1. Express 3 qrs. 8 lbs. @ 16*s.* 8*d.* per cwt. as decimals of cwt. and £'s.

2. Express 2 qrs. 5 lbs. @ £1, 13*s.* 4*d.* per cwt. as kilogrammes at £'s.

3. Find, using the tables, the cost of 3 qrs. 21 lbs. @ £1·7625 per cwt.

4. Find the cost of 1 ton 1 cwt. 1 qr. 13 lbs. @ £2·80417 per cwt.

5. Find the cost of 0·88393 of a cwt. @ 1*s.* 4*d.* per lb.

6. Find the cost of 16 pints @ £0·35834 per litre.

7. Find the cost of 54 grains @ 11*s.* 6*d.* per gramme.

8. Find the cost of 83 kilogrammes @ 2*s.* 6*d.* per lb.

9. A train travels at the rate of 90 kilometres per hour. Express the distance in miles per hour.

10. A stone falling from rest, falls 16 ft. in the first second. Express the distance in centimetres.

11. A cubic centimetre of water weighs 1 gramme. From this find the weight of a cubic foot of water.

12. A person buys 17 metres of cloth for 30 francs. If a franc is worth 0·04167 of a £, find the price per yard.

13. Find how many feet and how many metres there are in 0·5825 of a mile.

XVI.—PRACTICE.

127. Simple Practice is a method of calculating costs now very seldom used in actual business, its place being taken by multiplication. To understand the method we must first learn the "aliquot parts" of £1, 1*s*., &c. By "aliquot part" is meant any sum which is contained in the given sum an exact number of times, *e.g.* 10*s*. is an aliquot part of £1 since it is contained in it 2 times; 3*s*. 4*d*. is an aliquot part of 10*s*., being contained in it 3 times; and 2*d*. is an aliquot part of 3*s*. 4*d*., being contained in it 20 times. It is evident, therefore, that the cost of a number of articles at 10*s*. each will be half their cost at £1 each; and their cost at 3*s*. 4*d*. each will be one-sixth of the cost at £1, or one-third of the cost at 10*s*. each. By dividing the total cost at the larger sum by the number which represents the times the smaller is contained in the larger we obtain the cost at the smaller sum. The method is useful with sums not requiring to be broken up into more than two parts (see "Mental Arithmetic," §§ 67–73), but beyond this we do not advise its use.

128. Special attention should be paid to the part the whole of the shillings and pence form of £1, and from this the answer should be obtained by multiplication rather than by working with a number of aliquot parts.

Example—*Find the cost of* 345 *tons at* £2, 13*s*. 4*d*. *each.*

13*s*. 4*d*. = ⅔ of £1.

```
    £345 = cost @ £1.
      2⅔
 3 | 690
     230 = cost @ ⅓ of £1.
     690 =  "   @ £2.
    £920 = cost @ £2, 13s. 4d.
```

A question of this kind may be worked by decimalising the money part and multiplying as shown in Art. 99. This is the quicker method when the actual part the sum is of £1 cannot be seen at once.

129. In cases where the cost is slightly under an exact number of pounds, calculate for the exact number of pounds and deduct the cost at the difference.

Example—*Find the cost of* 853 *articles @ £5*, 19*s*. 10½*d*. *each.*
Method—*Calculate cost @ £6 and deduct cost @ 1½d., since £5*, 19*s*. 10½*d*. = £6, *less* 1½*d*.

```
853 articles @ £6 each          = £5,118  0  0
 "     "     @ 1½d.  "          =      5  6  7½
853 articles @ £5, 19s. 10½d.   = £5,112 13  4½
```

130. When a fraction occurs in the number of articles, any one of the following methods may be adopted.

(1) Calculate the cost of the integral number by any of the methods already explained, and then find the cost of the fractional part and add.

Examples—(i.) *Find the cost of* 45⅔ *articles @ £4, 15s each.*

45 articles @ £4 15 0 each	= £213	15	0
⅔ articles @ £4 15 0 ,,	= 3	3	4
∴ 45⅔ articles @ £4 15 0 each	= £216	18	4

(2) Find the cost of the number of articles (with fractions) at £1 each, and then calculate at the given price.

(ii.) *Find the cost of* 53⅖ *articles @ £7, 3s. 9d. each.*

3s. 9d = £$\frac{3}{16}$

53⅖ articles @ £1 0 0 each cost	£53	8	0
∴ 53⅖ ,, @ £7 0 0 ,, ,,	£373	16	0
53⅖ ,, @ £$\frac{3}{16}$,, ,,	10	0	3
53⅖ articles @ £7 3 9 each cost	£383	16	3

(3) Decimalise the fraction and multiply as for a whole number.

(iii.) *Find the cost of* 56¾ *articles @ £12, 1s. 8d. each.*

£56.75 × 12$\frac{1}{12}$
12$\frac{1}{12}$
681·00 (Multiply by whole number first.)
4·7291
685·7291
20
14·5820
12
6·98

56¾ articles @ £12, 1s. 8d. each cost **£685, 14s. 7d.**

(4) Decimalise the fraction and find the cost by aliquot parts. Select, as far as possible, parts that are tenths, twentieths, &c.

(iv.) *Find the cost of* 845⅝ *articles @ £6, 14s. 8d. each.*

	845·625 = cost at £1		
	6		
	5073·75	,,	£6
10s. = ½ of £1	422·8125	,,	10s.
2s. = $\frac{1}{10}$ of £1	84·5625	,,	2s.
2s. = $\frac{1}{10}$ of £1	84·5625	,,	2s.
8d. = ⅓ of 2s.	28·1875	,,	8d.
	5693·875	,,	£5693, 17s. 6d.

(5) Decimalise both and multiply by contracted method as explained in Arts. 51–55, taking care to obtain the answer correct to three places of decimals so as to ensure accuracy as far as the pence.

(v.) *Find the cost of* $84\frac{3}{16}$ *articles @ £10, 4s.* $7\frac{1}{2}d.$ *each* = 84·1875 *articles @ £10·23125 each.*

```
    84·18750
    10·23125
  ------------
   841875 | 0
    16837 | 5
     2525 | 6
       84 | 1
       16 | 8
        4 | 2
  ------------
 £861·343
       20
     ----
     6·86
       12
     ----
    10·32
```

∴ $84\frac{3}{16}$ articles @ £10, 4s. $7\frac{1}{2}d.$ cost **£861, 6s, 10d.**

(6) When the price given is not that of a single article, or is of a different unit from that of the article whose cost is required, simplify first by cancelling.

(vi.) *Find the cost of* 7320 *articles @* 17*s.* 6*d. per score.*

Write thus—7320 articles @ $\frac{17s.\ 6d.}{\text{score}}$. Cancel as shown on pp. 15, 16, obtaining 366 @ 17*s.* 6*d.* each or 7232 @ $10\frac{1}{2}d.$ each.

Proceed as in previous examples.

(vii.) *Find the cost of* 476 lbs. @ 18*s.* 8*d. per cwt.*

Write thus—476 ~~lbs.~~ @ $\frac{\overset{2}{\cancel{18s.\ 8d.}}}{\cancel{\text{cwt.}}\ \cancel{112}}$. (Cancel lbs. and cwts. (112) and money.)

Giving 476 @ 2*d.* each.

Examples 41.

1. Find the cost of 853 articles @ £2, 12s. 6d. each.
2. " " 459 " @ £10, 18s. "
3. " " $72\frac{1}{2}$ " @ £4, 7s. 6d. "

4. Find the cost of 329 articles @ £2, 19*s.* 11*d.* each.
5. " " $484\frac{1}{2}$ " @ £5, 19*s.* $10\frac{1}{2}$*d.* "
6. " " $57\frac{3}{8}$ " @ 18*s.* $4\frac{1}{2}$*d.* "
7. " " $74\frac{3}{4}$ cwts. @ £1, 3*s.* 4*d.* per cwt.
8. " " $581\frac{3}{16}$ lbs. @ 4*s.* $7\frac{1}{2}$*d.* per lb.
9. " " $94\frac{2}{3}$ yds. @ 5*s.* 8*d.* per yd.
10. " " $1219\frac{1}{4}$ yds. @ $3\frac{3}{4}$*d.* "
11. " " 87 articles @ £6, 9*s.* 6*d* each.
12. " " $534\frac{7}{8}$ quarters @ £2, 1*s.* 8*d.* per quarter.
13. " " 999 articles @ £999, 19*s.* $11\frac{3}{4}$*d.* each.
14. " " 2592 " @ 16*s.* $10\frac{1}{2}$*d.* each
15. " " 456 " @ £3, 18*s.* $4\frac{1}{2}$*d.* per doz.
16. " " 2840 " @ £1, 17*s.* $10\frac{1}{2}$*d.* per score.
17. " " 1240 yds. @ 7*s.* 6*d.* per ft.
18. " " $428\frac{1}{2}$ lbs. @ 20*s* per cwt.
19. " " $94\frac{3}{4}$ acres @ 10*s.* 9*d.* per rood.
20. " " 521 ozs. gold @ £47, 18*s.* 11*d.* per lb.
21. " " 7320 sq. yds. @ 13*s.* $7\frac{1}{2}$*d* per sq. ft.
22. " " $513\frac{3}{16}$ acres @ £25, 10*s.* 10*d.* per acre.
23. " " $94\frac{5}{8}$ cwts. @ £10, 7*s.* $4\frac{1}{2}$*d.* per lb.
24. " " $3847\frac{3}{8}$ bushels @ 13*s.* 10*d.* per bushel.

131. Compound Practice :—

In compound practice we have several units of both the goods and the money. When using it, we keep one of the sets of units fixed (the money) and take parts for the other. The following examples will show the various methods of dealing with such questions.

Examples—

(i.) *Find the cost of 3 tons 7 cwts. 2 qrs. 14 lbs. @ £2, 16s. per ton.*

(*a*) **Break up the cwts. qrs. lbs. into aliquot parts of tons, cwts. qrs., &c., and proceed as follows :—**

	£	s.	d.		
	2	16	0	=	cost of 1 ton
			3		
	8	8	0	=	" 3 tons
5 cwts. cost $\frac{1}{4}$ of 1 ton	0	14	0	=	" . . 5 cwts.
2 cwts. 2 qrs. cost $\frac{1}{2}$ of 5 cwts.	0	7	0	=	" . . 2 cwts. 2 qrs.
14 lbs. cost $\frac{1}{20}$ of 2 cwts. 2 qrs.	0	0	4·2	=	" . . . . 14 lbs.
	9	9	4·2	=	" 3 tons 7 cwts. 2 qrs. 14 lbs.

Decimalise the weight and find the cost as in simple practice.

	£3·38125 = cost of 3 tons 7 cwts. 2 qrs. 14 lbs. @ £1 per ton.
	2
	6·7625 *
10s. = ½ of £1	1·6906
5s. = ½ of 10s.	0·8453
1s. = $\frac{1}{10}$ of 10s.	0·1691
	9·4675
	20
	9·3500
	12
	4·2

£9, 9s. 4·2d. = cost of 3 tons 7 cwts. 2 qrs. 14 lbs.

(c) Decimalise both weight and money and obtain result by multiplication.

```
£3·38125
  2·8
 6 7625|0
 2 7050|0
 9·4675        Reduce as above.
```

£9, 9s. 4·2d. = cost of 3 tons 7 cwts. 2 qrs. 14 lbs.

* Carry multiplication and division to 4 places, making the necessary allowance.

(ii.) *Find the cost of railing a field whose perimeter is 534 yds. 2 ft. @ 4s. 6d. per yard.*

(a) Since 2 ft., when expressed as decimal of 1 yd., gives a repeating decimal, in working by the last method of (i.) we must use the method of § 54. To ensure accuracy in the pence, 4 places, and in the farthings, 5 places will be necessary.

```
Thus  534·6666 yds. @ £225 per yd.
        0·225
      106 9333|3
       10 6933|3
        2 6733|3
      120 300
          20
        6·00
        12
```

£120, 6s. 0d. = cost of 534 yds. 2 ft. of railing @ 4s. 6d. per yard.

(b) Such a question may be worked out by multiplication.

534⅔ yds. @ 4s. 6d. per yard.

```
     s.  d.                    2 | 534
      4  6                         267
       534⅔                       2136
      3  0                    20 | 2403
120   3  0                         120  3
```

£120 6 0 = cost of 534 yds. 2 ft. of railing @ 4s. 6d. per yard.

(c) The following additional method will be easily understood [see § 130 (iii.)].

(iii.) *Find the cost of* 34 *acs.* 2 *rds.* 10 *pls.* @ £13, 4*s.* 2*d. per acre.*

£34·5625 = cost of 34 acs. 2 rds. 10 pls. @ £1 per acre.
13¼⅛
48 | 345·625
7·2005 = ,, ,, ,, @ 4*s.* 2*d.* per acre.
449·3125 = ,, ,, ,, @ £13 per acre.
£456·513 = ,, ,, ,, @ £13, 4*s.* 2*d.* per acre.
20
10·260
12
3·12

£456, 10*s.* 3*d.* = cost of 34 acs. 2 rds. 10 pls. @ £13, 4*s.* 2*d.* per acre.

(d) Where the Metric System is employed use Multiplication.

(iv.) *Find the cost of* 34 *kilogrammes* 65 *grammes* @ £2, 13*s.* 6*d. per kilogramme.*

Decimalised, this reads 34·065 kilogrammes @ £2 675 per kilogramme.

34·0650
2·675
68 130|0
20 439|0
2 384|6
170|3
£91·124| = £91, 2*s.* 6*d.*

(v.) *Find the cost of* 52·658 *ares* @ £3, 5 *florins* 3 *centimes per are.*

This becomes 52·658 ares @ £3·53 per are (§ 137).

£3·53
52·658
176 500|0
7 060|0
2 118|0
176|5
28|2
£185·883| = £185, 17*s.* 8*d.*

Examples 42.

Find the values of—

1. 5 tons 3 cwts. 3 qrs. @ £2, 5*s.* per ton.
2. 64 mls. 6 furs. 110 yds. @ £3, 16*s.*, 8*d.* per mile.
3. 72 lbs. 12 oz. 8 drs. @ 13*s.* 6*d.* per lb.

Find the values of—

4. 27 yds. 2 ft. 6 ins. @ 7*s.* 4½*d.* per yard.
5. 150 acs. 3 rds. 20 pls. @ £2, 15*s.* 6*d.* per acre.
6. 3 lbs. 10 oz. 10 dwts. @ £65, 18*s.* 6*d.* per lb.
7. 45 qrs. 6 bush. 1 pk. @ £1, 6*s.* 6*d.* per quarter.
8. 20 acs. 3 rds. 12 pls. @ £1, 16*s.* 8*d.* per acre.
9. 2 tons 3 qrs. 23 lbs. @ £8, 19*s.* 8*d.* per cwt.
10. 2 lbs. 7 oz. 15 dwts. 20 grs. @ £46, 14*s.* 6*d.* per lb.
11. 3 tons 17 cwts. 2 qrs. 8 lbs. @ £32, 17*s* 4*d.* a ton.
12. 3 lbs. 4 oz. 13 dwt. 19 grains @ £3, 17*s.* 10½*d.* per oz.
13. 191 acs. 3 rds. 37 pls. @ £42, 3*s.* 4*d.* per acre.
14. 30 acs. 3 rds. 16 pls. @ £49, 13*s.* 4*d.* per acre.
15. 13 cwts. 1 qr. 10½ lbs. @ £2, 0*s.* 8*d.* per cwt.

16. Find the cost of constructing a railway 6 mls. 3 furs. 25 pls. in length at an average cost of £5, 5*s.* per yard.

17. Find, to the nearest penny, the feu duty of a piece of ground of 2 rds. 12 pls. 20 yds. at the rate of £20 per acre.

18. If 5 cwts. 3 qrs. 14 lbs. of a certain article cost £6, per cwt., find what will be the cost per lb. when the cost of the whole has been reduced by £7, 16*s.* 8*d.*

19. If the produce of an orchard, 1 ac. in extent, weighs 1 ton 16 cwts. 3 qrs. 20 lbs., find, by practice, the weight of produce that should be yielded by another orchard measuring 10 acs. 3 rds. 32½ pls.

20. Find, to the nearest penny, the cost for freight of 16 tons 15 cwts. 84 lbs. @ £1, 10*s.* 6*d.* per ton.

Examples 43.

Find the values, correct to the nearest penny, of—

1. 5 tons 3 cwts. 2 qrs. 8 lbs. @ £2, 1*s.* 6*d.* per ton.
2. 34 mls. 3 furs. 6 chs. 18 yds. @ 15*s.* 10*d.* per mile.
3. 74 lbs. 6 oz. 10 drs. @ 14*s.* 6½*d.* per lb.
4. 280 yds. 2 ft. 8 ins. @ 3*s.* 11*d.* per yard.
5. 16 acs. 3 rds. 25 pls. @ £3, 1*s.* 10*d.* per acre.
6. 37 cwts. 18 lbs. 10 oz. @ £5, 6*s.* 8*d.* per cwt.
7. 8 qrs. 3 bush. 5 galls. @ £2, 17*s.* 10*d.* per quarter.
8. 16 lbs. 5 oz. 10 dwts. @ £45, 13*s.* 4*d.* per lb. (Troy).
9. 3 lbs. 8 oz. (Avoir.) @ £50 per lb. (Troy).
10. 6 tons 15 cwts. 106 lbs. @ £5, 15*s.* 6*d.* per ton.
11. 3 mls. 6 furs. 175 yds. 2 ft. @ £1, 19*s.* 11*d.* per mile.
12. 154 acs. 2176 sq. yds. @ £3, 10*s.* 6*d.* per acre.

Find the values, correct to the nearest farthing, of—

13. 84 tons 16 cwts. 90 lbs. @ £3, 1*s.* 4*d.* per ton.
14. 72 cwts. 2 qrs. 15 lbs. 8 oz. @ 16*s.* 7½*d.* per cwt.
15. 325 yds. 2 ft. 10 ins. @ 3*s.* 6*d.* per yard.
16. 61 mls. 5 furs. 3 chs. 11 yds. @ £9, 17*s.* 6*d.* per mile.
17. 235 acs. 3 rds. 15 pls. @ £10, 1*s.* 9*d.* per acre.

Find the values, correct to the nearest farthing, of—

13. 372 sq. yds. 8 sq. ft. 100 sq. ins. @ 10*s.* 7½*d.* per square yard.
19. 3[illegible] cub. yds. 1[illegible] cub. ft. 576 cub. ins. @ £1, 14*s.* 10*d.* per cub. yd.
20. 64 bush. 3 pks. 2 galls. @ 3*s.* 4½*d.* per quarter.
21. 50 mls. 1056 yds. 2 ft. 10 ins. @ £7, 11*s.* 9*d.* per mile.
22. 13[illegible] tons 1729 lbs. @ 16*s.* 10*d.* per ton.
23. 58 lbs. 10 oz. 14 drs. @ 39*s.* 6*d.* per lb.
24. 15 lbs. 6 oz. 10 dwts. 14 grs. @ £19, 17*s.* 6*d.* per lb.

Decimal Practice. (See § 137.)

Find the cost of—

25. 84·376 kilogrammes @ £5, 6 fl. per kilogramme.
26. 2347 grammes @ £1, 7 fl. 5 c. per kilogramme.
27. 9413 centigrammes @ 3 mils per centigramme.
28. 325·621 kilometres @ 5 fl. 5 c. per metre.
29. 27·35 hectares of land @ £3, 5 fl. per square metre.
30. 384 hectares of growing corn @ 6 fl. 7 c. 5 mils per square metre.
31. 5739 litres of wine @ 2 fl. 5 c. per litre.
32. 7[illegible]5·25 metres of silk @ 5 fl. 2 c. 5 mils per metre.
33. 391 steres of wood @ 3 fl. 4 c. per stere.
34. 562 hectolitres of wine @ 3 fl. 2 c. per litre.
35. 315·75 cub. metres @ £1, 3 fl. 7 c. 5 mils per cub. metre.
36. 3500 c. cms. @ £1, 7 fl. 5 c. per litre.
37. 4·5 grammes of metal @ £520, 5 fl. per kilogramme.

XVII.—ALGEBRA.

132. We think no excuse is necessary for introducing the following short explanation of simple Algebraic Equations with a view to using such methods in Arithmetic. The time is surely gone when any attempt should be made to draw a hard and fast line between two subjects which from their very nature are bound up together. We would advise students to make every use possible of the algebraic method of solving questions in Arithmetic.

133. Every one will admit the truth of the following axioms:—

1. If equals be added to equals, the results are equal.
2. " taken from equals, " "
3. " multiplied by equals, " "
4. " divided by equals, " "

134. In algebra letters of the alphabet are used to represent figures and quantities, which, when known, may be substituted for these letters. Two letters written side by side, without any sign between them, indicate that their values are to be multiplied

together, e.g., xy implies that the value assigned to x is to be multiplied by the value assigned to y; xx is usually written x^2, the 2 indicating the number of x's to be multiplied together. So also, if a figure be written alongside a letter; thus, $3x$ indicates that the value assigned to x is to be multiplied by 3. Again, $x(y+z)$ indicates that the value assigned to x is to be multiplied by the sum of the values assigned to y and z. In all other cases the process to be undergone is indicated in the same way as in Arithmetic.

Now take such a statement as x is equal to y. This may be written

$$x=y$$

and means that whatever value you assign to y, the same value must be assigned to x. Such a statement is termed an equation. It is evident, then, that if y be known to be equal to 45, it follows that $x=45$. According to the above axioms we have—

$$\begin{aligned} x &= 45 \\ x+4 &= 45+4 = 49 \quad (\text{axiom } 1) \\ x-6 &= 45-6 = 39 \quad (\ \text{,,} \quad 2) \\ 3x &= 3 \text{ times } 45 = 135 \quad (\ \text{,,} \quad 3) \\ \frac{x}{5} &= \frac{45}{5} = 9 \quad (\ \text{,,} \quad 4) \end{aligned}$$

135. The following are typical examples of equations:—

(i.) $x=\frac{14\times 15}{100}$

by simplifying we get—

$x=\frac{210}{100}=2\cdot1.$

(ii.) $5x=\frac{16\times 10}{8}$

then $5x\div5=\frac{16\times10}{8}\div5$

i.e. $x=\frac{\overset{2}{16}\times\overset{2}{10}}{8\times5}=4.$

(iii.) $\frac{x}{5}=6$

$\frac{x}{5}\times5=6\times5$

i.e. $x=30.$

(iv.) $\frac{10}{x}=5$

$\frac{10}{x}\times x=5\times x$

$10=5x$

$\frac{10}{5}=\frac{5x}{5}$

$2=x.$

(v.) $\frac{24}{5x}=4$

$\frac{24}{5x}\times5x=4\times5x$

$24=20x$

i.e. $1\cdot2=x.$

(vi.) $\frac{3}{20}$ of $x=15$

$x=\frac{15\times20}{3}=100.$

136. Examples—

(i.) If $a=3$, $b=4$, and $c=5$, find the value of x, given $x=\frac{abc}{10}$.

Substituting the given values of a, b, and c, we get—

$$x=\frac{3\times4\times5^*}{10}=6.$$

(ii.) If $a=4$, $b=6$, and $c=8$, find the value of d given that $a=\frac{bcd}{36}$.

Substituting the values given, we get—

$$4=\frac{\overset{4}{\cancel{6}}\times\cancel{8}\times d}{\underset{\underset{3}{\cancel{6}}}{\cancel{36}}}$$

$$\therefore\ 12=4d$$

$$3=d.$$

(iii.) If $m=3$, $n=4$, $r=6$, find the value of y, given that

$$\frac{m}{r}=\frac{n}{y}$$

Substituting we get—

$$\frac{3}{6}=\frac{4}{y}$$

$$\frac{3y}{6}=4\ (\S\ 135,\ \text{iv.})$$

$$3y=24$$

$$y=8.$$

(iv.) If $F=70$, find the value of C, given that

$$\frac{F-32}{180}=\frac{C}{100}$$

Substituting we get

$$\frac{70-32}{180}=\frac{C}{100}$$

$$\therefore\ \frac{38}{180}=\frac{C}{100}$$

$$\frac{\overset{19}{\cancel{38}}\times10\cancel{0}}{\underset{9}{\cancel{18}\cancel{0}}}=C$$

$$21\cdot1=C.$$

(v.) If $A=330$, $R=2$, and $Y=5$, find the value of P, given that

$$A=P\left(1+\frac{RY}{100}\right)$$

Substituting we get—

$$330=P\left(1+\frac{2\times5}{100}\right)$$

$$330=P\left(1\tfrac{1}{10}\right)$$

$$330\times10=P\times11$$

$$\frac{\overset{30}{\cancel{330}}\times10}{\cancel{11}}=P$$

$$300=P.$$

* Care must be taken not to write thus: 345.

Examples 44.

1. Find the value of x in the following equations:—

(i.) $\frac{4x}{5}=16$

(ii.) $\frac{13}{x}=26$

(iii.) $\frac{24+x}{x}=1$

(iv.) $\frac{x-7}{x}=\frac{1}{8}$

(v.) $\frac{5}{6}$ of $x=35$

(vi.) $\frac{2\cdot5}{100}$ of $x=72$

(vii.) $\frac{5}{17}$ of $\frac{4\frac{1}{2}}{100}$ of $x=$ £67, 10s.

2. If $a=3$, $b=4$, $c=5$, find the values of x, given

(i.) $3x=\frac{abc}{2}$ | (iii.) $\frac{abx}{c}=24$

(ii.) $\frac{x}{a}=c-b+1$ | (iv.) $\frac{x-15}{4}=a+b+c$

3. If x is equal to the product of a, b, and c divided by 100, find the value of x when $a=400$, $b=3\frac{1}{2}$, and $c=4$.

4. If I is equal to the product of P, R, and Y divided by 100, find the value of R when $I=30$, $P=300$, $Y=2$.

5. Find the value of y in the equation $x=y\left(1+\frac{ab}{100}\right)$ given that $x=424$, $a=3$, and $b=2$.

6. Find the value of x in £'s, given that $\frac{3}{100}$ of $\frac{5}{8}$ of $x=$ £30.

7. Find the value of y in tons and cwts, given that

$$\frac{5}{12} \text{ of } y \times \frac{8}{100} = 8 \text{ cwts. } 14 \text{ lbs.}$$

8. If $m=40$, $n=5$, $x=6$, find the value of y, given that $y=\frac{mnx}{15}$. Prove your result by substituting your answer for y and finding the value of n.

9. Given that $x=y+z$, and $z=\frac{aby}{100}$ find the value of x when $a=3\frac{1}{3}$, $b=3$, and $y=250$.

10. Given that $I=\frac{PRY}{100}$. Express (i.) P (ii.) R (iii.) Y in terms of the other letters. Taking $P=600$, $R=3$, and $Y=4$, find the values of I. By substituting your answer for I in the equations for P, R, and Y respectively prove their values, in turn, to be those given.

11. If $s=ut$, find the value of s when $u=5$ and $t=4\frac{1}{2}$; also of t when $s=6$ and $u=1\frac{1}{3}$.

12. Given that $v=V+at$, find the value of V when $v=16$, $a=2$, and $t=1$.

13. If $u=\frac{V+v}{2}$, find the value of u when $V=0$ and $v=6$.

14. Given that $s=Vt+\frac{1}{2}at^2$, find the value of s when $V=4$, $t=2$ and $a=6$.

15. Find the value of v if $v^2=V^2+2as$ and $V=3$, $a=2$, $s=4$.

16. If $s=V^2\div 2g$, find its value when $V=200$ and $g=32$.

17. If $T=mng\div(m+n)$ find the value of g when $T=24$, $m=4$, and $n=3$.

18. If $a=g(m-n)\div(m+n)$ find its value when $g=16$, $m=8$, and $n=4$.

19. If $\omega=\frac{2\pi}{T}$ find the value of T when $\omega=4$ and $\pi=3·1416$.

20. Given that $F=m\left(\frac{2\pi}{T}\right)^2 R$, find the value of F when $m=2$, $R=3$, $T=\frac{1}{3}$, and $\pi^2=9\frac{8}{9}$.

XVIII.—MONEY SYSTEMS.

137. It has been already pointed out (§ 108) that most countries have adopted the metric system of weights and measures, and the advantage of the system will now be apparent to the student. From the following it will be seen that these countries have also adopted a decimal system of coinage. In the case of money, there is a disadvantage in all countries adopting the same nomenclature and values for their coins, since it would be difficult to get all countries to maintain the same standard values. This disadvantage ought not to prevent the adoption and use of a decimal system. For this reason such a system has been suggested for Great Britain, in which the £ would still remain the standard. The system will be most easily understood from the following table:—

10 mils = 1 cent.
10 cents = 1 florin.
10 florins = 1 pound.

From the table it will be seen that the only new coins required would be the cent and the mil. The value of the cent would be 2·4 pence, and the mil 0·24 pence, or very nearly one farthing. The following coins, in addition to the sovereign and florin, could be retained without any change in value:—

Half-sovereign (called 5 florins).
Four shillings („ 2 „).
One shilling („ 5 cents).

A 2-cent piece (value 4·8 pence) would take the place of the sixpence,
and a 5-mil piece („ 1·2 „) „ „ „ penny.

138. Another scheme for the decimalisation of the British coinage has recently been suggested, and the following paragraph appeared in the newspapers in October 1901:—

THE DECIMAL SYSTEM.

Proposed Division of the Florin into a Hundred Cents.

A committee of the Associated Chambers of Commerce has decided to recommend that the Chambers should unite in urging on the

Government the compulsory adoption of the metrical system of weights and measures, leaving matters of detail to be considered later. The committee is also of opinion that a British decimal system of coinage must be based on retaining the sovereign, with the florin as a unit, divided into a hundred cents or farthings. The committee further recommend that there should be nickel coins of five and ten cents, and bronze coins of one, two, and four cents or farthings. The Chambers of Commerce of the United Kingdom are asked carefully to consider these proposals.

139. The conversion from the present system to the proposed decimal system, and *vice versâ*, are examples of the **Decimalisation of Money** (§§ 89-91a).

E.g. (§ 89) £14, 17*s.* $5\frac{1}{4}$*d.* = £14·871875
i.e. £14, 8 fl. 7 c. 1·875 mils
or £14, 8 fl. 7 c. 2 mils
And (§ 90) £7, 6 fl. 5 c. 3·125 mils = £7·653125
= £7, 13*s.* $0\frac{3}{4}$*d.*

140. The following are the principal coins in use in the chief commercial countries. As the exchange value (§ 259) is not constant, the par value is given, or, in the case of silver standards, an approximate value.

141. DECIMAL SYSTEMS.

France.

100 centimes = 1 franc.

The value of the franc in British money varies from 25 to 27·45 francs per £. The par value is 1 franc = 9·51*d.* or

25·22 francs = £1.

The centime and franc are used also in **Belgium** and **Switzerland.** Coins with the same values but different names are used in **Italy** (100 centesimi = 1 lira Italiana); **Greece** (100 lepta = 1 drachme); **Roumania** (100 bans = 1 leu or lei nuovi); **Servia** (100 paras = 1 dinar); **Bulgaria** (100 stotinkis = 1 leva).

Germany.

100 pfennige = 1 reichsmark.

The par value of the mark is 11·745*d.* or

20·43 marks = £1.

Denmark.

100 ore = 1 krone.

The par value of the krone is 13·2*d.* or

18·16 krones = £1

The ore and krone are used also in **Norway** and **Sweden.**

Austria.

100 kreutzer = 1 florin.

The par value of the Austrian kreutzer is 9·985*d.* or

24·04 kreutzer = £1

Russia.

100 copeks = 1 rouble.

The par value of the rouble is 25·37*d.* or

9·46 roubles = £1

Spain.

100 centesimos = 1 peseta.
5 pesetas = 1 duro peso (piastre).

The English Exchange is quoted in pence per duro, the par value of which is 47·5785*d.* or

25·22 pesetas = £1.

Portugal.

1000 reis = 1 milreis.

The par value of the milreis is 53·28*d.* or

4·5 milreis = £1.

Netherlands.

100 cents = 1 florin.

The par value of the florin is 19·82*d.* or

12·11 florins = £1.

Turkey.

100 aspres = 1 piastre.
100 piastres = 1 pound.

The par value of the Turkish pound is 18*s.* 0·8*d.* or

110·7 Turkish piastres = £1.

America.

100 cents = 1 dollar.

The par value of the dollar is 49·3*d.* or

4·867 dollars = £1.

142. OTHER MONEY SYSTEMS.

India.

12 pie	=	1 anna
16 annas	=	1 rupee
100,000 rupees	=	1 lac

The value of the rupee is at present (October 1912) about 1*s.* 4*d.* The Indian coinage is being remodelled to one with a gold standard.

China.

1000 cash = 1 tael (silver).

The tael is not a coin, but a weight of about 580 grains (Troy) = about 3*s.* 4*d.*

Japan.

100 sen = 1 yen.

The yen varies from 2*s.* $0\frac{3}{8}$*d.* to 2*s.* $0\frac{7}{8}$*d.*, the par value being 2*s.* $0\frac{5}{8}$*d.*

143. FOREIGN WEIGHTS.

It has been stated (§ 108) that all countries, with a few exceptions, have adopted the metric system of weights and measures. In addition to the English-speaking nations, the chief exception is Russia.

Russia.

MEASURES OF WEIGHT.

96 zolotnik	=	1 funt
40 funts	=	1 pood
1 pood	=	36·113 lbs. (avoir.)

MEASURES OF LENGTH.

16 vershok	=	1 arshin
1500 arshin	=	1 verst
1 arshin	=	2 ft. 4 in.

In most Asiatic countries the measures are in course of change, and several systems are in use in each. The general tendency is to adopt the metric system.

144 Methods of Exchange:—

Nothing is so essential to successful foreign trading as the proper understanding of the different systems, and of the methods of expressing one set in terms of any other. When quotations for goods are offered, they ought to be expressed in the units of the country to which they are offered, not in terms of the unit of

the country applying for the order. British firms are sadly lacking in this, and many orders are lost through quotations not being properly expressed.

145. The change from one system to another can very speedily be accomplished by means of ordinary reduction (see §§ 85–88).

Examples *—

(i.) *Express £12, 10s. in francs, given £1 = 25·2 francs.*

TABLE.

£		Francs
1	=	25·2

$\therefore$ £12, 10s. $= 12·5 \times \frac{25·2}{1} = 315$ **francs.**

(ii.) *Express £1, 6s. 8d. in francs (1 franc = 9½d.).*

Here pence acts as the bridge between £'s and francs.

£		d		Franc
1	=	240		
		9·5	=	1

$\therefore$ £1, 6s. 8d. $= 1\frac{1}{3} \times \frac{240}{1} \times \frac{1}{9·5}$

$= \frac{4}{3} \times \frac{240}{1} \times \frac{1}{9·5}$

= **33·68 francs**

or **33 frs. 68 c.**

(iii.) *Express 425 francs 60 centimes in £ s. d., taking 25·1 francs = £1.*

TABLE.

Francs		£
25·1	=	1

$\therefore$ 425 *f.* 60 *c.* $= \frac{425·6 \times 1}{25·1}$

$= 16·956 =$ **£16, 19s. 1½d.**

20

19·120

12

1·44

(iv.) *Express 850 dollars in francs, taking the dollar as equivalent to 4s. 2d., and 25·2 francs = £1.*

TABLE.

Dollars		Pence		£		Francs
1	=	50				
		240	=	1		
				1	=	25·2

$\therefore$ 850 dollars $= 850 \times \frac{50}{1} \times \frac{1}{240} \times \frac{25·2}{1}$ (cancelled: 240 → 2, 25·2 → 2·1)

= **4462·5 frs. or 4462 frs. 50 c.**

* In these examples the names are omitted in the working parts, as in every case they would all cancel out except the last one, which belongs to the answer. They may, however, be put in at first as a safeguard.

146. Sometimes the exchange has to be calculated *viâ* the coinage of several intermediate countries.

Example—

(v.) *If £1 = 24·5 francs; 24 American dollars = £5; 1·25 francs = 1 rouble; 26 Austrian florins = 10 roubles; 12 marks (in Hamburg) = 8·3 Austrian florins, find how many dollars are equal to 1000 marks.*

TABLE.

Marks		Florins		Roubles		Francs		£		Dollars
12	=	8·3								
		26	=	10						
				1	=	1·25				
						24·5	=	1		
								5	=	24.

$$\therefore 1000 \text{ marks} = 1000 \times \frac{8\cdot3}{12} \times \frac{10}{26} \times \frac{1\cdot25}{1} \times \frac{1}{24\cdot5} \times \frac{24}{5}.$$

$$= 65\cdot15 \text{ dollars.}$$

147. The same principle may be applied to weights and measures. The metric system, however, is so generally used on the Continent that the exchange is greatly simplified.

(vi.) *How many metres are equivalent to* 50 *yds. Take the metre* = 39·37 *ins.*

TABLE.

Yds.		Ins.		Metre
1	=	36		
		39·37	=	1

$$\therefore 50 \text{ yds.} = 50 \times \frac{36}{1} \times \frac{1}{39\cdot37} = 45\cdot72 \text{ metres.}$$

(vii.) *Express* 14·625 *kilos. in lbs. oz., given* 1 *kilo.* = 2·2 *lbs.*

TABLE.

Kilo.		Lbs.
1	=	2·2

∴ 14·625 kilos. = 14·625 × 2·2
= 32·175 = 32 lbs. 2 oz. 13 drs.

```
 16
2·800
 16
12·8
```

After working out several examples the writing out of the tables may be omitted.

148. The following questions involve reduction :—

(i.) *Express 6s. 8d. a yard as francs per metre to two places of decimals.*

In this example we wish to express a metre in francs—that is, to obtain the equivalent of a metre in francs (see § 88).

TABLE.

Metre.	Inches.	Yard.	Pence.	£.	Francs.
1 =	39·37				
	36 =	1			
		1 =	80		
			240 =	1	
				1	25·2.

$$\therefore 1 \text{ metre} = 1 \times \frac{39·37}{1} \times \frac{1}{36} \times \frac{80}{1} \times \frac{1}{240} \times \frac{25·2}{1}$$

$$= \mathbf{9·19 \text{ francs.}}$$

(ii.) *A piece of land is rated in Germany @ 104 marks per hectare. Express this in £s per acre.*

In this example we wish to obtain the equivalent of 1 acre in £s.

TABLE.

Acres.	Hectares.	Marks.	£.
2·47 =	1		
	1 =	104	
		20·35 =	1.

$$1 \text{ acre} = 1 \times \frac{1}{2·47} \times \frac{104}{1} \times \frac{1}{20·35}$$

$$= \mathbf{£2·069 = £2,\ 1s.\ 4\tfrac{1}{2}d.}$$

(iii.) *Find a multiplier to convert francs per metre into shillings per yard.*

In this case take a cost of 1 franc for 1 metre and express it as shillings per yard, *i.e.* find the equivalent of 1 yard in shillings.

TABLE.

Yard.	Inches.	Metre.	Francs.	£.	Shillings.
1 =	36				
	39·37 =	1			
		1 =	1		
			25·2 =	1	
				1 =	20

$$\therefore 1 \text{ yard} = 1 \times \frac{36}{1} \times \frac{1}{39·37} \times \frac{1}{1} \times \frac{1}{25·2} \times \frac{20}{1}.$$

$$= \mathbf{0·7257 \text{ shilling.}}$$

0·7257 is the required multiplier; hence, given any number of francs per metre, multiply by 0·7257, and the answer is the corresponding number of shillings per yard.

149. Tables similar to those of §§ 124–125 for conversions from one system of money to another may be drawn out. Below we give a table for the conversion of pounds, shillings, and pence into francs—the rate of exchange being 25·2817 francs for £1.

Pence.	Francs.	Shillings.	Francs.	£	Francs.
1	0·1053	1	1·2641	1	25·2817
2	0·2107	2	2·5282	2	50·5634
3	0·3160	3	3·7923	3	75·8451
4	0·4214	4	5·0563	4	101·1268
5	0·5267	5	6·3204	5	126·4085
6	0·6320	6	7·5845	6	151·6902
7	0·7374	7	8·8486	7	176·9719
8	0·8427	8	10·1127	8	202·2536
9	0·9481	9	11·3768	9	227·5353

Table of Francs, at par value, to £s (25·22 *francs per £*).

Francs.	£
1	039651
2	·079302
3	·118953
4	·158604
5	·198255
6	·237906
7	·277557
8	·317209
9	·3[illegible]6860

Table of Marks, at par value, to £s (1 *mark*=11·745 pence).

Marks.	£
1	0·0489375
2	0·0978750
3	0·1468125
4	0·1957500
5	0·2446875
6	0·2936250
7	0·3425625
8	0·3915000
9	0·4404375

Such tables may be used to simplify reduction.

Example—

Find a multiplier to convert marks per kilogramme into shillings per lb.

$$1 \text{ mark per kg.} = \frac{1 \text{ mark}}{1 \text{ kg.}} = \frac{0{\cdot}0489375 \times 20 \text{ shillings}}{2{\cdot}2046 \text{ lbs.}}$$

$$= 0{\cdot}4439 \text{ shillings per lb.}$$

See also § 126.

150. In offices where any of the non-decimal systems are in constant use, it is often found convenient to draw out tables showing the decimal values of the chief unit. Below we give table of Annas, Pie, in decimals of the Rupee—Bengal currency.

Annas.	Pie.	Decimal of Rupee.	Annas.	Pie.	Decimal of Rupee	Annas.	Pie.	Decimal of Rupee	Annas	Pie.	Decimal of Rupee.
	1	0052	4	1	2552	8	1	5052	12	1	7552
	2	0104		2	2604		2	5104		2	7604
	3	0156		3	2656		3	5156		3	7656
	4	0208		4	2708		4	5208		4	7708
	5	026		5	276		5	526		5	776
	6	03125		6	28125		6	53125		6	78125
	7	0365		7	2865		7	5365		7	7865
	8	0417		8	2917		8	5417		8	7917
	9	0469		9	2969		9	5469		9	7969
	10	0521		10	3021		10	5521		10	8021
	11	0573		11	3073		11	5573		11	8073
1	—	0625	5	—	3125	9	—	5625	13	—	8125
	1	0677		1	3177		1	5677		1	8177
	2	0729		2	3229		2	5729		2	8229
	3	0781		3	3281		3	5781		3	8281
	4	0833		4	3333		4	5833		4	8333
	5	0885		5	3385		5	5885		5	8385
	6	09375		6	34375		6	59375		6	84375
	7	099		7	349		7	599		7	849
	8	1042		8	3542		8	6042		8	8542
	9	1094		9	3594		9	6094		9	8594
	10	1146		10	3646		10	6146		10	8646
	11	1198		11	3698		11	6198		11	8698
2	—	125	6	—	375	10	—	625	14	—	875
	1	1302		1	3802		1	6302		1	8802
	2	1354		2	3854		2	6354		2	8854
	3	1406		3	3906		3	6406		3	8906
	4	1458		4	3958		4	6458		4	8958
	5	151		5	401		5	651		5	901
	6	15625		6	40625		6	65625		6	90625
	7	1615		7	4115		7	6615		7	9115
	8	1667		8	4167		8	6667		8	9167
	9	1719		9	4219		9	6719		9	9219
	10	1771		10	4274		10	6771		10	9271
	11	1823		11	4323		11	6823		11	9323
3	—	1875	7	—	4375	11	—	6875	15	—	9375
	1	1927		1	4427		1	6927		1	9427
	2	1979		2	4479		2	6979		2	9479
	3	2031		3	4531		3	7031		3	9531
	4	2083		4	4583		4	7083		4	9583
	5	2135		5	4631		5	7135		5	9635
	6	21875		6	46875		6	71875		6	96875
	7	224		7	474		7	724		7	974
	8	2292		8	4792		8	7292		8	9792
	9	2344		9	4844		9	7344		9	9844
	10	2396		10	4896		10	7396		10	9896
	11	2448		11	4948		11	7448		11	9948
4	—	25	8	—	5	12	—	75	16	—	

Examples 45.

1. Using the table of § 149, find the equivalents in French money of £10, 7*s.* 6*d.*; £83, 14*s.* 7*d.*; £59, 17*s.* 10*d.*; £733, 13*s.* 3*d.*; £85, 6*s.* 4*d.*; £55, 16*s.* 11*d.*

2. Convert the following into the proposed decimal system of money: £35, 16*s.* 4*d.*; £424, 13*s.* 6*d.*; £9, 17*s.* 6½*d.*; £5, 6*s.* 10¼*d.*

3. Convert the following into the present system: £6, 7 f. 3 c. 5 m.; £83, 4 c. 7 m.; 16 f. 3 c.; 325 f. 5 m.

Express—

4. £5, 15*s.* in francs, given £1 = 25·18 francs.
5. £17, 7*s.* 6*d.* in francs, given £1 = 25·24 francs.
6. 63 francs 50 centimes in £ *s.* *d.*, given £1 = 25·22 francs.
7. 319 francs in £ *s.* *d.*, given 1 franc = 9½*d.*
8. £12, 14*s.* 6*d.* in marks, given £1 = 20·4 marks.
9. 2000 marks in £ *s.* *d.*, given £1 = 20·37 marks.
10. £5, 16*s.* in krones, given £1 = 18·2 krones.
11. £44, 7*s.* 6*d.* in Austrian florins, given £1 = 12·1 florins.
12. 40 roubles in francs, given £1 = 6·3 roubles, and £1 = 25·31 francs.

13. A man in Lisbon has a bank draft for £20. If he is paid in Portuguese money at the par value, what will he receive?

14. A man leaves a legacy of 4060 dollars. What is the value in British money if £1 = 4·86 dollars?

15. If £1 = 25·83 francs and 83 francs = 85 lire, find how many lire should be paid for £15.

16. The exchange between London and Paris is 25·30 francs per £; between Berlin and Paris, 0·82 marks for 1 franc; between Berlin and London, 20·10 marks per £. How much per cent. would a merchant gain or lose by remitting money to Berlin through Paris, instead of direct?

17. A person contracts to do a certain work for £10,000, and agrees to receive payment in francs at the rate of 25 francs per £1. If the rate of exchange is 25·6, how much does he lose by the agreement? Answer in £ *s.* *d.*

18. If £23 = 469 marks and 335 francs = £13, how many marks are equivalent to 115 francs?

19. If American dollars are worth 4*s.* 7½*d.* each and £1 is equivalent to 25·4 francs, find the value of 1000 dollars in francs and centimes.

20. Using the table of § 150, convert the following into decimal of rupees, and find their value when a rupee is equal to 15·96875*d.*

(*a*) 76 rupees 10 annas 5 pie. (*c*) 34 rupees 15 annas 7 pie
(*b*) 456 " 5 " 11 " (*d*) 145 " 7 " 3 "

and taking a rupee as equal to 1*s.* 4*d.*

(*e*) 6 rupees 5 annas 9 pie. (*g*) 25 rupees 4 annas 3 pie.
(*f*) 10 " 8 " 6 " (*h*) 30 " 12 " 9 "

21. If silver is sold at 41⅞*d.* per oz., what is the value of 1000 taels?

22. What is the rate of exchange if a person receives 300 francs 75 centimes for £11, 16*s.*?

23. What is the value of a yen in £ *s. d.*, if a person owing 1500 yens can pay his debt with 1425 dollars, when 4·87 dollars = £1 ?

24. If £5 = 127 francs and 6 francs = 1 milreis, how many milreis are equal to £1 ?

25. A person leaving for France changed £20 of his money into French money at the rate of 25 francs per £. He then travelled to Germany and changed his French money (195 francs) into marks at the rate of 13 francs for 10 marks. What did he lose if the rates of exchange were £1 = 25·32 francs or 20·54 marks ? Answer in £ *s. d.*

Examples 46.

Using the values given in §§ 112–116, 141–142, find a multiplier to convert—

1. Shillings per yard into francs per metre.
2. Pounds per mile into marks per kilometre.
3. Dollars per yard into marks per metre.
4. Francs per kilogramme into shillings per lb.
5. Pence per ounce (Avoir.) into centimes per gramme.
6. Francs per kilogramme into dollars per cwt.
7. Marks per metre into pounds per yard.
8. Roubles per kilogramme into pounds per lb.
9. Lire per are into dollars per acre.
10. Pounds per square yard into roubles per are.
11. Shillings per mile into centimes per kilometre.
12. Francs per litre into shillings per gallon.
13. Marks per cubic metre into pounds per cubic yard.

Examples 47.

1. A kilogramme weighs 2·20462 lbs. avoirdupois. Express the weight of a gramme in grains and decimals of a grain.

2. A merchant buys in Paris velvet at 6 francs the metre. At what price per yard must he sell it in London so as to make at least 10 per cent. profit on his outlay ?

3. Find how many feet and how many metres there are in 0·581257 of a mile, having given 1 metre = 1·093638 yards.

4. A kilometre being 1093·638 yards, find to four places of decimals how many kilometres there are in 100 English miles.

5. Given that a gallon equals 277·274 cubic inches and that a litre equals 1·7608 pints, find to the nearest hundredth part the number of litres in a cubic foot.

6. Express (to two decimals) 7280 square metres in square yards. 1 metre = 3·28092 feet.

7. Assuming that "a pint of pure water weighs a pound and a quarter," and that a gallon contains $277\frac{1}{4}$ cubic inches, find how much

the weight of a cubic foot of water differs from 1000 ozs., and express the latter quantity in litres and in grammes. 1 litre = 1·76 pints; 1 kilogramme = 2·2 lbs.

8. If the rent of land in France be 125 francs per hectare of 2·47 acres, find the rent per acre in English money, assuming £1 to be worth 25 francs 20 cents.

9. A metre being equal to 39·371 inches and a franc being equal to 9·38 pence, what is the value in English money of a yard of silk worth $7\frac{1}{2}$ francs a metre?

10. A metre being equal to 39·37 inches and a gramme being equal to 15·43 grains, find the weight in grammes of a cubic metre of air when 100 cubic inches of air weigh 31 grains.

11. Find to the nearest penny the value of 900 kilogrammes of a material which costs £25, 14*s.* 6*d.* per ton. 1 kilogramme = 2·2046 lbs.

12. If a 4-lb. loaf be worth 6*d.*, find to the nearest centime the value of a loaf of 2 kilogrammes. 1 kilogramme = 2·205 lbs., and £1 = 25 francs 30 centimes.

13. Find how many grammes of a substance worth 12·8 francs per kilogramme should be given in exchange for $11\frac{3}{4}$ lbs. of a substance worth 5*s.* $10\frac{1}{2}$*d.* per lb. 1 franc = $9\frac{1}{2}$*d.*; 1 kilogramme = 2·205 lbs.

14. If the average cost for the construction of railways in Russia in 1882 was 62,500 roubles per verst, what was the cost in English money per mile? 1 verst = 3500 ft.; and 6·37 roubles = £1.

15. In 1896 the northern ports of Russia imported 10,571,290 poods of coal and culm. Express this quantity in tons, correct to 4 significant figures, having given that 1 Russian pood = 36·11276 lbs. avoirdupois.

XIX.—PROPORTION.

151. The relation which one number bears to another in respect to magnitude is termed **ratio.** Thus, the relation which 3 bears to 4 is called the ratio of 3 to 4, and may be written $\frac{3}{4}$ or $3 \div 4$, or $3 : 4$. When two ratios, $\frac{3}{4}$ and $\frac{6}{8}$, are said to be equal, a proportion exists between them. This equality of ratios is one of the fundamental principles underlying much of our reasoning in arithmetic.

152. Taking the two ratios above we complete the proportion by stating $\frac{3}{4} = \frac{6}{8}$, or $3 : 4 = 6 : 8$.

Each number in the above constitutes a **term,** so that there are four terms, viz.: 3, 4, 6, and 8. The 3 and the 8 being the outside terms are called the **extremes,** and the other terms, the 4 and 6, are called the **means.** It will be observed from the above, and from all other proportions set down as the above, that the

product of the means equals the product of the extremes, *i.e.* $4 \times 6 = 3 \times 8$. From this we have an easy method of determining any one number of a proportion when the other three are given, *e.g.* if the 6 were required. Putting x for the 6 we get—

$$4 \times x = 3 \times 8$$
$$x = \frac{3 \times 8}{4} = 6. \quad (\S\ 135.)$$

Questions involving proportion may be worked out on this principle.

Example—

If 34 books cost £1, 5s., what will 51 books cost?

It is evident that 34 books bear the same relation to £1, 5s. as 51 books bear to other cost, say x.

Thus, we have $\frac{34 \text{ books}}{£1,\ 5s.} = \frac{51 \text{ books}}{x}$

From which $x = \frac{£1,\ 5s. \times \overset{3}{\cancel{51}}}{\underset{2}{\cancel{34}}} = £1,\ 17s.\ 6d.$

153. More satisfactory results, however, are obtained by using what is termed the **Unitary Method.** Take the above example.

Fully reasoned out we might set it down thus:—

If 34 books cost £1, 5*s.*

then 1 book will cost £1, 5*s.* ÷ 34, or $\frac{£1,\ 5s.}{34}$.

∴ 51 books " $\frac{£1,\ 5s.}{\underset{2}{\cancel{34}}} \times \overset{3}{\cancel{51}} = £1,\ 17s.\ 6d.$

154. The above may be shortened by the omission of the middle line. Since the numbers of the books and the amounts of the prices vary **directly,** *i.e.* the more books bought the more money must be paid, we know that 51 books will cost more than 34, and hence the price of 51 will bear the same relation to the given sum as 51 bears to 34.

155. Take, for example, the number 20 and suppose we have also the numbers 4 and 5. If we require to multiply the 20 by one of these numbers and divide by the other in order to obtain an answer greater than the 20, it is obvious that we must multiply by the 5 and divide by the 4; conversely, if the answer is to be less than the 20 we must multiply by the 4 and divide by the 5.

156. Applying this to the above question we may write out the solution thus :—

* If 34 books cost £1, 5s. then 51 „ ? $= \frac{£1, 5s. \times 51}{34}$ = £1, 17s 6d.	The answer required will be greater than £1, 5s., and will bear the same relation to it as 51 bears to 34. ∴ to obtain it we multiply by 51 and divide by 34.

* See note on cancelling, p. 15.

157. Example—

(ii.) *If 16 men can complete a piece of work in 40 days, how long would 20 men take to complete the same piece of work ?*

In this case the greater the number of men employed the shorter the time taken. The number of men and the time taken are said to vary **inversely**, or the one varies as the *reciprocal* of the other. Using the reasoning of § 155, the solution of this is—

If 16 men complete the work in 40 days then 20 „ „ „ ? „ $\frac{40 \times 16}{20} = 32$ days.	The required result is fewer days, and ∴ we multiply by 16 and divide by 20.

158. Should fractional quantities be introduced proceed as directed in §§ 27–29 It is usual to convert the mixed numbers into improper fractions, or directly into decimals.

Example—

(iii.) *If $34\frac{1}{2}$ lbs. are carried 34 miles for a certain sum, how far ought $45\frac{1}{4}$ lbs. to be carried for the same amount ?*

If $34\frac{1}{2}$ lbs. are carried 34 miles
then $45\frac{1}{4}$ „ „ ? „

$$\frac{34 \times 34\frac{1}{2}}{45\frac{1}{4}} = \frac{34 \times 69}{2} \times \frac{4}{181} \text{ or } \frac{34 \times 34\cdot5}{45\cdot25}$$
$$= 25\cdot92 \text{ miles.}$$

Examples 48.

1. If magazines may be bought at the rate of 14 for 3s. 6d., find the cost of 8 dozen.

2. Coals are bought at the rate of 15s. 6d. per cart of 24 cwts. Find the price of 75 cwts. at this rate.

3. Three men working all day can plant a field in 10 days, but one of them, having other employment, can work only half time. How long will it take them to complete the work ?

4. If 12 days' labour is worth £1, 11s. 6d., find how much will pay the wages of 4 men who have worked 6, 9, $10\frac{1}{2}$, and 11 days respectively.

5. If $56\frac{1}{2}$ lbs. of a certain article cost £1, 10*s.* 7*d.*, find to the nearest penny the cost of 240 lbs.

6. The price of diamonds per carat varies as the square of their weight. If a diamond of 3 carats is worth £75, what is the value of a diamond of 2 carats?

7. Assuming that an express train runs 40 miles an hour, and an ordinary train 30 miles an hour, and that the express fare is $\frac{1}{4}d.$ a mile more than the ordinary fare, find how much a man's time is worth if it cost him the same to travel by the one as by the other.

8. If 38·64 kilogrammes of a certain substance cost 20·5 francs, find to the nearest centime the cost of 7846 grammes.

9. If 3·85 kilometres of wire weigh 56·4 kilogrammes, find to the nearest gramme the weight of 69 metres of it.

10. If 3 quintals of corn be required to feed 60 horses for a certain time, find in kilogrammes the weight of the corn required to feed 14 horses for the same time.

159. It is important at this stage that the student should clearly understand the difference between **abstract** and **concrete numbers**. If we divide 60 pence by 12 pence, the answer is 5. The 5 obtained is an abstract number, *i.e.* it has no reference to any particular set of units; the 60 pence and 12 pence are concrete numbers. The 5 only tells us that 12 pence are contained in 60 pence 5 times, or that we can take 5 sets of 12 out of 60. Now since ratio is merely a relation, the ratio of £6, 10*s.* to £19, 10*s.* cannot be expressed as so many pounds, shillings, and pence, but as £6, 10*s.* : £19, 10*s.*, or as $\frac{£6,\ 10s.}{£19,\ 10s.} = \frac{130s.}{390s.} = \frac{1}{3}$, *i.e.* as the number 1 is to the number 3.

160. It is evident from the last paragraph that (1) no ratio can exist between quantities differing in kind, *e.g.* between horses and men; and (2) when quantities of the same kind differ in name, both should be reduced to the same common name in order to obtain the ratio, *e.g.* £4, 7*s.* 6*d.* and £2, 3*s.* 1*d.* should both be expressed as pounds, as shillings, or as pence, but not as a mixture of all three.

It is sufficient in many cases to reduce such mixed quantities each to **one**, though not **the same** name; thus, one may be expressed entirely as pounds, while the other is expressed in shillings or as pence. When this method is adopted cancelling will take place between the names, as explained in § 26a, page 15, care being taken to write down all the different names in the process working. This is not essential where the quantities have been reduced to the same names.

As this process of reduction has to be applied very frequently, it is better to do it before inserting the terms into the proportion. The necessity for this does not apply to the term which corresponds to the unknown, as the result is always obtained in exactly the same form as this term; see example (v.). Reduction of this term is sometimes useful, but only as an aid to cancelling.

Examples—

(iv.) *If 19 articles cost £3, 3s. 4d., how many ought to be obtained for £4, 6s. 8d.?*

If 19 articles cost £$3\frac{1}{6}$ or If £$3\frac{1}{6}$ buy 19 articles
then ? ,, ,, £$4\frac{1}{3}$ then £$4\frac{1}{3}$ will buy ?

$$\frac{\cancel{19}\times 13\times \overset{2}{\cancel{6}}}{\cancel{19}\times \cancel{3}} = 26 \text{ articles.}$$

(v.) *If 4 tons 3 cwts. are carried a certain distance for £3, 7s. 8d., how much ought to be paid for the carriage of 1 ton 15 cwts. 64 lbs. for the same distance?*

4 tons 3 cwts. = 9296 lbs.
1 ton 15 cwts. 64 lbs. = 3984 lbs.
Hence, if 9296 lbs. cost £3, 7s. 8d. for carriage,
then 3984 lbs. cost ? for carriage.

$$\frac{\overset{9s.\ 8d.}{\cancel{£3,\ 7s.\ 8d.}}\times \overset{3}{\cancel{3984}}}{\underset{7}{\cancel{9296}}} = £1,\ 9s.$$

(vi.) *If 12 acres 2 roods cost £480, find the cost of 25 square poles.*

12 acres 2 roods = $12\frac{1}{2}$ acres.
Hence, if $12\frac{1}{2}$ acres cost £480
25 square poles ,, ?

$$\frac{£\overset{3}{\cancel{480}}\times \overset{2}{\cancel{25}}\ \cancel{\text{square poles}}}{\cancel{12\frac{1}{2}}\ \cancel{\text{acres}}\ \underset{\cancel{160}}{}} = £6$$

The cancelling must be carefully watched in cases like the following:—

(vii.) *If 5 men do a piece of work in 16 days, how long will 8 boys take to do it, 2 men being able to do as much as 3 boys.*

If 5 men do the work in 16 days
then 8 boys ,, ?

$$\frac{16 \text{ days}\times \overset{3}{5}\ \cancel{\text{men}}}{\underset{2}{8}\ \cancel{\text{boys}}} = 15 \text{ days.}$$

161. In many instances it will be found more convenient to decimalise the quantities to be changed, and make use of contractions when possible.

(viii.) *If 6 tons 5 cwts. cost* £3, 2*s.* 6*d.*, *how much ought to be obtained for* £15, 7*s.* 6*d.*?

If 6·25 tons cost £3·125

then ? „ £15·375

$$\frac{6{\cdot}25 \times 15{\cdot}375}{3{\cdot}125} = \mathbf{30{\cdot}75\ tons = 30\ tons\ 15\ cwts}$$

Examples 49.

1. If 24 men can perform a piece of work in 105 days, in what time would 60 men do it?

2. A man travelling at 18 miles a day can perform a journey in 25 days, at what rate must he travel to complete it in $22\frac{1}{2}$ days?

3. Find the cost of 31 cwts. 3 qrs. when 127 cwts. cost £6, 11*s.*

4. If 2 ac. 3 roods of turnips are sold for £25, 6*s.*, what should I pay for 30 ac. 15 poles?

5. What is the height of a tree whose shadow is 234 ft. 8 ins. long, when a pole of 8 ft. casts a shadow of 18 ft. 4 ins.?

6. How many lbs. of plums @ 7*d.* per lb. are equal in value to 21 yards of cloth @ $7\frac{1}{2}d.$ per yard?

7. If 16 cwts. 2 qrs. 12 lbs. of sugar costs £30, 3*s.* 9*d.*, what will be the cost of 18 cwts. 3 qrs. 8 lbs.?

8. The rates of a railway train and a coach are as 7 : 2. If the train travels 56 miles per hour, how long will it take the coach to go 398 miles?

9. If 5 men can do a piece of work in 8 days, how long will it take 7 boys to do it, calculating that 3 men are equal to 8 boys?

10. Find the price of $\frac{4}{7}$ of a quarter, when $\frac{5}{8}$ tons cost $£6\frac{2}{3}$.

11. A family of 14 persons has provisions for 38 days. After 26 days 4 more persons arrive. How long will the food last?

12. If 21 horses cost £490, find the value of 18 cows, taking 5 horses as equal in value to 8 cows.

13. If 3 men and 2 boys do a piece of work in 24 days, how long will it take 2 men and 3 boys to do it, the work of 2 men being equal to that of 3 boys?

Examples 49a.

1. Find the ratio which $\frac{2}{3}$ of £27, 1*s.* 5$\frac{3}{4}$*d.* bears to 0·6 of £42, 10*s.* 10$\frac{3}{4}$*d.*

2. Express in the simplest form (*a*) the ratio of 2$\frac{1}{4}$ to 7$\frac{1}{5}$; (*b*) the ratio of $\frac{3}{4}$ of 53 cwts. 3 qrs. 3 lbs. to 0·4 of 65 cwts. 11 lbs.

3. A lump of gold weighing 6 lbs. 6 oz. 2 dwts. 22 grains, is valued at £298, 7*s.* 6*d.*; it is made into rings weighing 10 dwts. 2 grs. each. Find (1) the number of rings, (2) the value of each.

4. A coach going 9 miles an hour runs a certain distance in 11 hrs. 36 min., and a train does the same distance in 5 hrs. 13 min. 12 sec. What is the speed of the train per hour?

5. A pound of powder costs 3*s.*, and the charge for a gun is 2$\frac{1}{4}$ drams. How many shots will 6*s.* 9*d.* worth of powder furnish?

6. If 3$\frac{1}{5}$ lbs. cost 8$\frac{4}{7}$*s.*, what will 97$\frac{3}{10}$ lbs. cost?

7. A man pays £10 for the first 100 books, and so much per book for each additional book. He pays £12, 10*s.* for 150 books. How much should he pay for 200?

8. If a merchant be offered 40 yards of silk at 10*s.* per yard, which is worth only 9*s.* 6*d.*, how should he rate his tweed which is worth 3*s.* 2*d.* per yard? How many yards of it should he give in exchange for the silk?

9. A cyclist has to run 23 miles 5$\frac{1}{2}$ fur.; his machine has two wheels differing in circumference in the ratio of 10 : 15. If the smaller wheel measures 5 feet in circumference, find how many revolutions the wheels will respectively make in running the distance.

10. If the carriage of 5 tons 6 cwts. 56 lbs. for a certain distance is £20, 10*s.* 6*d.*, find to the nearest pound what weight will be carried the same distance for £18, 7*s.* 6*d.*

11. If the income of a school amounted to £1, 13*s.* 1$\frac{1}{2}$*d.* per week when the children paid 1$\frac{1}{2}$*d.* per week, what would it have amounted to if the fee had been raised to 2$\frac{3}{4}$*d.* and the attendance was reduced by 23?

12. Calculate the cost of a truck of coals weighing 4 tons 3 cwts. 21 lbs., if another weighing 2 tons 5 cwts. was sold for 37*s.* 6*d.*

13. If 16 journeymen earn £26, 8*s.* per week, how much will be required to pay the wages of 30 apprentices who each earn $\frac{2}{3}$ of a journeyman's pay?

14. Given that 60 iron bars weigh 3 cwts. and that 8 sheets of zinc weigh as much as 5 iron bars, find the weight of 92 sheets of zinc.

15. If 7 boys can do as much work as 10 girls, how long will it take 50 girls to do what 34 boys can do in 35 days?

16. If 16 men and 10 women earn £27, what will 40 men and 17 women earn, supposing 14 men earn as much as 25 women.

(Express 16 men and 10 women either as an equivalent number of men or women. Similarly with the other—they need not be the same. See § 166.)

162. Compound Proportion :—

When the ratio between two numbers is equal to the ratio compounded of the ratios of several pairs of numbers, the proportion is said to be **compound**. Compound proportion consists, therefore, of a series of simple proportion questions. It may, however, be conveniently reduced to one question, the several related terms being taken together and the answer modified to suit each in turn.

Example—*If* 16 *tons be carried* 27 *miles for* £1, 10*s*., *how far ought* 24 *tons to be carried for* £5.

Write out thus :—If 16 tons be carried 27 miles for £1, 10*s*.,
then 24 . . ? . . £5

$$\frac{27 \times 16 \times 5}{24 \times 1\frac{1}{2}}$$

$$= \frac{27 \times 16 \times 5 \times 2}{24 \times 3}$$

(cancelled: 27 to 9 to 3; 16 to 2; 24 to 3; 3 to 1)

$= 60$ miles.

Method—Dealing with the terms in tons, we find 24 tons would be carried a shorter distance for any sum than 16 would ; hence, multiply by 16 and divide by 24. Coming next to the money, any weight would be carried further for £5 than for £$1\frac{1}{2}$; hence, multiply by 5 and divide by $1\frac{1}{2}$.

Examples 50.*

1. If 9 men working eight hours a day can earn £36 in twenty days, how much will 19 men working ten hours a day earn in thirty days?

2. If 200 men can make an embankment 5 miles long in 25 days, how much overtime per day must 60 men work to finish an embankment 2 miles long in 32 days, 12 hours being a day's work?

3. After a certain number of men had been employed on a certain work for 12 days and had half finished it, their number was increased to 48, and half the remainder was completed in 4 days. How many men were employed at first?

4. If the carriage of $3\frac{1}{2}$ tons for a distance of 39 miles cost 14*s*. 7*d*., what will be the cost of the carriage of 25 tons for a distance of 156 miles at half the former rate?

5. 36 men were engaged to finish a piece of work in 2[illegible] days, but at the end of 15 days [illegible] of the work remained undone. How many additional men must be employed in order to finish it in the prescribed time?

6. A trench, 1000 yds. long, 5 ft. wide, and 3 ft. deep, takes 20 labourers 5 days of 8 hrs. each to dig. At the same rate of work, what is the length of a trench, [illegible] ft. wide and 4 ft. deep, 30 labourers could dig in 4 days of 9 hrs. each?

7. If 16 men, in 8 days of 10 hrs. each, build a wall 10 ft. high and 118·3 yds. long, how many days will a dozen men—of whom six work 8 hrs. per day and the rest 9 hrs.—take to build a wall 1065 ft. long, 4 ft. 3 ins. high?

* See page 15.

8. A contractor having engaged to make a railway $58\frac{1}{2}$ miles long in 40 weeks employed 2160 men upon the work. At the end of 13 weeks $19\frac{1}{2}$ miles were finished. How many men had he then to pay off so that the work might not be completed before the stipulated time?

9. If 30 men can perform a piece of work in 13 days of $8\frac{1}{2}$ hrs. each, how many men, working the same number of hours a day, can do one-third as much again in one-sixth of the time, three of the second set being equal to four of the first?

10. A garrison of 4500 men is provisioned for 15 weeks at the rate of 13 oz. per day per man. How many men must leave that the same provisions may last those who remain 27 weeks, at 10 oz. per day per man?

11. A colliery employs 120 men, working 8 hrs. a day for 5 days a week. How many men can it employ for 4 days a week, working 6 hrs. a day, if only half as much coal is required to be raised?

12. A contractor engaged to remove 4725 cub. yds. of earth in 75 days; he employed 60 men, but at the end of $52\frac{1}{2}$ days he finds only 2700 cub. yds. gone. Find the least number of extra men he must put on to complete the work within the specified time.

13. If the rent of a field containing 26 acs. 2 rds. 23 pls. be £50, 8*s.* 9*d.*, what would be the rent of another containing 17 acs. 3 rds. 2 pls., supposing 6 acs. of the latter are worth 7 acs. of the former?

14. If it takes 60 slabs 4 ft. long and $2\frac{1}{2}$ ft. broad to pave a certain courtyard, how many slabs 3 ft. long and $1\frac{1}{2}$ ft. broad would cover the same?

15. If 9 men can mow a square meadow in 4 days, in what time can 7 men mow a square meadow one-sixth as long again as the former?

16. Glycerine is 1·27 times as dense as water, and mercury is 13·596 times, and a pint of water weighs 20 oz. How many pints of glycerine would weigh as much as 3·76 pints of mercury? (See also § 88.)

17. A garrison of 1500 men has provisions for 18 days at the rate of $1\frac{1}{2}$ lbs. per man per day; 300 men leave and the rations are reduced to 20 oz. How long should the remainder hold out?

18. A contractor engages to make a road in 72 days, and employs 75 men. After one-fifth of the work is done, he finds that they have taken one-sixth of the time. How many men may he now dismiss and yet finish the road at the date required by his contract?

19. What will be the cost of the paper required to print 4000 copies of a book of 304 pages? The paper costs $3\frac{1}{2}$*d.* per lb., a sheet contains 16 pages, and a ream of 480 sheets weighs 65 lbs. (See also § 88.)

20. A man agrees to supply potatoes in return for bread. If he sends one stone and a half of potatoes every day for 20 days, how many loaves should he get in return on each of 15 days, the price of potatoes being 8*d.* per stone and the bread 4*d.* per 2 lb. loaf?

21. If the railway carriage for 3 tons 2 cwts. for a distance of 250 miles is 15s. 6d., what will be the charge for conveying 15 tons 10 cwts. for 600 miles, calculating half-rate for the last 100 miles?

22. Railway companies convey parties of 10 and upwards at a reduced rate of single fare and a quarter for the return journey, whereas parties of 50 get the double journey at single fare. If it costs £5, 10s. for a party of 10, what ought to be the charge for a party of 50?

23. If on a certain occasion it costs £4, 7s. 6d. to purchase 60 metal bars each 5 ft. 4 ins. long, what ought to be the length of 45 other bars which cost £5, 10s. when the metal had increased in value one-tenth?

24. If 30 horses and 296 sheep can be kept for $4\frac{1}{2}$ days for £75, 15s., what sum will keep 5 horses and 66 sheep for 8 days, if 5 horses eat as much as 84 sheep?

25. If 35 men earn £270 in 3 weeks, what will 42 women earn in 7 weeks, the wages of 8 men being equal to that of 15 women?

26. A contractor undertook to make 15 miles of roadway in 40 weeks. In 10 weeks, 3 miles were completed by 180 men working 8 hours a day; then the men agreed to work 1 hour a day overtime, and, some boys being engaged to assist them, the work was finished in the stipulated time. How many boys were employed, if the work of 3 boys was equal to that of 2 men?

27. A man on horseback and a motor car start together on a journey of 22 miles, the motor travelling uniformly. The horseman while on the footpath travels at the rate of 38 yards to 33 yards of the motor, but on the road at the rate of only 10 yards to 11 yards of the car. If they finish together, what distance of the journey did the horseman ride on the footpath?

XX.—PERCENTAGES.

163. Consider the meaning of the fraction $\frac{3}{4}$. While called three-fourths it indicates that a certain unit has been divided into four equal parts, and that three of those parts have been taken. Thus $\frac{3}{4}$ of £20 means that £20 is divided into fourths, and three of these taken; or better, 3 taken out of every four contained in £20 = £15.

164. Similarly, the fractions $\frac{1}{100}$, $\frac{2}{100}$, or $\frac{3}{100}$ of any number indicate 1, 2, or 3 out of every hundred contained in the number. The term **Per Cent.** is used to indicate such parts. Thus 3 per cent. (usually written 3%) of £600 indicates that 3 is given out of every 100 in the 600, hence 3% of 600 = $\frac{3}{100}$ of 600 = £18.

165. Similarly 15% of 9800 $= \frac{15}{100}$ of 9800 = 1470,

and 6% of 7835 $= \frac{6}{100}$ of 7835 = 470·10.

Note.—Never cancel out the 0's of the 100 in the denominator except with 0's in the numerator, as it is much easier to divide by 100 than by 50 or 25 (see § 25). Move the decimal point in the answer one place to the left for each 0 remaining in the denominator. When, however, the percentage is a simple fraction of 100, cancelling may be allowed, *e.g.*—

$$12\tfrac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \tfrac{1}{8}; \quad 8\tfrac{1}{3}\% = \frac{8\frac{1}{3}}{100} = \tfrac{1}{12}; \quad 33\tfrac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \tfrac{1}{3}, \text{ \&c.}$$

Even in many similar cases it is better to leave the 0's, *e.g.* 4%, while $\frac{4}{100} = \frac{1}{25}$ is better left as $\frac{4}{100}$, 20% $= \frac{20}{100} = \frac{1}{5}$ is better left as $\frac{2}{10}$.

166. With fractional percentages as $3\frac{3}{4}\%$, $4\frac{1}{2}\%$, $6\frac{1}{3}\%$, &c., write down as $\frac{3\frac{3}{4}}{100}$, $\frac{4\frac{1}{2}}{100}$, $\frac{6\frac{1}{3}}{100}$, and then simplify into $\frac{15}{400}$, $\frac{9}{200}$, $\frac{19}{300}$, or they may be written in the latter form at once.

Examples—

(i.) $4\frac{1}{2}\%$ of £642 $= \frac{4\frac{1}{2}}{100}$ of £642 $= \frac{9}{200}$ of £642 $= \frac{321}{100}$ ... =

£28·89
20
17·80
12
9·6

£28, 17s. 9·6d.

In many cases it will be found expedient to calculate the required percentage in two or more steps, using aliquot parts when possible.

(ii.) $7\frac{3}{4}\%$ of £850, 10s. 6d. (to nearest penny)

$(7\frac{3}{4} = 5 + 2\frac{1}{2} + \frac{1}{4})$

$\frac{5}{100}$ of £850·525 = 42·52625

$\frac{2\frac{1}{2}}{100}$,, $\left(\frac{1}{2} \text{ of } \frac{5}{100}\right)$ = 21·263125

$\frac{\frac{1}{4}}{100}$,, $\left(\frac{1}{10} \text{ of } \frac{2\frac{1}{2}}{100}\right)$ = 2·1263125

65·9156875

= **£65, 18s. 4d.**

167. The following are typical examples of Percentages:—

Examples—

(i.) *The capital of a company amounts to £60,000. If an increase of 15% is made on it, what will the new capital be?*

Old capital = £60,000

Increase = $\frac{15}{100}$ of 60,000 = 9,000

∴ New capital = **£69,000**

(ii.) *The output of a mine was 350 tons of coal per day, of which 6% was worthless. Calculate the amount of good coal.*

Total output = 350 tons.

Worthless $\frac{6}{100}$ of 350 = 21·0

Good coal **329 tons.**

(iii.) *The population of a town in 1891 was 65321. During the next ten years it increased almost 20%. Calculate the increase.*

Increase $\frac{20}{100}$ of 65321 = 13064·2

Ans. **13064.**

Examples 51.

1. Calculate 3 per cent. of £840, 12*s.* 6*d.* to the nearest penny.
2. ,, 15 ,, £945, 8*s.* 7*d.* ,, ,,
3. ,, 14 ,, 68432 cwts. to two places of decimals.
4. ,, 30 ,, 98476 people.
5. Increase £52, 10*s.* 6*d.* by 15% of itself.
6. Deduct $2\frac{1}{4}$ per cent. from £860, 10*s.*, correct to nearest penny.
7. ,, $12\frac{1}{2}$,, ,, 9476 cwts.
8. Find the value of $7\frac{1}{4}$% of £125, 4*s.* 10*d.*
9. Of 3960 gallons of milk 5 per cent. was useless. How much was left?
10. The annual output of a mine is 856240 tons, of which 4% is waste. Find the amount of waste.
11. £70,500 is invested in a concern, and, at a later period, 15% of this is withdrawn. How much remained?
12. A cask contains 80 gallons of spirits. 4% is withdrawn, then 4% of the remainder, and then 5% of the second remainder. How much now remains?
13. There are two schools, one containing 1300 and the other 1600 children. 10 per cent. of the former are generally absent, and $14\frac{1}{2}$ per cent. of the latter. Find the total number usually present in both.
14. A horse and cart cost together £65, 10*s.*; the cart costs $7\frac{1}{2}$ per cent. of this. Find the price of the horse.

15. If a man travels 360 miles in 12 days of 8 hrs. each, how many hours a day less need he travel if he increase his rate 20 per cent. in order to accomplish other 450 miles during the next 20 days?

16. The annual consumption of wine in a district is 8000 gallons, and the duty 10*s.* 5*d.* per gallon. If the duty were reduced 4 per cent. and the consumption increased $4\frac{1}{2}$ per cent., how would the revenue be affected?

17. The population of a country was 4,314,000 and increased 3·168 per cent. every year for 3 years. What was the population at the end of 3 years?

18. A man may buy 1650 metres of cloth at 3 francs per metre, but if he increase his order $33\frac{1}{3}$ per cent. he will get 5 per cent. reduction in price. Find the total cost of the increased order.

19. A vat contains 250 litres of wine. If 8 per cent. be drawn off and the vat filled up with water and again 8 per cent. drawn off, how much of the original wine is left?

168. It is often necessary to calculate what percentage one number is of another, *i.e.* we are to find out what fraction of 100 is represented by the relation between the given numbers. Thus, to find what per cent. 3 is of 4, we work out as follows:—

3 out of 4 is written as $\frac{3}{4}$, and the fraction of 100 is obtained in the usual way—$\frac{3}{4}$ of $100 = 75$

Hence 75 per cent. is the required percentage.

Examples—

(i.) *A farmer owned 60,000 sheep and sold 25,000. What percentage did he sell?*

He sold 25,000 out of 60,000

$$\therefore \text{Percentage} = \frac{25000}{60000} \text{ of } 100 = 41\tfrac{2}{3}.$$

(ii.) *The value of a mine 3 years ago was estimated @ £32,000, while now it is worth only £12,000. Calculate how much per cent. the decrease is.*

Original value = £32,000 $\therefore$ decrease = £20,000

$$\therefore \text{the percentage decrease} = \frac{20000}{32000} \text{ of } 100 = 62\tfrac{1}{2}.$$

(iii.) *A merchant allows his customers 2s. 6d. in the £ as discount for cash. What per cent. is this?*

$$\text{Percentage discount} = \frac{2s.\ 6d.}{£1} \text{ of } 100 = 12\tfrac{1}{2}.$$

(iv.) 12 *gallons of a 30 per cent solution of nitric acid are added to 14 gallons of a 40 per cent solution and the whole poured into 14 gallons of pure water. Find the percentage strength of the resulting solution.*

$\frac{30}{100}$ of 12	gallons	solution gives	3·6	gallons nitric acid.
$\frac{40}{100}$ of 14	,,	,,	5·6	,, ,,
14	,,	water ,,	0	,, ,,
40	,,	solution contains	9·2	,, ,,

$\therefore$ Percentage of solution $= \frac{9 \cdot 2}{40}$ of $100 = \underline{23}$

168a Questions, similar to the following, frequently occur in connection with calculations and experiments In dealing with such, it must be remembered that the percentage is calculated on the true (*i.e.*, correct) value.

Example.—

A student in practically determining the composition of a compound finds 90 grammes of one substance and 30 of another The correct values are 91 and 29 respectively. Find the percentage error in each case

First substance; error is 1 in 91.

Percentage error $= \frac{1}{91}$ of $100 = $ **1·099**

Second substance; error is 1 in 29

Percentage error $= \frac{1}{29}$ of $100 = $ **3·488.**

Examples 52.

1. What per cent. is 85 of 2125; 189 of 6300, $7\frac{1}{2}$ of 10500; 10*s.* 6*d.* of 3 guineas; 13*s.* 4*d* of £75?

2. What percentage of 1 ton is each of the following quantities.—(*a*) $1\frac{1}{2}$ cwts.; (*b*) 1 qr. 15 lbs. 12 oz.; (*c*) 2 lbs. $12\frac{4}{5}$ oz.?

*3. A person owned an estate of 2040 acres, of which 68 acres was moorland. What per cent. was this?

4. Out of a legacy of £15,300, a man got £5100 as his share. What per cent. was this?

5. An article that once cost 17*s.* 6*d.* is now sold for 15*s.* What percentage decrease is there in the price?

6. In a school of 760 pupils, 450 are girls. What per cent. are boys?

7. In a school of 740 children, 5 failed in reading, 16 in writing, and 24 in arithmetic. What is the percentage of passes in each?

8. Prove the following rule for finding any percentage of a given whole number of pounds sterling. "Multiply the number of pounds by twice the rate per cent.; set off one decimal place, and the result represents in shillings the percentage required." Exemplify the rule by making use of it to find $3\frac{1}{2}\%$ of £25, and $4\frac{1}{2}\%$ of £1247.

9. A manufacturer combines 3 gallons of a mixture which contains 15 per cent. of water, with 2 gallons of one containing 10 per cent. of water, and adds one gallon of water. Find the percentage of water in the resulting mixture.

10. A railway which used to pay 5 per cent. duty on all its passage traffic, has now to pay nothing on its third-class traffic, and only 2 per cent. on the remainder. If the whole amount of duty payable is now one-tenth of what it was before, find what per cent. the third-class traffic is of the whole.

11. One liquid contains $22\frac{1}{2}$ per cent. of water; another 27 per cent. A glass is filled 5 parts of the one liquid and 7 parts of the other. What percentage of water is in the glass?

12. On a farm of 850 hectares, 40 hectares are barren. What per cent. is fit for cultivation?

13. If a value of 53 is obtained in place of a correct value of 54, what is the percentage error?

14. If a value of 61·08 is found in place of a correct value of 60·04, what is the percentage error?

15. In taking the atomic weight of silver as 108 in place of the more correct value of 107·93, what is the percentage error?

16. A substance on analysis gives sodium 39·18 per cent., chlorine 60·70 per cent. The correct values are sodium 39·31 per cent., chlorine 60·59 per cent. Find the percentage error in each case.

17. A cubic foot of water is usually taken as weighing 62·5 lbs. (Avoir.). The true weight is 453590 grains. Find the percentage error.

18. For rough calculations a kilometre is taken as $\frac{5}{8}$ of a mile. The correct value is 0·62138 of a mile. Find the percentage error.

19. In a series of values an error of less than 0·02 per cent. is neglected. In which of the following cases does the error exceed that amount, and by how much: (*a*) 58·4 in place of 58·5; (*b*) 637 in place of 635; (*c*) 38·13 in place of 38·125?

20. The correct lengths of the sides of a rectangle are 5·427 and 4·636 cms. A person measuring them obtains 5·42 and 4·63 cms. respectively. Find the percentage error in the measurements of the sides and of the area calculated to these measurements.

169. Given one number and the percentage it is of another number, to find that other.

Examples—

(i.) *A man's income is increased by £25 per annum, which represents an increase of 20%. Find his original income.*

Increase is $\frac{20}{100}$ of original income and is equal to £25.

$$\therefore \frac{20}{100} \text{ of original income} = £25.$$

$$\therefore \text{original income} = \frac{25 \times \overset{5}{100}}{20}$$

$$= £125.$$

(See Ex. vi., p. 84.)

(ii.) *The manufactures of a firm increased last year by £6165, an increase of 15% on the previous year. Find last year's total.*

$$\text{Increase} = \frac{15}{100} \text{ of previous year} = £6165.$$

$$\text{Previous year} = \frac{\overset{411}{6165} \times 100}{15}$$

$$= 41100$$

$$\therefore \text{Last year's total} = 41100 + 6165 = £47265.$$

(iii.) *A man spends annually £875, and saves $12\frac{1}{2}$% of his income. Calculate his income.*

Since he saves $12\frac{1}{2}$% he must spend $87\frac{1}{2}$% of it.

$$\therefore \frac{87\frac{1}{2}}{100} \text{ of income} = £875$$

$$\therefore \text{Income} = \frac{\overset{5}{875} \times 100 \times 2}{175}$$

$$= £1000.$$

Examples 53.

1. An addition of 5% to the price of an article brings in 2*s.* 6*d.* more profit. Find the original price of the article.

2. On an increase of 4% being made on the income tax, a man has to pay £16, 10*s.* extra. Find what he now pays.

3. The income from a certain patent this year amounts to £140 more than it was last year. This is found to be an increase of $12\frac{1}{2}$%. What is this year's income?

4. In an examination A gets a second class with 45 per cent., whilst B fails with 23 per cent. of full marks. Between them they obtain 408 marks. Find the number of full marks.

5. The gross receipts of a railway for a certain year were apportioned as follows:—43 per cent. to pay working expenses; 52 per cent. to pay a dividend of $3\frac{1}{4}$ per cent. on the paid-up capital; the remainder, £25,000, to be added to the reserve fund. Find the paid-up capital.

6. If $37\frac{1}{2}$ per cent. of the candidates in an examination are girls, and if 75 per cent. of the boys and $62\frac{1}{2}$ per cent. of the girls pass and 342 girls fail, how many boys fail?

7. If 20 per cent. be added to a certain sum, and 30 per cent. be deducted from the new total, the result is 210. Find the original amount.

8. A leakage of 3·5 litres is discovered to have taken place from a certain vessel in which 85 per cent. of the original amount still remains. Find the capacity of the vessel in c.cms.

9. If cloth is raised in price 0·3 franc per metre, a merchant has to pay $3\frac{1}{3}$ per cent. more than he used to do. Calculate the cost of 20 metres at the new price.

10. The profits of an American firm for 1900 showed a net increase of 345·6 dollars on the preceding year's, representing an increase of 20 per cent. Find the profits for 1900.

170. Given percentage increase or decrease and resulting value, to find original value:—

Examples—

(i.) *A man's income for this year is £650, being an increase of 4% on last year's. Find last year's income.*

Since there has been an increase of 4%, it is obvious that we have 104 out of every 100 of last year's income.

$$\text{Hence } \frac{104}{100} \text{ of last year's income} = £650$$

$$\therefore \text{last year's income} = £650 \times \frac{100}{104}$$

$$= £625.$$

(ii.) *The population of a town in 1901 was 216788, showing an increase of $33\frac{1}{3}$% on last census. Find the population in 1891.*

$$\frac{133\frac{1}{3}}{100} \text{ of the population in 1891} = 216788$$

$$\therefore \text{population of 1891} = 216788 \times \frac{100}{133\frac{1}{3}}$$

$$= \frac{216788}{1} \times \frac{3}{4}$$

$$= 162591.$$

(iii.) *After allowing $7\frac{1}{2}\%$ for depreciation in a piece of machinery, it is valued at £596, 12s. 6d. Find its original value.*

Since value has decreased $7\frac{1}{2}\%$

$$\frac{92\frac{1}{2}}{100} \text{ of original value} = £596{\cdot}625$$

$$\therefore \text{ Original value} = £596{\cdot}625 \times \frac{100}{92{\cdot}5}$$

$$= \mathbf{£645.}$$

(iv.) *The value of a house has increased 5% annually during the last three years. The increase in value last year was £55, 2s. 6d. Find its value 3 years ago.*

£55, 2s. 6d. $= \frac{5}{100}$ of its value at end of s[illegible]

$$\therefore \frac{£55,\ 2s.\ 6d. \times \overset{20}{100}}{5} = £1102,\ 10s., \text{ value at end of second year}$$

$$\text{Value at end of first year} + \frac{5}{100} \text{ of same value} = £1102,\ 10s.$$

$$\frac{105}{100} \text{ of value at end of first year} = £1102,\ 10s.$$

$$\therefore \text{ Value at end of first year} = \frac{£1102,\ 10s. \times 100}{105} = £1050.$$

$$\text{Similarly, value at beginning of first year} = \frac{£\overset{10}{1050} \times 100}{105} = \mathbf{£1000.}$$

(v.) *A mixture of 60 gallons of wine and water contains 5 per cent. of water. How much water must be added to make the water 10 per cent. of the resulting mixture?*

Note.—Work with the substance whose total quantity remains constant—in this case, wine.

$$\text{1st mixture, wine} = \frac{95}{100} \text{ of 60 gallons} = 57 \text{ gallons}$$

$$\text{2nd} \quad ,, \quad ,, = \frac{90}{100} \text{ of mixture} = 57 \quad ,,$$

$$\therefore \text{ mixture} = \frac{57 \times 100}{90} = 63\tfrac{1}{3} \quad ,,$$

Hence $3\frac{1}{3}$ gallons of water must be added.

Examples 54.

1. Between 1871 and 1881 the population of a certain town increased $24\frac{3}{4}$ per cent., and in the latter year was 206087. What was it in the former?

2. The population of a town is 6806. What was its population a year ago, if during the year there has been an increase of 3·75 per cent.?

3. A timber merchant during four years' running incurs losses which he estimates at, on the average, equal to $3\frac{3}{4}$ per cent. per annum on his original capital. At the end of the four years his capital is found to be £2783, 15*s*. Find what capital he began with?

4. The population of a county is 3,278,181. If it has increased at the rate of 3 per cent. during each of the last three years, what was it before that time?

5. $12\frac{1}{2}$ per cent. is written off the value of a piece of machinery for depreciation. If it is now valued at £6644, 15*s*., find its original cost.

6. A piece of land measuring 340 acs. 2 rds. has increased in value 25%. Find its original cost per acre, if the total value is now £6384, 7*s*. 6*d*.

7. A merchant bought a quantity of tea at 2*s*. 6*d*. per lb. Had he bought 20 per cent. more, the cost would have been £12. Find the amount he bought.

8. A piece of ground has been sold three times, and each time has resulted in a loss of 4 per cent. to the seller. If the last loss amounted to £45, find the original value of the ground.

9. A piece of flannel has shrunk 5 per cent. and now measures 5·7 metres. Find its original cost @ 2·75 francs per metre.

10. The revenue of a railway company is $650,000, being an increase of $2\frac{1}{2}$ per cent. Find the exact amount of the increase.

11. A man buys three-fourths of a property, which afterwards rises 15 per cent. in value. He then sells five-sixths of his share for £575. What was the property worth?

12. A man spends annually one-third of his income. Had he spent $6\frac{2}{3}$ per cent. more, he would have saved £300 in all. Find his total income.

13. A mixture of 80 gallons contains 8 gallons of water and the rest wine. How much water should be added to make the water $12\frac{1}{2}$ per cent. of the mixture?

14. In a mixture of 70 gallons of two liquids A and B, A is 40 per cent. of total. How much of B must be added to reduce A to 35 per cent.

15. Pure milk contains 89 per cent. of water. A sample taken from a cask containing 22 gallons is found on being analysed to contain 90 per cent. of water. Find how much water had been added to the pure milk originally in the cask.

16. After deducting $8\frac{1}{3}$ per cent. from a certain weight there still remained 34·5 kilogrammes. What would have been the weight had $8\frac{1}{3}$ per cent. been added instead of deducted?

17. For a monthly return ticket 25 per cent. more is paid than for a single ticket. At the end of the month an extension of the time, for a week, is obtained by paying 5 per cent. on the monthly ticket. The whole sum paid is £2, 12s. 6d. Find the price of a single ticket.

18. In 85 lbs. of a mixture of coffee and chicory the latter is found to be 10 per cent. of it. How much chicory must be added to raise its percentage to 25?

19. Genuine milk contains 88·75 per cent. by bulk of water, 2·75 of fat, and the remainder is non-fatty solids. A purchaser buys 7 gallons of milk at 3d. a quart; the milk on being analysed is found to contain 90·84 per cent. of water and 2·24 of fat, the residue being non-fatty solids. Find whether anything besides water has been added to it, and find the sum of which the purchaser has been defrauded.

XXI.—AVERAGES AND STATISTICS.

171. When the rate at which anything is done is not uniform, we cannot speak of any of the rates as *the* rate. For convenience we take a rate which is as near as possible to all the rates, viz., the average rate. This rate, when applied to the total number of items, will give the same result as the total obtained when each individual rate is applied to each item. *Example:*—One horse is sold for £33, 10s., a second for £32, and a third for £34; total, £99, 10s. The average price per horse is £33, 3s. 4d., *i.e.* the total £99, 10s. ÷ 3 (the total number of horses). The average is always found by adding all the rates and dividing by the number of items.

Examples.—

(1.) *The numbers of readers in a public library during a certain week are as follows:—Monday,* 639; *Tuesday,* 745; *Wednesday,* 530; *Thursday,* 693; *Friday,* 813; *Saturday,* 1116. *Find the daily average.*

Total for week $= 4536$
Number of days $= 6$

$$\text{Average per day} = \frac{4536}{6} = 756 \text{ readers}$$

(ii.) *A railway train travels 402 miles at the following rates:—The first 110 miles in $2\frac{1}{4}$ hours, 75 miles in $1\frac{1}{2}$ hours, 160 miles in 3 hours, and the remainder in 1 hour. Find its average speed.*

Total distance $= 402$ miles

Number of hours $= 8$

Average speed $= \frac{402}{8} = 50\frac{1}{4}$ miles per hour.

171a. The Median.—In those cases where the given observations are of equal weight, *i.e.* ought to affect the result to the same degree, the average is often very unsatisfactory. In its place it has been proposed to substitute the **Median** (that point which has as many of the given observations above as below it).

Example—

The averages of the heights of two sets of ten men (1st set, 65, 64, 65, 66, 66, 67, 65, 66, 76, 66 inches; 2nd set, 65, 64, 65, 66, 66, 67, 65, 66, 66, 65 inches) are 66·6 and 65·6 inches respectively. It is quite evident from these results that the very tall man in the first set ought not to outweigh the influence of the other men. The difficulty and unfairness can be removed by using the median. Arranged in order the first set becomes 64, 65, 65, 65, 66, 66, 66, 66, 67, 76, and the median which lies between the 5th and 6th is 66. The second set gives the same result. The median for n values is that one which is the $\frac{n+1}{2}$th in order of size from either extreme.

"Experience has shown that the median is often a better value than the average in many statistical and psychological measurements and in many other measurements where psychological factors come into play, *e.g.* in the so-called errors of observation of astronomical and physical measurements. Strangely enough such phenomena as the fluctuating price of corn or the varying records of the barometer are better represented by the median than by the average. Of course in cases where the results are reckoned by the number of units and not by the number of things, *e.g.* meat by the pound, the average is the proper values." *

In certain cases the **Mode**, or the case which occurs oftenest, is taken

* *The New Psychology*, Appendix III., E. W. Scripture.

Examples 55.

Find the average in each of the following cases :—

1. 453, 397, 519, 479, 386, 537.
2. 16·8, 17·9, 13·3, 14·6, 16·7, 18·3.
3. 0·06, 0·15, 0·13, 0·08, 0·07, 0·14.
4. 13·3615, 14·024, 13·734, 12·936, 13·735, 14·732.
5. The numbers of visitors at an exhibition for one week were as follows: Monday, 85,399; Tuesday, 65,570; Wednesday, 86,047; Thursday, 81,074; Friday, 76,894; Saturday, 105,475. Find the daily average.
6. The number of hours of bright sunshine at Glasgow for the last ten days of August 1901, were 7, 7, 6, 8, 3, 12, 10, 0, 8, 12. Find the average and median number of hours' sunshine per day.
7. The rainfalls on thirteen days in the same month were 0·15, 0·31, 0·07, 0·08, 0·39, 0·05, 0·03, 0·01, 0·37, 0·00, 0·03, 0·86, 0·25 inches. Find the average and median rainfall for the period.
8. The money taken at the turnstiles of an exhibition on nine successive days was as follows: £459, 12*s.* 3*d.*, £575, 5*s.* 0*d.*, £796, 16*s.* 6*d.*, £630, 19*s.* 6*d.*, £604, 13*s.* 0*d.*, £1946, 18*s.* 11*d.*, £685, 16*s.* 0*d.*, £473, 5*s.* 6*d.*, £1090, 16*s.* 6*d.* Find the average per day for that time.
9. The following figures show the amount of bills and cheques which passed through the Bankers' Clearing-house during the week ending September 18th, 1901: £48,354,000; £25,219,000; £24,280,000; £24,071,000; £20,356,000; £24,911,000. Find the daily average, correct to five significant figures.
10. Complete the following statistical paper, correct to three significant figures.

Country.	Area in Acres.	Population, 1911.	No. of Acres per Person.
England	32,586,910	34,017,659	
Wales	4,750,720	2,027,620	
Scotland	19,070,466	4,759,521	
Ireland	20,327,947	4,381,951	
United Kingdom .			

11. The number of letters delivered during a certain year were as follows: England and Wales, 856,042,400; Scotland, 91,120,700; Ireland, 71,792,100. If these represent an average of 35 letters per person for England and Wales, 26 for Scotland, and 13 for Ireland, find the population of each in the year and the average number of letters to each person in the United Kingdom.

172. Care must be taken, when the rates and items are not stated separately, to include all in finding the totals.

Examples—

(iii.) *The number of hours of sunshine during a certain month were as follows:—On 5 days 0, on 4 days 3, on 7 days 7, on 12 days 10, and on 3 days 11 hrs. Find the average number of hours of sunshine per day.*

Days.		Hours.		Total Hours.
5	@	0	=	0
4	@	3	=	12
7	@	7	=	49
12	@	10	=	120
3	@	11	=	33
31 days	@	?	=	214.

Average number of hours of sunshine per day $=\frac{214}{31}=6\cdot9$.

(iv.) *A grocer buys 12 lbs. of tea @ 1s. 6d., 15 lbs. @ 2s., and 9 lbs. @ 3s. per lb. Calculate the average price per lb.*

Lbs.	Price per lb.		Total cost.
12	@ 1s. 6d.	=	18s.
15	@ 2s.	=	30s.*
9	@ 3s	=	27s.*
36 lbs.	@ ?	=	75s.*

∴ Average price per lb $=75s.\div36=2s.\ 1d.$

Note.—If the grocer were to mix the teas together the answer would be the value per lb. of the mixture.

173. A very common mistake in averages is to overlook the fact that the average over the whole is not necessarily the same as the average of the averages over the various parts into which the whole may be divided. This will be seen from the following example:—

(v.) *The attendances in a school, having accommodation for 1200 pupils, during four weeks were as follows:—1st week, 10,895 (10 meetings); 2nd week, 9347 (9 meetings); 3rd week, 5610 (6 meetings); 4th week, 11,236 (10 meetings). Find the average attendance each week, the average of these averages, and the average for the four weeks.*

1st week,	10895 attendances,	10 meetings,	average	=1089·5
2nd ,,	9347 ,,	9 ,,	,,	=1038·55
3rd ,,	5610 ,,	6 ,,	,,	= 935·
4th ,,	11236 ,,	10 ,,	,,	=1123·6

Total of averages . =4186·65
No. of averages . =4
Average of averages =1046·66
Total number of attendances for the four weeks = 37,088
Total ,, meetings during ,, = 35
Daily average for four weeks = 1059·66.

* Keep these values in shillings, as it can be readily seen that the average will be in shillings, and so the double reduction is unnecessary. The student should keep a look-out for similar examples.

Example—

(vi.) *The following results are obtained from a Reference Library. During the month of March the average number of persons per day who consulted historical works was 73. If the daily averages for each week were—1st week (6 days) 75; 2nd week (5 days) 70, 3rd week (6 days) 67; 4th week (6 days) 76,—find the average per day for the last three days of the month.*

No. of days on which the library was open = 26.

∴ No. of persons who consulted historical works—

During the months = 73 × 26 = 1898
During 1st week = 75 × 6 = 450
" 2nd " = 70 × 5 = 350
" 3rd " = 67 × 6 = 402
" 4th " = 76 × 6 = 456
Total . = 1658

∴ No. of persons who consulted historical works during last 3 days = 1898 − 1658 = 240

and daily average for last 3 days $= \frac{240}{3} = 80$.

Examples 56.

1. The revenue and expenditure of Bermuda for the five years ending 1900 were as follows:—Revenue, £34,256; £35,965; £38,923; £39,955; £40,124. Expenditure, £34,717; £35,704; £39,102; £39,243; £47,532. Calculate the average revenue and expenditure.

2. Complete the following statistical paper:—

Total Imports to Bermuda during Five Years ending 1900.

Year.	Total.	United Kingdom	Canada.	Other British Possessions.	United States of America.	Other Countries.	Per centage to United Kingdom.
	£	£	£	£	£	£	£
1896	305,495	90,625	20,886	7,756	184,391	1,837	
1897	323,074	95,424	24,550	6,517	195,167	1,416	
1898	351,274	104,974	30,611	7,346	205,308	3,035	
1899	394,888	104,408	33,119	7,238	246,447	3,176	
1900	397,136	104,009	31,488	9,932	250,729	978	
Total							
Yearly Averages							

3. A horse-dealer buys 7 horses at £26, 3 at £31, and 4 at £38 each. Find the average cost per horse.

4. The population of Scotland, according to Census returns, from 1851 to 1901 was as follows —

1851 . 2,888,742; 1871 . 3,360,018; 1891 . 4,025,647;
1861 . 3,062,294; 1881 . 3,735,573; 1901 . 4,472,000.
1911 . 4,759,521.

Calculate the average increase for a decade, and the percentage increase since 1851 and 1891.

5. Complete the following statistical paper:—

	Population in 1911	Number of Members of Parliament.*	Average Number of Persons represented by one Member.	Number of Members of Parliament a cording to Population.	Excess Number of Members of Parliament.
England	34,047,659	460			
Wales	2,027,620	30			
Scotland	4,759,521	70			
Ireland .	4,381,951	101			
Totals		661		661	

* Excluding members for universities.

6. A railway train travels 90 miles at the average rate of 40 miles per hour, 120 miles at 45 miles per hour, 180 miles at 54 miles per hour, and 80 miles at 65 miles per hour. Find the difference between the average of these averages and the general average.

7. In an examination of a class of 49 pupils: 2 pupils obtained 190 marks each, 12 obtained 180, 20 obtained 150, 15 obtained 120. What was the average number of marks obtained and the average percentage, if the highest possible number of marks was 220?

8. During the year 1900 the Bank of England rate was as follows:—6 per cent. for 1 week, 5 per cent. for 1 week, 4½ per cent. for 1 week, 4 per cent for 17 weeks, 3½ per cent. for 3 weeks, 3 per cent. for 5 weeks, and 4 per cent. for 24 weeks. What was the average rate during the year?

Complete the following statistical tables :—

9. *Number and Net Tonnage of Sailing and Steam Vessels built in England and Wales.*

	Number.				Tonnage.			
Year.	Sail.	Steam.	Total.	Percentage Steam.	Sail.	Steam.	Total.	Percentage Steam.
1896	338	307			23,684	273,062		
1897	423	309			32,328	246,459		
1898	589	415			34,553	380,677		
1899	450	394			30,633	404,838		
1900	428	395			30,144	378,166		
Totals								
Averages								

10. *Number and Net Tonnage of Sailing and Steam Vessels built in Scotland.*

	Number.				Tonnage.			
Year.	Sail.	Steam.	Total.	Percentage Steam.	Sail.	Steam.	Total.	Percentage Steam.
1896	47	223			31,255	149,408		
1897	80	210			34,082	122,026		
1898	63	273			7,047	205,197		
1899	108	265			14,208	220,[illegible]93		
1900	68	251			8,071	225,498		
Totals								
Averages								

11. At a distribution of prizes at a public school 3 prizes are worth 10*s.* each, 20 worth 6*s.*, 42 worth 5*s.*, 83 worth 3*s.* 6*d.*, 37 worth 2*s.* 6*d.*, and 17 worth 1*s.* 6*d.* Find to the nearest farthing the average value of a prize.

12. Find, correct to one decimal place of pence, and of miles, the entries for the blank spaces in the following extract from a railway company's report.

	No of Passengers.	Aggregate Miles Carried.	Fares Earned.	Average Fare per Passenger.	Average Distance Travelled.
1st Class	26,912	2,151,519	£22,762	*s.* *d.*	miles.
2nd Class	64,827	3,319,671	26,478	*s.* *d.*	miles.
3rd Class	294,838	10,760,739	50,790	*s.* *d.*	miles
Totals				*s.* *d.*	miles

13. The total rainfall at Glasgow for the month of August 1901, was 3·61 inches, and this is 0·25 inches below the average for 33 years. What was the average rainfall during the months of August for the preceding 32 years?

14. The average weight of the crew of an eight-oared boat, including the coxswain, is 10 stone 12 lbs., but, excluding the coxswain, it is 11 stone 2 lbs. Find the weight of the coxswain.

15. The following are details as to the number of hours sunshine at Glasgow in August 1901:—Daily average for first ten days, 3·2 hours; next eight days, 4·75 hours, next six days, 7·16 hours; the daily average for the month is 5·35 hours. Find the daily average for the last seven days.

16. Complete the following statistical papers:—

Income.	Abatement Allowed.	Dutiable Income.	Income Tax at 9*d.* in the £.	Rate per £ on Total Income.
£200	£160			
£250	£160			
£300	£160			
£350	£160			
£400	£160			
£450	£150			
£500	£150			
£550	£120			
£600	£120			
£650	£70			
£700	£70			
£750				

17.

Year of Enumeration	Population in England and Wales	Population in London	Persons in London to 100 in England and Wales to 4 significant figures
1801	8,892,536	959,310	
1831	13,896,797	1,655,582	
1861	20,066,224	2,808,494	
1891	29,002,525	4,228,317	
1901	32,526,075	4,536,063	

18. *Divisions of the Population in* 1891 *and* 1901.

District	Population 1891	Population 1901	Percentage 1891	Percentage 1901
England and Wales—				
Urban	21,743,977	25,054,268		
Rural	7,258,145	7,471,242		
Neither Urban nor Rural	403	565		
Total . .				
Scotland—				
Town Districts (2000 and upwards) . . .	2,925,080	3,367,280		
Mainland Rural Districts	974,841	983,274		
Insular " "	125,726	121,446		
Total . .				

19. A debt was to be discharged thus —$\frac{1}{4}$ in ready money, $\frac{1}{8}$ in 3 months, $\frac{3}{8}$ in 5 months, and the balance in 7 months. What is the equated time for paying the whole?

20. A bought goods from B on 1st January 1901, to the amount of £2000, and agreed to pay £200 that day, £600 on 10th February, £800 on 2nd March, and the remainder on 1st April. On what day ought a single payment of £2000 to be made to clear the whole debt?

21. If a person owes £20 due in 3 months, £20 in 6 months, and £40 in 8 months, when ought a single payment to discharge these debts?

22. A man owes £200, of which £40 is due in 2 months, £60 in 4 months, and the rest in 8 months. If he desires to pay the debt all at once, when is it due?

23. A debt of £1000 is due in instalments as follows:—£200 at present time, £200 in 1 month, £200 at end of 3 months, and £300 at the end of 4 months, the remainder to be paid in 6 months. When ought a single payment to discharge the whole debt?

24. A person buys furniture valued at £50 on the instalment principle, paying £10 down and £5 per month thereafter till the whole is paid. If he offers to pay the full £40 as his second payment, when is it due?

25. A person buys an organ valued at £15, on condition of paying £2, 10*s.* down and 12*s.* 6*d.* per month thereafter till all is paid. When ought a second payment of £12, 10*s.* to be accepted in discharge of the full debt?

26. A person buys a house valued at £1200, to be paid as follows:— £500 at once and the remainder in equal yearly instalments of £175. When ought a full payment of £1200 to be made in lieu of the above arrangement?

XXII.—RATES AND TAXES.

174. In all towns the money required for municipal purposes is raised by means of taxes or rates. The method of taxation varies, but, as a general rule, each individual who rents a house, &c., is charged so many pence or shillings on every £1 paid for rent.

175. We have seen (§ 164) that 3% may be written $\frac{3}{100}$, thereby indicating £3 out of £100, &c. Similarly, such a statement as 5*s.* out of, or in, £1 may be written $\frac{5s.}{£1} = \frac{5}{20} = \frac{1}{4}$, which last form we make use of as a fraction without name of any kind. Students should accustom themselves to the double reading of these expressions.

Thus:—

$\frac{7s.}{£30} = \frac{7}{600}$ reads 7s. out of, or in, £30; or seven six-hundredths.

$\frac{5d.}{1s.} = \frac{5}{12}$ reads 5d. out of, or in, the shilling; or five-twelfths.

$\frac{4 \text{ cwts. } 2 \text{ qrs. } 21 \text{ lbs.}}{6 \text{ cwts. } 1 \text{ qr.}} = \frac{4\cdot6875 \text{ cwts.}}{6\cdot25 \text{ cwts.}}$ or $\frac{4\frac{11}{16}}{6\frac{1}{4}} = \frac{75 \times 4}{16 \times 25} = \frac{3}{4}$ reads 4 cwts. 2 qrs. 21 lbs. out of, or in, 6 cwts. 1 qr.; or three-fourths. Observe that, before naming as a fraction, all names of concrete quantities must be cancelled out (§ 160)

176. By making use of the above principle, we can calculate rates and taxes very rapidly.

Examples—

(i.) *A man's rent is £150, 10s. Calculate his tax at 3s. 6d. in the £1.*

He pays as tax 3s. 6d. in £1, which may be written $\frac{3s.\ 6d.}{20s.} = \frac{7}{40}$.

His total tax is therefore this fraction of the total rent—

$$\text{Tax} = \left(\frac{3s\ 6d.}{20s.} = \frac{7}{40}\right) \text{ of } £150\cdot5$$
$$= \frac{105\cdot35}{4}$$
$$= £26\cdot3375$$
$$= £26,\ 6s.\ 9d.$$

(ii.) *The valuation of a town is £143256, 10s. How much money could be raised by a tax of 4d. in the £1?*

$$\text{Tax} = \frac{4d.}{£1}\ i.e.\ \frac{1}{60} \text{ of } £143256.\ 10s. = £2387.\ 12s.\ 2d.$$

(iii.) *A man pays £5, 12s. as tax at the rate of 1s. 3d. in the £1. Calculate his rent.*

$$\text{Tax} = \left(\frac{1s.\ 3d.}{£1} = \frac{1}{16}\right) \text{ of rental} = £5,\ 12s.$$
$$\text{rental} = £89,\ 12s.$$

(iv.) *£26, 3s. 9d. is charged as tax on a rental of £838. Find the rate.*

$$\text{Rate per } £1 = \frac{£26,\ 3s.\ 9d.}{£838} \text{ of } £1 = \frac{26\cdot1875}{838} \text{ of } £1 = 7\tfrac{1}{2}d.$$

177. Income Tax is calculated in the same way—the rate being so much on every £1 of income.

Examples—

(v.) *Calculate a man's income tax @ 8d. in the £, if his dutiable income is £345, 10s.*

$$\text{Tax} = \frac{8}{£1} \text{ i.e. } \frac{8}{240} \text{ of } £345, 10s. = £11, 10s. 4d.$$

(vi.) *A man pays an income tax of 9d. in the £. How much has he left out of a dutiable income of £650, 16s. 8d. after paying tax?*

If he pays 9*d.*, he has left 231 out of 240*d.*

$$\therefore \text{ he has left } \frac{231}{240} \text{ of } £650, 16s. 8d. = £626, 8s. 6\tfrac{1}{2}d.$$

178. *Note.*—In calculating Income Tax, the following reliefs are made to small incomes :—

Not exceeding £160, total exemption.			
,, ,,	£400, abatement of duty on		£160.
,, ,,	£500,	,, ,,	£150.
,, ,,	£600,	,, ,,	£120.
,, ,,	£700,	,, ,,	£70.

(vii.) *When the income tax is 1s. in the £, at what rate does a man whose income is £380 pay when the correct allowance is made?*

$$\text{Taxable income} = 380 - £160 = £220$$

$$\therefore \text{Income tax payable} = \frac{1}{20} \text{ of } £220 = £11.$$

$$\therefore \text{Rate per } £ = \frac{11}{380} \text{ of } £1 = 6\frac{18}{19}d.$$

(viii.) *After paying income tax at 1s. in the £ on all above £160 a man has £331 left. Find the amount of tax paid.*

£331 − £160 = £171 is net result of taxed part of income.

$$\frac{19s}{£1} \text{ of whole of taxed part of income} = £171$$

$$\therefore \text{ whole } ,, \quad ,, \quad = £171 \times \frac{£1}{19s.} = £180$$

Gross income = £180 + £160 = £340
Net income = £331
Tax = £9.

(ix.) *A man whose gross income is £275 has £270, 13s. 9d. left after paying the usual income tax on all above £160. At what rate per £ is he taxed?*

£275 − £270, 13s. 9d. = £4, 6s. 3d. is the tax on £275 − £160

$$\text{Rate per } £ = \frac{£4, 6s. 3d.}{£115} \text{ of } £1 = 9d.$$

179. Frequently it happens that when a certain sum is to be raised by taxation no even rate per £ will raise the exact amount, and the calculation must be made to find the rate per £, to raise the required amount at least.

(1) *A town, the rateable value of which is £756,281, 10s., requires £87,564, 15s. for municipal purposes. Find to the nearest farthing the rate per £ that must be charged, and the surplus to obtained if all rates are paid.*

$$\text{Rate per £} = \frac{87564 \cdot 75}{756281 \cdot 5} \text{ of £1} = \cdot 1157 = 2s\ 4d.$$

$$\text{Amount collected} = \frac{2s.\ 4d.}{£1} = \frac{\overset{7}{\cancel{28}}}{\underset{60}{\cancel{240}}} \text{ of £756281·5} = £88232,\ 16s.\ 10d.$$

$$\text{Surplus} = £88,232,\ 16s.\ 10d. - £87,564,\ 15s. = £668,\ 1s.\ 10d.$$

Examples 57.

(In income tax questions, no abatements are to be made unless stated in the question.)

1. Find (*a*) the income upon which the income tax, levied at the rate of 7*d*. in the £, amounts to £29, 3*s*. 4*d*, and (*b*) the income which, after deducting the tax at the same rate, becomes £932.

2. After paying income tax at 8*d*. in the £, and his other rates which exceed the income tax by 3*d*. in the £, a man has £1105 left. Find his gross income.

3. What is the real income of a person whose income tax at 7*d*. in the £ amounts to £22, 16*s*. 5½*d*.?

4. After paying a tax of 4*d*. in the £ a man has £826, 12*s*. 3½*d*. left. Find his total income.

5. A man spends one-ninth of his income on life insurance, and on this he is not charged any tax. If the tax on the remainder, at 7½*d*. in the £, amounts to £13, 10*s*. 5*d*., find his total income.

6. A man's income is £300 a year. He pays no income tax on the first £120, and 6*d*. in the £ on the rest. His house rent is £43, 3*s*. 3*d*., and he spends otherwise at the rate of 10*s*. 3½*d*. per day. How much would he save in the year 1892?

7. A man spends one-twelfth of his income on life insurance, and on this part he is not charged income tax. If income tax charged on the remainder of his income, at the rate of 6*d*. in the £, amounts to £14, 17*s*. 11*d*., find his total income.

8. A man's income amounts to £650 before paying rent and taxes. If his rent amounts to £60, and poor rate, municipal taxes, &c. (charged on rental), amount to 2*s*. 6*d*. in the £, find what he has left after paying

all, including an income tax of 6*d.* in the £, supposing he pays the latter only on his net income and is allowed a relief of £150.

9. On account of income tax a man pays £15, 1*s.* 9*d.* less when the tax is 3*d.* per £ than when it is 6*d.* per £. Find his income.

10. A man's net income after paying an income tax of 7*d.* in the £ is £291, 5*s.* What would his net income be if the tax were 1*s.* 4*d.* in the £?

11. A man's income is increased by £200, but the income tax being reduced from 6*d.* to 5*d.*, he pays the same amount of tax as before. What is his income?

12. A man's income is diminished by £100, but the income tax being raised from 8*d.* to 9*d.* in the £, he pays the same amount of tax as before. Find his original income?

13. The rateable value of a village is £85733; an assessment has to be made to raise £3156. What rate per £ must be charged in pence and hundredths of a penny to secure this? What surplus will result if all is paid up?

14. An amount of money has to be raised in a town by a tax on the rental. It is found that a tax of $4\frac{1}{2}d.$ in the £ produces £20 too little, while 5*d.* in the £ brings in £60 too much. Find the money required, and what will be obtained if a tax of $4\frac{3}{4}d$ in the £ be imposed.

15. If persons whose incomes are under £500 deduct £120 from these incomes in calculating the income tax, while those whose incomes are between £500 and £600 deduct £80, which income is really greater, a gross income of £501 or a gross income of £499, when the tax is at the rate of 1*s.* in the £?

16. A man's net income after paying £11 for life insurance and income tax at 9*d.* on the £ on all above £160 plus cost of life insurance amounts to £248, 11*s.* Find his gross income.

17. A man earns £500 a year, of which £100 is from money invested. If he is allowed an abatement of £160, +£10 for each child under 16 years, and is taxed at the rate of 1*s.* 2*d.* in £ and 9*d.* in £ on unearned and earned portions respectively, find his net income, his family consisting of five children under 16 years

18. A man earns £350 a year, pays £20 for life insurance, and is allowed the usual abatements in paying income tax. If his net income before paying insurance is £341, 10*s*, find the rate of income tax.

19. A person's salary is £320 a year and he receives £150 a year from a property which he owns. His own life is insured for £300 at £2, 14*s.* 7*d.* per cent. and his wife's for £200 at £3, 1*s.* 6*d.* per cent. He has three children under 16 years of age. Calculate the amount of his income tax rated at 9*d.* in the £ on earned income and 1*s.* 2*d.* in £ on unearned income, allowing an abatement of £160 on earned income and deducting the total cost of life insurance together with a further reduction from his taxable earned income of £10 for each child under 16 years of age.

20. A man's income amounts to £400 a year, of which only £280

is earned. Calculate his net income after paying income tax of 9*d.* and 1*s.* 2*d.* in £ on earned and unearned incomes respectively, allowing the usual abatement of £160 on earned income.

21. A person whose gross income is £310 pays £6, 10*s.* as income tax calculated at 9*d.* in £ on his earned income and 1*s* 2*d.* on unearned income. He was allowed as deductions from his earned income £160 abatement and £10 for life insurance. Find his earned and unearned incomes respectively.

XXIII.—BUSINESS DISCOUNT.

180. In business it is customary to deduct a certain amount from an account for ready cash payment. This deduction is termed **Discount**, and is usually calculated at a percentage rate.

Examples—

(i.) *A owes B £52, 7s. 6d., but is allowed $2\frac{1}{2}\%$ discount. Find the net amount to be paid.*

Total account =	£52	7	6
Discount $\frac{2\frac{1}{2}}{100}$ of £52, 7*s.* 6*d.* =	1	6	$2\frac{1}{4}$
Net	£51	1	$3\frac{3}{4}$

181. In connection with these small discounts great use may be made of the methods indicated in the "Mental Arithmetic," p. 43.

(ii.) *An account for £725 was settled for £580. What percentage discount was allowed?*

In this the actual discount was £725 – £580 = £145 out of £725.

$$\text{Percentage discount} = \frac{\text{£}145}{\text{£}725} \text{ of } 100 = 20.$$

Examples 58.

1. Calculate the discount allowed on £35, 7*s.* $10\frac{1}{2}$*d.*, at the rate of $2\frac{1}{2}$ per cent., correct to nearest farthing.

2. Which is better for the customer, a discount of $3\frac{1}{2}$ per cent. or a discount of $\frac{1}{2}$d. on the shilling? Find how much the difference would amount to on an account of £5, 7*s.* 6*d.*

3. A firm allows a discount of $1\frac{1}{2}$ per cent. on all accounts paid within a month of the purchase being made. What is the net amount of an account for £21, 13*s*. 8*d*. so paid?

4. A man's account amounts to £35, 10*s*., but on discount being allowed he pays £34, 10*s*. 6*d*. Calculate (1) discount allowed; (2) rate per cent. of discount.

5. Find the cash value of goods marked at £5, 17*s*. 6*d*. when the following discounts are allowed:—(*a*) 5%, (*b*) 4%, (*c*) 3%, (*d*) $2\frac{1}{2}$%, (*e*) 1%.

6. An article is sold for £10, 12*s*. $0\frac{3}{4}$*d*., which represents its cash value after a discount of $2\frac{1}{2}$ per cent. has been deducted. Find the price at which it was marked.

7. A certain magazine may be bought at the rate of 3*s*. per dozen; but on buying two or more dozen a customer receives 14 magazines to each dozen. Calculate the rate of discount allowed in this way.

8. Apples are sold at 2*d*. per lb., or 7 lbs. for a shilling; calculate the discount per cent. allowed to the purchase of 7 lbs.

9. Many firms rather than alter the price of an article according to the fluctuation of the market, prefer to alter the rate of discount. Find the difference in the net price of an article marked £21, 10*s*. when the discount is changed from $33\frac{1}{3}$ per cent. to 20 per cent.

10. I paid £1, 14*s*. for an article when the rate of discount was 20 per cent. What ought I to pay for it when the discount is altered to 3*d*. in the shilling?

11. One firm, A, offers to pay the carriage of all goods bought at cash price, while another, B, gives a discount of $\frac{1}{2}$*d*. on the shilling but pays no carriage. If I reside 20 miles from both and desire to purchase 3 cwts. of a certain article at $4\frac{1}{2}$*d*. per lb., which firm ought I to go to, the carrier's charge being at the rate of 1*s*. 8*d*. per cwt. for a distance of 20 miles?

12. If I buy goods at a certain village I must pay net prices marked. The town prices are $1\frac{1}{4}$ per cent. lower than the country prices, and purchasers in town are allowed $2\frac{1}{2}$ per cent. discount, but require to pay carriage. What would I save by buying articles in the town which would cost £10 in the country, the carriage amounting to 15*s*. 6*d*.?

XXIV.—INVOICES.

182. When a merchant sends goods to a customer he usually encloses with them, or sends after them, a detailed statement of the nature, quantity, and price of the various articles supplied, chiefly to enable the customer to check the order. Such a statement is termed an **Invoice**.

183. **The following will indicate the general appearance of an Invoice.**

Example—

(i.) *Make out an invoice of the following goods sent on 3rd February* 1901, *by Messrs. Arthur & Matheson, Brunswick Street, Glasgow, to Mr. Allan Macdonald, Paisley:—*40½ *lbs. Roast Beef @* 8½*d. per lb.;* 20 *lbs. Rump Steak @* 8*d. per lb.;* 10¾ *lbs. Roasted Gigot @* 1*s.* 4*d. per lb.;* 3 *doz. shapes Potted Meat @* 4*s. per doz.;* ¼ *doz. glasses Beef and Tongue @* 10*s.* 10*d. per doz.;* 6 *Smoked Hams,* 0 *cwt.* 2 *qrs.* 27 *lbs. @* 70*s. per cwt.;* 3 *Gigot Hams,* 0 *cwt.* 1 *qr.* 22 *lbs. @* 78*s. per cwt. Bags,* 1*s.; Hampers,* 10*s. Less* 1¼% *discount; cash in* 1 *month.*

Telegrams, "*Armath,*" *Glasgow.*
Telephone, 6539.

BRUNSWICK STREET,
GLASGOW, 3*rd Feb.* 1901.

MR. ALLAN MACDONALD.

Bought of ARTHUR & MATHESON,
Wholesale and Retail Butchers.

		£	s.	d.
40½ lbs. Roast Beef	-/8½	1	8	8
20 lbs. Rump Steak	-/8		13	4
10¾ lbs. Roasted Gigot	1/4		14	4
3 doz. shapes Potted Meat	4/-		12	0
¼ doz. glasses Beef and Tongue	10/10		2	8½
6 Smoked Hams cwts. qrs. lbs. 0 2 27	70/-	2	11	10½
3 Gigot Hams . . 0 1 22	78/-	1	14	10
Bags, 1*s.*; Hampers, 10*s.*			11	
		£8	8	9
Less 1¼% Discount, Cash in 1 month.				

184. **When the Account for the above is rendered,** *i.e.* **is sent to the customer for payment, either an exact duplicate of the invoice is sent, or, as is more common, the following brief statement:—**

Feb. 3.	To goods as per Invoice	£8	8	9

185. Should goods supplied on several different occasions be included in the one account, then the totals of each invoice will appear as above, in separate lines.

186. If it is necessary to give a duplicate of an invoice when an account is rendered, the account is said to be **detailed.**

187. When an account is being paid, the individual who receives the money deducts the discount and any other allowances for returned empties, &c., and acknowledges receipt of the money by writing at the bottom "Received by Cash" (or "by cheque" if payment is made by the latter), signing his name, and writing the date. Should the account amount to £2 or over, a penny stamp must be affixed and then defaced. This is usually done by writing the date, &c., over the stamp, thus—

Received by Cash
Pro Arthur & Matheson
J. W.
5th Feb. 1901.

188. The following is an example of an invoice sent to a customer abroad, to whom the goods were shipped. This kind of invoice is termed a **Shipping Invoice.**

Note.—The following terms are frequently used on Invoice forms, contracted thus:—E. & O. E., errors and omissions excepted; f.o.b., free on board; c.i.f., cost, insurance, freight.

Example—(ii.) *Invoice of 34 cases and 70 bdls. Tea Garden Stores shipped by Campbell, Lang & Co., per s.s. "Jelunga," London, viâ Suez Canal to Calcutta, for a/c and risk of The Diamond Mint Tea Estate, per Messrs. Graham & Co.. 84 William St., there.*

				£	s.	d.	£	s.	d.
[illegible]	1/4	Four Cases 19″ × 12″ × 10″, each 2 qrs.							
		6 Quart Botts. Soldering Fluid . . .	9/–	1	16	0			
	5	One Cask 24″ × 22½″ {5 cwts. 0 qrs. 12 lbs. / 4 cwts. 3 qrs. 10 lbs.}		0	6	7			
		100 Elephant Hoes, No. 5	19/–	7	18	4			
		Advance 10% . .		0	15	10			
	6/13	Eight Ingots Tin 3 cwts. 3 qrs. 6 lbs.	138/–	26	4	11			
	14/20	Seven Iron Kegs each 1 cwt. 0 qrs. 8 lbs.,							
		1 cwt. Hooping Nails, 1″	22/6	7	17	6			
	21/25	Five Iron Kegs each 1 cwt. 0 qrs. 8 lbs.							
		1 cwt. F.V. Nails, 1¾″	16/9	4	3	9			
	26	One Case 19″ × 13″ × 10″ {1 qr. 2 lbs. / 18 lbs.}		0	1	6			
		6 Soldering Irons	1/8	0	10	0			
		1 Salter's Balance to 200 lbs. . . .	19/–	0	19	0			
	27/34	Eight Drums each	3/–	1	4	0			
		10 galls. Lubricant	2/–	8	0	0			
	No Nos	Seventy bdls. Hoop Iron 35 cwts. 1 qr.	17/3	30	8	1			
				90	5	6			
		Less 2½% . .		2	5	2			
		Charges—					88	0	4
		Freight, Primage, and Dues		5	2	5			
		Bills of Lading, Stamps, and Sundry Expenses .		0	2	6			
		Insurance £160 @ 15s % and Duty 6d. . . .		1	4	6			
		Commission on £94, 9s. 9d. @ 2½%		2	7	3			
							8	16	8
		E. & O. E. GLASGOW, 10th Sept. 1901. CAMPBELL, LANG & CO.					96	17	0

Examples 59.

Make out the following invoices with appropriate headings, date, &c., and complete the necessary calculations.

			s.	d.
1.	14 lbs. ground white pepper		1	2
	7 lbs. ground black pepper		0	9
	14 lbs. ground ginger		0	10
	7 lbs. mixed spice		1	0
	10½ lbs. whole black pepper		0	10
2.	14½ yds. silk		10	0
	16 yds. serge		4	11
	27 yds. flannels		1	1
	19 yds. velvet		2	8
	3½ dozen reels linen thread		2	6
3.	2 cases Armour's tongues, 2 lb. tin	4 dozen	28	0
	4 cases tinned apricots	8 "	7	0
	2 cases tinned peas	4 "	8	6
	1 case lunch tongues	4 "	22	9
4.	2 cwts. 1 qr. sugar		0	2
	3 cwts. sugar		0	2½
	4 cwts. 3 qrs. sugar		0	3
	2 chests fine tea, each 40 lbs.		1	10½
	3 tins special tea, each 20 lbs.		2	3½
5.	6 boxes French plums net, 28 lbs. each = 1 cwt. 2 qrs.		42	0
	12 7-lb. bags natural figs	net, 3 qrs.	38	0
	2 boxes pale soap			

	cwts.	qrs.	lbs.		lbs.
gross,	1	0	16	tare,	8
	1	0	19		9
	2	1	7		17
			17		
	2	0	18		

			s.	d.
	(2 boxes pale soap, net 2 cwts. 0 qrs. 18 lbs.)		21	0
	5 bags St. T. granulated sugar	net, 10 cwts. 2½% discount.	17	0
6.	20 lbs. steak sausages		0	6½
	15 lbs. pork sausages		0	8
	14 lbs. black puddings		0	3
	12½ lbs. boiled bacon		0	11½
	6 steak pies		1	2
	2 dozen pork pies		1	6
	2 hampers		5	0

7. 3 tins oat cakes @ 2*s.* 6*d.* each; 3 doz. packets oat cakes @ 2*s.* 6*d.* each; 6 apple tarts @ 1*s.* 2*d.* each; ½ doz. glasses lunch tongue @ 14*s.* per doz. Case, 1*s.* Hamper, 5*s.*

8. 5 boxes Sultana raisins, gross, 1 cwt. 1 qr. 15 lbs.; tare, 15 lbs. @ 48*s.* per cwt. 4 ¼-cases Bostizza currants, gross, 1 cwt. 1 qr. 20 lbs.; tare, 28 lbs. @ 37*s* per cwt. 2 bags Patna rice, net, 4 cwts. @ 16*s.* per cwt. 3 bags small sago, gross, 3 cwts. 3 qrs. 12 lbs.; tare, 6 lbs. @ 14*s.* per cwt. 5 bags flake tapioca, gross, 5 cwts. 1 qr. 8 lbs.; tare, 10 lbs. @ 21*s.* 6*d.* per cwt. Less 1¼% discount. Terms, cash in 1 month.

9. 2 cwts. 3 qrs. sugar @ 1½*d.* per lb.; 1 qr. 12 lbs. tea @ 4*s.* 4*d.* per lb.; 20¼ lbs. cheese @ 10*d.* per lb.; 19¾ lbs. salt @ 3 lbs. a penny; 6¾ lbs. ham @ 10*d.* per lb.

10. 3 pieces scarlet flannel, each 57 yds., @ 4*s.* 6*d.* per yard; 96 yds. white flannel @ 4*s.* 11*d.* per yard; 75 yds. chintz @ 7½*d.* per yard; 11 doz. handkerchiefs @ 7¾*d.* each; 3½ doz. pairs gloves @ 3*s.* 2*d.* per pair; 100 reels @ 2*s.* 6*d.* per doz. Less 5% discount for cash.

11. 3½ cwts. flour @ 2*s.* 8*d.* per stone; 17½ lbs. cheese @ 8½*d.* per lb.; 3 firkins butter, each ½ cwt., @ 1*s.* 7*d* per lb.; 96 lbs. tea @ 2*s.* 7½*d.* per lb.; 5 boxes oranges, each 9 doz., @ ½*d.* per orange. Add 2½% for 6 months' credit.

12. 1¼ cwts. apples @ 2¼*d.* per lb.; 35½ lbs. pears @ 4½*d.* per lb.; 1 box oranges weighing 3½ qrs. @ 2*d.* per lb.; 2 stones grapes @ 10½*d.* per lb.; preserved prunes 3*s.* 10*d.* Deduct 5 per cent. discount for cash in one month.

13. 4 boxes Sultana raisins, gross, 1 cwt. 0 qrs. 10 lbs.; tare, 12 lbs. @ 50*s.* per cwt. 8 boxes French plums, each 28 lbs. net, @ 42*s.* 6*d.* per cwt. 7 cases tinned peas, each containing 2 doz., @ 8*s.* 10½*d.* per doz. 5 bags sago, gross, 6 cwts. 1 qr. 20 lbs.; tare, 10 lbs. @ 14*s.* 6*d.* per cwt. 3 boxes soap, gross, 1 cwt. 0 qrs. 15 lbs., 1 cwt. 0 qrs. 19 lbs., and 1 cwt. 0 qrs. 17 lbs.; tare, 7 lbs., 10 lbs., 8 lbs. @ 22*s.* per cwt.

14. A draper sold 120 yds. silk @ 6*s.* 3*d.* per yard; 80 yds. cloth @ 3*s.* 6*d.* per yard; 36 yds. calico @ 2½*d.* per yard; 3½ doz. pairs socks @ 1*s.* 10*d.* per pair. He received in exchange a horse and cart worth £15, 2*s.* 6*d.*, and allowed a discount of 5 per cent. on the balance. Make out the draper's account and write a receipt for the amount still due to him.

15. 12⅛ yds. silk @ 7*s.* 8*d.* per yard; 1 cwt. 3 qrs. 2 lbs. of potatoes @ 8¾*d.* per stone; 101 lbs. tea @ $2\frac{1}{16}d.$ per oz.; 12 parcels of sugar, each weighing 4¼ lbs., @ 14*s.* per cwt.; 151 yds. of cotton @ 4*s.* 6*d.* per dozen yards; sundries, £1, 5*s.* 11¾*d.* Deduct 3¾ per cent. for ready money.

16. Find the cost of building a house on which 7 bricklayers are employed @ 7½*d.*, 7 labourers @ 5½*d.*, 9 carpenters @ 10*d.* per hour for 49 days of 10 hours each; 40,000 bricks are required @ 32*s.* 6*d.* per 1000, and the other cost is £79. Make out a detailed statement.

17. 9½ yds. flannel @ 1*s.* 5½*d.* per yard; 26 yds. calico @ 2*s.* 1¾*d.* per yard; 23 yds. muslin @ 3*s.* 7*d.* per yard; 18 yds. linen @ 2*s.* 9¼*d.* per yard; 15 yds. ribbon @ 11½*d.* per yard; ½ doz. pairs of gloves @ 2*s.* 3½*d.* per pair. Deduct 2½ per cent. off the complete pounds for cash.

18. 47 lbs. 4 oz. of tea @ 1*s*. 10*d*. per lb.; 38 lbs. 8 oz. of sugar @ 2¼*d*. per lb.; 18 yds. 18 in. of linen @ 8½*d*. per yard; 282 boxes of matches @ 2½*d*. per doz. Deduct 5 per cent. for ready money payment.

19. Make out an invoice for 14 pkgs. bobbins and pressing rollers shipped by the Quarrie Co., Ltd., per s.s. *City of Athens*, Clyde *via* Mersey and Suez Canal to Calcutta, and consigned to Messrs. Smith and Young there for transmission to Serampore.

From JAS. WHYTE.

8	gross	10″ × 5″ × 1$\frac{1}{16}$″	Rove Bobbins, pd. and oiled,		30*s*.
5	,,	6¾″ × 4⅝″	Warp ,,	,, ,,	18*s*.
4	,,	6″ × 3¼″	Spinning ,,	,, ,,	13*s*.
10	,,	5″ × 3¼″	,, ,,	,, ,,	11*s*. 9*d*.
21½	,,	4″ × 2⅝″	,, ,,	,, ,,	10*s*. 6*d*.
13⅝	,,	Planetree Pressing Rollers			35*s*.
	3 Cases				18*s*.

1 Cask @ 5*s*. and 10 @ 3*s*.

Railway carriage from Inverness, £2, 13*s*. 0*d*.

Charges—

Freight, primage, and dues, £16, 15*s*. 10*d*.

Bills of lading and entries, 1*s*.

Insurance, £110 @ 6*s*. 8*d*. %, and duty 6*d*.

Less 10% on 7*s*. (net premium).

20. Make out a statement of account sales of 334 chests of tea received per s.s. *City of Oxford* from Messrs. Smith & Young, Calcutta, and sold by Strand & Co. for a/c of The Sirocco Tea Estate.

44	chests	Fanngs	4277 lbs.	4½*d*.
51	,,	Pek. Soug.	4234 ,,	5½*d*.
40	,,	Pekoe	3517 ,,	6¼*d*.
19	,,	,,	1679 ,,	6*d*.
70½	,,	Bro. Or. Pekoe	3350 ,,	7½*d*.
23	,,	Fanngs	2233 ,,	4¼*d*.
22	,,	Pek. Soug.	2120 ,,	4¾*d*.
28	,,	Pekoe	2706 ,,	5½*d*.
37½	,,	Bro. Or. Pekoe	1784 ,,	7*d*.

Loss in weight, including draft, &c., 800 lbs.

Charges—	£	*s*.	*d*.
Freight @ 40*s*. per 50 ft.	53	4	10
Import charges	53	1	11
Interest on freight and charges	1	12	8
Fire Insurance, £776 @ 2*s*. 6*d*. %			
Public sale charges	2	4	3
Postages and petties	0	3	3
Brokerage @ 1% on gross proceeds			
Commission @ 1½% ,, ,,			

21. Make out an invoice of 41 pkgs. and 50 bdls. Tea Garden Stores, shipped by Strand & Co. per *Jelunga*, London, *viâ* Suez Canal to Calcutta for a/c and risk of the Ark Lane Tea Estate, per Messrs. Small & Stewart, 20 Villiers Street, there.

3/7. 5 Reels 17″ × 17″ × 12″ each 1 cwt. 0 qr. 4 lbs.
Each 1 cwt. Barbed Wire . . . 16*s.* 6*d.*
8/12. 5 Ingots Tin . . . 2 cwts. 1 qr. 26 lbs. . 140*s.*
13/16. 4 Wood Kegs . . each 1 cwt. 0 qr. 5 lbs.
Each 1 cwt. Triangular Nails, 1¾″ . . 17*s.*
17/18. 2 Wood Kegs . . each 1 cwt. 0 qr. 5 lbs.
Each 1 cwt. Tacks, ¾″ . . . . . 16*s.*
19/32. 14 Drums . . . . each 3*s.*
10 galls. Engine Oil . . 140 galls. 2*s.*
33. 1 Cask 22″ × 20″ 3 cwts. 1 qr. 25 lbs. and 3 cwt. 1 qr. 1 lb.
100 flat Hoes . . . . 13*s.*
34/35. 2 Casks, each 26″ × 20″ { 5 cwts. 1 qr. 18 lbs. / 5 cwts. 0 qr. 14 lbs. } @ 6*s.* 6*d.*
100 flat Hoes, B. ribd., No. 7 . . 19*s.*
Advance 10%.
36. 1 Case 43″ × 43″ × 9″ . . { 1 cwt. 1 qr. 14 lbs. / 3 qrs. 7 lbs. } 9*s.* 6*d.*
6 Tins Stencil Ink . . . . 1*s.*
3 Stencil Brushes . . . . 6*d.*
2 Canvas Cloths, 18′ × 12′ . . . . 45*s.*
1 Sheet Asbestos Millboard, ¼″, ... 15½ lbs. 9*d.*
5 lbs. Asbestos Packing . . . . 2*s.* 6*d.*
37. 1 Case 33″ × 15″ × 14″. { 1 cwt. 1 qr. 21 lbs. / 1 cwt. 0 qr. 23 lbs. }
59′ 6″ Brass Wove Wire, No. 4 . . × 30″ 148′ 9″ 1*s.*
38. 1 Keg (contg. 14 lbs. Green Paint) . . . . 3*s.*
39/40. 2 Kegs (each contg. ½ cwt. White Paint) . . . 10*s.* 6*d.*
41. 1 Wood Keg contg. 1 cwt. Galv. Staples . . . 21*s.*
42. 1 Case 56½″ × 24″ × 14″ { 1 cwt. 2 qrs. 14 lbs. / 3 qrs. 16 lbs. }
43. 1 Case 36″ × 24½″ × 15½″ { 2 cwts. 2 qrs. 16 lbs. / 1 cwt. 3 qrs. 26 lbs. }
1 Portable Platform Weighing Machine to 2 cwts., £10.
No Nos. 50 Bdls. Hoop Iron, ½″ 24 cwts. 3 qrs. 0 lb. . 16*s.* 9*d.*
Less 2½%.

Charges— £ *s.* *d.*
Freight, Primage, and Dues 6 10 2
Bills of Lading, Stamps, and Sundries . . 0 3 3
Insurance on to Factory, £180 @ 15*s.* %.
Commission on £ (gross proceeds), @ 2½%.

22. Make out an invoice 16 cases mill furnishings shipped by the Temple Co, Ltd., per *City of Oxford*, Clyde *viâ* Mersey and Suez Canal to Calcutta, and consigned to Messrs. Hogg & Dixon there for transmission to Surat.

From John Smith.

H 1. 31 Sacking Beech Shuttles @ 1*s*. 1½*d*.
57 Hessian Beech Shuttles with Drag Sprgs. @ 1*s*. 1¾*d*.
Less 5%.

From Thomas Henry.

H 2. 449 pairs Batavia Hide Buffalo Pickers, 469 lbs. @ 1*s*.
Less 5%.

From John Lawson.

H 3/16.	8¼ gross	10″ × 5″ × 1⅛″	Rove Bobbins,	pin^d	and oiled,		30*s*.
	5 ,,	6¾″ × 4⅝″	Warp	,,	,,	,,	18*s*.
	4 ,,	6″ × 3¾″	Spinning	,,	,,	,,	13*s*.
	10 ,,	5″ × 3¾	,,	,,	,,	,,	11*s*. 9*d*.
	20 ,,	4″ × 2⅝	,,	,,	,,	,	10*s*. 6*d*.
	12 ,,	Planetree Pressing Rollers					35*s*.

3 cases @ 18*s*., 2 casks @ 5*s*, 9 casks @ 3*s*
Railway freight on H 3/16 from Perth, £2, 9*s* 1*d*.

Charges—	£	*s*.	*d*.
Freight, Primage, and Dues	17	19	6
Bills of Lading and Entries	0	2	0
Insurance, £200 @ 6*s* 8*d*. %, and duty 6*d*.			
Less 10% on 12*s*. 8*d*. (net premium).			

23. 3 5 kilogrammes butter @ 1 fl. 6 cents per kilogramme; 6 bars soap, each 4 75 kilogrammes, @ 4 cents 5 mils per kilogramme; 4 quintals sugar @ £3, 7 f. 5 c. per quintal; 1 barrel apples containing 1 quintal 45 kilogrammes @ 1 c. 2 m per kilogramme. Deduct 5 per cent. for cash.

24. 82 metres cloth @ 3 fl 5 c. per mètre; 356·5 metres tweed @ 1 fl. 2 c. 5 m. per metre; 3 webs cloth, each 0 285 of a kilomètre long, @ 5 fl. 2 c. per metre; 3 95 metres @ 6 fl. 5 c. per decametre.

XXV.—COMMISSION, BROKERAGE, &c.

189 In most cases where money is paid through a third party, *e.g.* agents employed to collect accounts, a percentage of the money is paid to that party. This is termed Commission, and is applied to all moneys paid to agents in return for service of any kind, as, for example, buying and selling articles for another, factoring

properties, collecting premium for Insurance, dealing in "Stocks," &c. The agent is sometimes termed a **Broker**, and his fee **Brokerage**.

Examples—

(i.) *An agent who is allowed 1% commission collects accounts valued at £840. What is the amount of the agent's commission?*

$$\text{Agent gets as commission } \frac{1}{100} \text{ of £840} = £8{\cdot}4 = \textbf{£8, 8s.}$$

(ii.) *A gentleman sold a house for £5000 through an agent who charged 5s. per cent. What money did he actually receive for the house?*

Total value of house . .	= £5000	0	0
Less $\left(\frac{5s.}{£100} = \frac{1}{400}\right)$ of £5000 =	12	10	0
Net receipt . .	= **£4987**	**10**	**0**

(*Note*—While $\frac{5}{100}$ may be read as 5 out of 100, *i.e.* £5 out of £100, 5*s*. out of 100*s*., 5 pence out of 100 pence, &c., care must be taken not to confuse it with 5*s*. per cent., which always means 5*s*. out of £100; similarly 2*s*. 6*d*. per cent. $= \frac{2s.\ 6d.}{£100} = \frac{1}{800}$ &c.

(iii.) *A man bought an estate for £16,281, which included the agent's charge of ½ per cent. For what price was the estate sold?*

$$\frac{100\frac{1}{2}}{100} \text{ of original value} = £16{,}281$$

$$\frac{201}{200} \text{ times original value} = £16{,}281$$

$$\therefore \text{ original value} = £16{,}281 \times \frac{200}{201}$$

$$= \textbf{£16,200.}$$

(iv.) *A factor who collected rents to the amount of £683, 6s. 8d. paid out £662, 16s. 8d., keeping the difference as his commission. What rate per cent. did he charge?*

Actual commission = £683, 6*s*. 8*d*. − £662, 16*s*. 8*d*. = £20, 10*s*.

$$\therefore \text{ Percentage commission} = \frac{£20,\ 10s.}{£683,\ 6s.\ 8d.} \text{ of } £100$$

$$= \frac{20\frac{1}{2}}{683\frac{1}{3}} \text{ of } 100$$

$$= \frac{41 \times 3}{2 \times 2050} \times 100$$

$$= \textbf{3.}$$

Examples 60.

1. An agent charges $2\frac{3}{4}$ per cent. on all moneys collected by him. Calculate his commission on an account for £850.

2. A man purchases a house, the upset price of which is £4856, through an agent who charges one-quarter per cent. commission. If the other expenses connected with the purchase amount to one-half of the agent's charges, what is the real cost of the house?

3. A man sells a property through an agent who charges 2*s.* 6*d.* per cent. for the sale. With the proceeds he purchases another property through an agent who charges 3*s.* 4*d.* per cent. commission. If the first property was sold for £140,500 and the second cost £140,000, find how much cash the man has left.

4. A flour traveller obtains a commission of one farthing per sack when sold at 23*s.* 6*d.* What rate per cent. does his commission correspond to?

5. A traveller is offered a salary of £150 a year and no commission or £90 a year and a commission of $1\frac{1}{4}$ per cent. Which ought he to accept? The estimated average sales amounted to 15 guineas per day (313 days = one working year). Find, to the nearest penny, what his average sales would require to amount to to make both bargains alike.

6. A man sells 35 bullocks by auction at an average price of £13, 7*s.* 9*d.* The auctioneer charges 4 per cent. commission on the sale. Previous to the sale an offer of £450 had been made privately for the lot. Which would have proved the better sale?

7. An agent charges $2\frac{1}{4}$ per cent. commission for disposing of a property. If the expenses of the sale amounted to £25, 10*s.*, and the agent after deducting this and his own charges, hands over £707, 12*s.* 6*d.*, for what was the property sold?

8. Find the broker's commission on selling 560 shares, each of a nominal value of £10, at the rate of 6*d.* a £.

9. A broker sells an article for £386, 5*s.* After deducting 16 guineas for expenses, and also his own charges, he hands over £343, 14*s.* What rate per cent. did he charge for commission?

10. An article is sold at a commission of 7*s.* 6*d.* per cent. Calculate the net return if the article is sold for £1575 and the expenses amounted to 1 per cent. over and above the commission.

11. A partner in a firm received a certain percentage of the drawings as wages. If his salary during a year amounts to £875 and the average weekly drawings amount to £134, 10*s.*, and that of the last day of the year to £6, calculate the rate he was paid at.

12. A building changed hands 3 times, each agent who sold it charging 25*s.* per cent. as commission. If each time it was sold for the net amount obtained at the previous sale, calculate its original value if the third sale realised £4622, 4*s.* 10·05*d.*

190. Insurance :—

When a person insures anything against loss, he takes out what is termed a **policy** for so much, and pays a **premium** on it, *i.e.* a sum calculated as a certain percentage of the amount, as indicated in the policy, for which the insurance is made.

Examples—

(i.) *Find the premium for insuring a life for £400 @ £2, 16s. 3d. per cent.*

$$\text{Premium} = \frac{\pounds 2,\ 16s.\ 3d.}{\pounds 100} \times 400 = \pounds 11,\ 5s.$$

(ii.) *If the cost of insuring for £750 be £15, 3s. 9d., what rate per cent. has been charged?*

£15, 3s. 9d. is charged on £750.

$$\therefore \text{rate per cent.} = \frac{\overset{\pounds 1,\ 0s.\ 3d.}{\cancel{\pounds 15,\ 3s.\ 9d.}}}{\underset{15}{\cancel{\pounds 750}}} \times \overset{2}{\cancel{100}} = \pounds 2,\ 0s.\ 6d.$$

(iii.) *The premium charged on a house, insured for ⅘ths of its value @ 3s. per cent., is £4. Find the value of the house.*

$$\frac{3s.}{\pounds 100} \text{ of } \frac{4}{5} \text{ of value of house} = \pounds 4$$

$$\frac{3}{2000} \text{ of } \frac{4}{5} \quad ,, \quad ,, \quad = \pounds 4$$

$$\frac{3}{2500} \quad ,, \quad ,, \quad = \pounds 4$$

$$\therefore \text{Value} = \frac{4 \times 2500}{3}$$

$$= \pounds 3333,\ 6s.\ 8d.$$

191. Sometimes an effort is made to effect an insurance in such a way as to repay both value insured and cost of insurance in case of loss. It is evident that to accomplish this the house or ship must be *over* insured. Suppose the premium required is at the rate of 5% and the house is valued at £100. A common though erroneous method would be to insure the house for £105, which, at first sight, appears to be equal to the value of the house and the cost of insurance. It is evident, however, that £5 will not pay the premium on £105 (the amount insured for). The correct method is to insure a house worth £95 as if it were valued at £100 so that in case of loss the £100 returned by the company would replace the house value (£95) and the £5 paid as insurance on the £100. It should be noted, however, that this kind of question is more theoretical than practical, as the Insurance Companies can prevent such action by themselves replacing the articles lost, and by taking all precautions to prevent over-insurance.

The following is an example of this type of question:—

Example—

(iv.) *For what must a ship valued at £142,125 be insured to recover value of vessel and cost of insurance in case of loss, the rate for insuring being 5 guineas per cent.? Find also the cost of insuring.*

If a ship valued at £94, 15s. is insured for £100,
then „ „ £142,125 „ ?

$$= \frac{100 \times 142125}{94.75} = £150,000$$

Cost of insurance $= \frac{5\frac{1}{4}}{100}$ of £150,000 = £7875

or Cost of insurance = £150,000 − £142,125 = £7875.

Examples 61.

1. Find the cost of insuring a life for £350 at £3, 0s. 6d. per cent., and the total number of payments being 40.

2. Calculate the cost of insuring a property at 3s. 6d. per cent., its value being £4384 (to nearest penny).

3. A person insures his life for £1000 at the rate of £2, 15s. 9d. per cent. If he dies after paying 15 premiums, how much does the insurance company lose?

4. A vessel valued at £35,600 is insured at three-fourths of its value at 4 per cent. Find the total loss, including cost of insurance, if the ship is wrecked.

5. A man over-insured a property, valued at £6275, to the extent of five-sixteenths per cent. If the insurance costs 3s. 4d. per cent. calculate the amount of actual loss if the building is completely destroyed.

6. A ship worth £1800 is wrecked; $\frac{1}{3}$ of her belonged to A, $\frac{1}{4}$ to B, and the rest to C. Find what loss each of them will sustain if the ship is insured to the extent of $\frac{2}{3}$ of her value.

7. For what sum will a man insure property worth £9100, at an annual premium of four guineas per cent., so that in case of loss he may recover the value of both the property and the premium? Calculate also the cost of insurance.

8. The goods in a store are valued at £3600. If a merchant owns $\frac{1}{18}$ of the whole, what sum should he insure at $2\frac{1}{4}$ per cent. to cover the value of his goods and the premium paid?

9. What premium did a shopkeeper pay who insured his stock worth £1933, 6s. 8d. at $3\frac{1}{4}$ per cent. so as to cover both stock and cost of insurance?

10. What premium at 3 per cent. will be paid for insuring a ship worth £12,000? What sum must it be insured for so that in case of loss the value of the ship and the premium paid may be recovered?

11. Find how much a house worth £5122 should be insured for so that in case of loss the value of the house and the insured premium at $2\frac{1}{2}$ per cent. may be repaid.

12. What sum must be insured to cover cargo valued at £1500, premium being 7 guineas, policy 5*s.* 6*d.* and commission $\frac{1}{8}$ per cent., and what will the expense of insuring come to?

XXVI.—PROPORTIONAL DIVISION, PARTNERSHIPS, &c.

192. When a sum of money, or any total whatever, has to be divided among several individuals, it is usual to make the shares proportional to some numbers. Thus, if it is desired to divide £86, 16*s.* between A, B, and C in the proportion of 1, 2, and 4, it is intended that whatever A gets, B is to get double and C four times that amount. If now the sum be divided into 7 (1 + 2 + 4) equal shares, and 1 share is given to A, 2 to B, and 4 to C, the money will evidently be divided as desired.

A then gets 1 out of 7 shares $= \frac{1}{7}$ of £86, 16*s.* = £12, 8*s.* (see § 163).
B „ 2 „ 7 „ $= \frac{2}{7}$ of £86, 16*s.* = £24, 16*s.*
C „ 4 „ 7 „ $= \frac{4}{7}$ of £86, 16*s.* = £49, 12*s.*

193. Such division often occurs in connection with business firms carried on conjointly by two or more individuals. Each individual is termed a **partner**, and the amount of money each has applied to the business is termed his **capital**. As a rule, the profits are divided in proportion to the capitals.

Examples—

(i.) *A and B enter into business together; A invests £1000 and B £1500. How should they share a profit of £480?*

Of a total capital of £2500 A owns £1000.

$$\therefore \text{ his claim is } \frac{1000}{2500} \text{ of the profits} = \frac{\overset{2}{\cancel{1000}}}{\underset{5}{\cancel{2500}}} \text{ of } £\overset{96}{\cancel{480}} = £192.$$

$$\text{B's claim is } \frac{\overset{3}{\cancel{1500}}}{\underset{5}{\cancel{2500}}} \text{ of } £\overset{96}{\cancel{480}} = £288.$$

or, more simply, B's claim = 480 − 192 = £288.

194. When a person invests money in a business, but does not become actively engaged in it, he is termed a sleeping partner, and his position as such affects his share of the profits.

Examples—

(ii.) *A invests £6000 in a concern, B £5000, and C £3000. A and B are working partners. C is a sleeping partner. By arrangement A and B are to receive each 10% of the total profits, and the remainder is to be divided between all three in proportion to their respective capitals. If the profits amounted to £3500, find C's share.*

Total profits = £3500
Deduct for A and B, 20% = $\frac{1}{5}$ = 700
Remainder to be divided = 2800

C's share = $\frac{3000}{14000}$ of £2800 = £600.

195. When the time during which the different partners have their capitals invested varies, the capitals must be reduced to equivalent sums for the same time in each case, before the shares of the profits are calculated.

(iii.) *A starts business with a capital of £600, and three months later he is joined by B with £400. At the end of other three months C enters into the partnership with £500. How ought a profit of £414 to be divided at the end of the year?*

In this case it would obviously be unfair to divide the profits in proportion to the capitals as they stand.

A has invested £600 for 12 months equivalent to £7200 for 1 month.
B „ £400 for 9 „ „ £3600 for 1 „
C „ £500 for 6 „ „ £3000 for 1 „
∴ The total capital is equivalent to £13,800 for 1 „

A claims $\frac{7200}{13800}$ of profits = $\frac{7200}{13800}$ of £414 = £216.

B claims $\frac{3600}{13800}$ of £414 = £108.

C claims $\frac{3000}{13800}$ of £414 = £90.

Total = £414.

Of a somewhat similar type are the following:—

Examples.—

(iv.) *A, B, and C rent a field for grazing for £29, 2s. A puts in 15 cattle for 4 weeks; B, 27 for 3 weeks; C, 30 for 5 weeks. Determine the share of the rent which each should pay.*

A has 15 cattle	for 4	weeks	equivalent to	60	cattle	for 1	week.
B has 27	,, 3	,,	,,	81	,,	1	,,
C has 30	,, 5	,,	,,	150	,,	1	,,
		Total cattle	equivalent to	291	,,	1	,,

∴ A should pay $\frac{60}{291}$ of £29·1 = £6.

B ,, $\frac{81}{291}$ of £29·1 = £8, 2s.

C ,, $\frac{150}{291}$ of £29·1 = £15.

Total = £29, 2s.

(v.) *A sum of £35, 10s. is made up of half-crowns, florins, and shillings. If the number of coins of each kind are to one another in the proportion of 5, 7, and 9, how many coins are there altogether?*

5 coins	each value	2s. 6d.	=	12s. 6d.
7 ,,	,,	2s.	=	14s.
9 ,,	,,	1s.	=	9s.
		Total value	=	35s. 6d.

∴ Half-crowns equal 5 out of 35s. 6d.

$\frac{5 \text{ half-crowns}}{\cancel{35 \cdot 5} \text{ shillings}}$ of £$\overset{20}{\cancel{35 \cdot 5}}$ = 100 half-crowns

$\frac{7 \text{ florins}}{\cancel{35 \cdot 5} \text{ shillings}}$ of £$\overset{20}{\cancel{35 \cdot 5}}$ = 140 florins

$\frac{9 \text{ shillings}}{\cancel{35 \cdot 5} \text{ shillings}}$ of £$\overset{20}{\cancel{35 \cdot 5}}$ = 180 shillings

Note.—Since the number of each kind of coin and not its value is used as the numerator of the fraction, care must be taken to have the sum divided reduced to the same name as the denominator; in this case, to shillings.

Examples 62.

1. A man, his wife, and three children earn £2, 1s. 3d. per week. His wife earns twice as much as each child and the man three times as much as his wife. How much does the man earn per week?

2. An estate of 1315 acres is to be divided among 4 tenants in proportion to their rentals. If the rents be £425, £376, £485, and £292 respectively, find the acres each gets.

3. Divide £59, 6s. $3\frac{3}{4}d$ amongst 15 persons, giving each of 4 of them twice as much as each of the others.

4. If £250 be divided amongst 100 children, $\frac{3}{8}$ of whom are boys, so that the sum divided among the boys be to the sum divided among the girls as 2 : 3, find the amount given to each boy and girl.

5. Divide £150 between A, B, and C, so that A may receive £7 as often as B gets £8 and B £4 as often as C gets £5.

6. Divide £4, 0s. 6d among a man, woman, and child, so that for every guinea given to the man the woman receives a pound and the child a crown.

7. Two men bought an ox weighing 5 cwt. 2 qrs. 20 lbs and sold it at the rate of $7\frac{1}{2}d$ per lb. Find the shares of the selling price, the one having given half-a-crown for every two shillings of the other in the purchase-money.

8. A force of police 1921 strong is to be distributed among 4 towns in proportion to the number of inhabitants in each, the population being 4150, 12450, 24900, and 29050 respectively. Determine the number of men sent to each.

9. A tailor hires a workshop for a year at a rent of £20. After $5\frac{1}{2}$ months he admits another to an equal share of it. How much rent should each pay?

10. A starts business at the beginning of the year with a capital of £400. In two months he is joined by B with £300, and 6 months later than this by C with £550. If the profits at the end of the year are £166, 19s 3d, how much should each receive?

11. A and B rent a field for £35. A puts in 6 horses for the whole year, B puts in 5 horses for 11 months and 20 more for 5 months. Find how they must share the rent.

12. A, B, and C entered into partnership. A had £175 and B had £210, their profits amounted to £422, 16s, of which C's share was £172, 5s. Find (1) what sum C put into the business; (2) what share of the profits was due to A and B respectively.

13. A, B, and C pay the rent of a field amounting to £5, 11s. A puts in 72 cattle for 3 months, B 48 cattle for 4 months, and C 60 cattle for 8 months. What will each pay?

14. Three partners have capitals of £7400, £6000, and £4600 invested in a business. The first charge on the profits is 5 per cent.

interest on capital; after this, £200 is paid to each of the partners for management, and the remainder is divided proportionately between the partners. In a certain year the profits were £2310. What did each partner receive?

15. A and B are partners in a business in which A has invested £3889 and B £1500. B receives 15 per cent. of all the profits for acting as manager, and the remainder is divided between the partners in proportion to the sums invested. What will each receive out of profits amounting to £726, 9s. 2d.?

16. A starts business with a capital of £3000. After 5 months he takes B, with a capital of £2000, into partnership, and one month later C, with a capital of £4500. At the end of the year the profits are divided and B finds that he has to pay 5 guineas as income tax. Find the total profits of the concern, the tax being at the rate of 6d. in the pound.

17. In a certain business A invests £200 more than B, but B has his money in for 5 months, while A has his for only 4, and if, out of a total profit of £210, A receives £30 more than B, find the amount invested by each.

18. A begins on February 1 with £4500; B joins him on May 5 with £7000. Divide the year's profit of £956, 17s. 8d.

19. A goldsmith is required to supply 90 grains of an alloy containing gold and silver in the proportion of 11 to 1. He happens to have an alloy of these metals in the proportion of 15 to 1. How much of this should he take, and how much silver should he add? (See Ex. v., p. 115.)

20. A sum of money amounting to £123, 19s. 6d. is made up of a certain number of crowns, three times as many half-crowns, five times as many florins, twelve times as many shillings, and eighteen times as many sixpences. How many are there of each?

21. A sum of money amounting to £105, 3s. 9d. is made up of an equal number of threepences, sixpences, shillings, florins, half-crowns, and crowns. How many are there of each?

22. The sum of £23 is made up of half-sovereigns, half-crowns, and sixpences. There are three times as many half-crowns as half-sovereigns, and the number of half-sovereigns is to the number of sixpences as 3 : 10. How many of each coin are there?

23. The sum of £65, 5s. consists of half-sovereigns, half-crowns, and sixpences, the number of these coins being as 3 : 4 : 7. How many are there of each?

24. Two merchants, A and B, enter into partnership. A advances £500 and B £650. Each is to receive 4 per cent. on his capital, and the remainder is to be divided between them in proportion to their capital. They gain £690. Find each man's share. Had the total gain been divided in proportion to the capital, what difference would it have made in A's share?

196. Division proportional to Fractions.—

Examples—

(i.) *Divide £2, 18s. 7d. between A, B, and C in the proportion of the fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$.*

The fractions must be reduced to equivalent fractions of the same denominator.

$$\left.\begin{array}{l} \text{A gets } \frac{1}{2} = 6 \\ \text{B gets } \frac{1}{3} = 4 \\ \text{C gets } \frac{3}{4} = 9 \end{array}\right\} \text{12ths.}$$

Total equivalent = 19

A gets 6 out of 19 = $\frac{6}{19}$ of £2 18 7 (3, 1) = £0 18 6

B gets 4 out of 19 = $\frac{4}{19}$ of £2 18 7 (3, 1) = £0 12 4

C gets 9 out of 19 = $\frac{9}{19}$ of £2 18 7 (3, 1) = £1 7 9

Total . = £2 18 7

(ii.) *Divide £2, 10s. 11d. between A and B, so that $\frac{3}{4}$ of A's share will equal $\frac{5}{9}$ of B's share.*

Since $\frac{3}{4}$ of A's share = $\frac{5}{9}$ of B's share

A's share = $\frac{5}{9}$ of B's share × $\frac{4}{3}$ (see § 136)

= $\frac{20}{27}$ of B's share

Giving B a share equal to 1, A will get $\frac{20}{27}$ of that = $\frac{20}{27}$.

$$\left.\begin{array}{l} A = \frac{20}{27} = 20 \\ B = \ \ 1 = 27 \end{array}\right\} \text{27ths}$$

Total = 47

A gets $\frac{20}{47}$ of £2 10 11 (1, 1) = £1 1 8

B gets $\frac{27}{47}$ of £2 10 11 (1, 1) = £1 9 3

£2 10 11

Examples—

(iii.) *Divide £9, 5s. among 4 men, 3 women, and 6 boys, giving each woman half as much again as each boy, and each man double each boy.*

Suppose one boy gets a share = 2,* then a woman will get a share = 3, and a man a share = 4.

∴ 4 men	get	$4 \times 4 = 16$ shares
3 women	get	$3 \times 3 = 9$ shares
6 boys	get	$6 \times 2 = 12$ shares
	Total shares	= 37

A man gets 4 out of $37 = \frac{4}{37}$ of £9 5 0 = £1 0 0

A woman gets 3 out of $37 = \frac{3}{37}$ of £9 5 0 = £0 15 0

A boy gets 2 out of $37 = \frac{2}{37}$ of £9 5 0 = £0 10 0

(iv.) *Divide £98 between A, B, and C, so that A will get 5 shares to B's 6 shares and B 4 shares to C's 9.*

Here we alter B's share to suit A and C at the same time.

A = 5		= 10
B = 6	4	= 12
C	9	= 27
Total		49

A gets $\frac{10}{49}$ of £98 = £20

(Finish as in previous examples.)

(v.) *Divide £50 between A, B, and C, giving A ⅚ of B's share and C £2 more than A.*

First deduct £2 for C, leaving £48 to be divided between the three, giving A and C each ⅚ of B's share.

A gets 5 shares	A gets $\frac{5}{16}$ of £48	= £15
B ,, 6 ,,	B ,, $\frac{6}{16}$ of £48	= £18
C ,, 5 ,,	C ,, £2 more than A	= £17.
Total, 16		

* When possible, select as the equivalent of the lowest share a number that will obviate the necessity of expressing the equivalents of the other shares in fractional form; or, having expressed them in fractional forms, raise them to proportional integral forms (§ 196) before proceeding with the division.

Examples 63.

1. A, B, and C undertake to do a piece of work for 35*s*. If the money is divided between them in the proportion of $\frac{1}{7}$, $\frac{1}{8}$, and $\frac{1}{56}$, find the share of each.

2. Divide 25 tons 11 cwts. 1 qr. in the proportion of 1, ·5, ·$\dot{3}$, ·25.

3. Three persons claim respectively ·5, ·1$\dot{6}$, and ·$\dot{6}$ of an estate of 1150 acres. How may it be divided so that they may receive shares in proportion to their claims?

4. Divide 5 guineas among 3 persons so that they may have shares proportional to $\frac{1}{2} : \frac{1}{3} : \frac{1}{8}$.

5. A, B, and C do a piece of work for £3, 8*s*. 9*d*. Suppose A's, B's, and C's labour is valued in proportion to the fractions $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$, how much should each receive?

6. The total area of three estates is 1768 acres. If the areas of the two smaller estates be respectively three-fifths and two-thirds of the largest, find the acreage of each.

7. Divide £15, 5*s* 4*d*. among 73 children, 81 women, and 63 men, giving each woman $\frac{3}{4}$ of a man's share, and each child $\frac{2}{3}$ of a woman's share.

8. The sum of $7\frac{1}{2}d$. was divided between A, B, and C in such proportions that A receives $1\frac{1}{2}d$ more than C, and B $2\frac{1}{4}d$. less than C. Had a sovereign been divided amongst them in the same proportion, what would each have received?

9. Divide £3, 18*s*. 4*d*. between A, B, and C, so that B may get seven-eighths as much as A, and C five shillings more than B.

10. Divide £90, 6*s*. between A, B, and C, so that A may receive $\frac{3}{4}$ as much as B gets, and C $\frac{1}{5}$ as much as A and B together.

11. Divide £250, 14*s*. 4*d*. between A and B, so that $\frac{5}{6}$ of A's share may equal $\frac{4}{7}$ of B's share.

12. Divide £28, 18*s*. $10\frac{1}{2}d$. into 3 shares, so that the second share may be $\frac{2}{3}$ of the first, and the third $\frac{4}{5}$ of the second.

13. Divide £34, 2*s*. between A, B, and C so that A will get 1 shilling for every half-crown B gets, and B will get 3 shillings for every florin C gets.

14. Divide £369, 2*s*. 2*d*. between A, B, and C so that B's share will be $\frac{2}{3}$ of A's and C's share $\frac{5}{8}$ of A's.

XXVII.—BANKRUPTCIES.

197. When any person or firm is unable to pay their debts, the person or firm becomes **bankrupt** or **insolvent**. The books are then handed over to an agent, who, on behalf of the creditors, calculates the total amount of money that the bankrupt estate can now yield. This amount is termed the **Assets**, whereas the debts contracted are called the **Liabilities**. He then divides the assets among the creditors in proportion to their shares of the

debts. This is usually done by calculating the **Dividend**, *i.e.* what the bankrupt estate can yield for each £1 of debt, and then allotting the assets in proportion to the shares of the debts.

Examples—

(i.) *A bankrupt's assets amount to* £345, 10s., *while his liabilities are* £1727, 10s. *What dividend can he pay?*

He can pay £345, 10s. out of £1727, 10s.

$\therefore$ Dividend $= \frac{345\cdot5}{1727\cdot5}$ (cancelled to $\frac{1}{5}$) of £1 = **4s. in the £.**

(ii.) *A bankrupt's debts amount to* £17,325, *and his assets to* £1155. *What does a creditor receive to whom he owes* £832, 12s. 6d.?

Dividend $= \frac{1155}{17325}$ (cancelled to $\frac{1}{15}$) of £1 = 1s. 4d.

$\therefore$ creditor gets 1s. 4d. × 832⅝ = **£55, 10s. 2d.**

or *directly*, the creditor gets $\frac{1155}{17325}$ of £832, 12s. 6d. = **£55, 10s. 2d.**

(iii.) *A bankrupt estate yields* 1s. 3d. *in the £. What does a creditor lose to whom* £845, 10s *is owing?*

He gets 15 pence out of every 240 pence

$\therefore$ he gets $\frac{15}{240}$ of £845, 10s. = £52, 16s. 10½d.

and he loses £845, 10s. − £52, 16s. 10½d. = **£792, 13s. 1½d.**

or, directly, since he gets 15 pence out of 240 pence, he loses 225 pence out of 240 pence

$\therefore$ he loses $\frac{225}{240}$ (cancelled to $\frac{15}{16}$) of £845, 10s. = **£792, 13s. 1½d.**

198. In dealing with a bankrupt's affairs there are certain claims which must be paid in full before any dividend is declared. These preferential claims include rent, taxes, servants' wages.

(iv.) *A bankrupt's assets amount to* £1350, 8s. 4d., *and his liabilities to* £7455, *of which* £129, 10s. *is due for rent, taxes, servants' wages, and must accordingly be paid in full. What dividend can be paid to the remaining creditors?*

Assets available for dividend
= £1350, 8s. 4d. − £129, 10s. = £1220, 18s. 4d.

and the liabilities after paying preferential claims
= £7455 − £129, 10s. = £7325, 10s.

$\therefore$ Dividend $= \frac{£1220,\ 18s.\ 4d.}{£7325,\ 10s.}$ of £1 = **3s. 4d.**

(v.) *The assets of an insolvent estate amount to £676, 15s. 1d., and the liabilities to £49,725, 10s 6d. £350, 10s. 6d. is the amount of the preferential claims. If the legal expenses of winding up the concern amount to £120, 10s., what dividend can be declared?*

The assets are reduced by the amount of the preferential claims, and also by the charge for winding up before the amount available for dividend is known.

∴ Assets available for dividend
= £676, 15s. 1d. − £350, 10s. 6d. − £120, 10s. = £205, 14s. 7d.

and liabilities (after paying off preferential claims)
£49,725, 10s. 6d. − £350, 10s. 6d. = £49,375

$$\therefore \text{ Dividend} = \frac{£205, 14s. 7d.}{£49,375} \text{ of } £1 = 1d. \text{ in the } £.$$
240

(vi.) *A bankrupt whose liabilities amount to £8300 has assets £720. After paying his preferential claims and a lawyer's fee of £20 he declares a dividend of 1s. in the £ to the ordinary creditors. What was the value of the preferential claims?*

After payment of lawyer's fee the assets equal £720 − £20 = £700.
Had he paid all claims alike 1s. in £, he would pay $\frac{1}{20}$ of £8300 = £415.

£700 − £415 = £285 remain to pay the other 19s. in £ to the preferential claimants, thus giving them 20s. in £.

$$\text{Hence } \frac{19s.}{£1} \text{ of preferential claims} = £285.$$

$$\therefore \text{ preferential claims} = £285 \times \frac{£1}{19s.} = £300.$$

Examples 64.

1. If a bankrupt's liabilities amount to £789, 3s. 4d. and he can pay 15s. 4½d. in the £, what is the loss sustained by his creditors?

2. A bankrupt owes £900 to three creditors, and his assets are £675. The claims of two of the creditors amount to £375 and £125 respectively. Find how much the third creditor will get for his dividend.

3. A bankrupt's debts amount to £9500. After paying legal expenses amounting to £138, his assets suffice to pay his creditors a dividend of 4s. 6½d. in the £. Find the amount of his assets.

4. A bankrupt's assets amount to £5438, 7s. 10½d. After paying legal expenses amounting to £160, he declares a dividend of 1s. 3d. in the £. Find his total liabilities.

5. A bankrupt has assets £600 and liabilities £939; of which latter amount £35 are preferential claims for rent, taxes, &c., and must be paid in full. Find how much in the £ the creditors ought to receive.

6. A bankrupt's debts amount to £9579. After paying legal expenses amounting to £157, his assets suffice to pay the creditors a dividend of 3s. 10½d. in the £. Find the amount of his assets.

7. A bankrupt, whose total assets amount to £309, 7*s.* 6*d.*, can pay a dividend of 1*s.* 6¾*d.* in the £. What is the amount of his debts?

8. A bankrupt's debts amount to £10,659, 5*s.*, and his effects when realised amount to £888, 5*s.* 5*d.* Find (1) what rate of dividend per £ he can pay; (2) how much a creditor for £150 will lose.

9. A bankrupt's debts amount to £5729, 10*s.*, and his assets to £893. Find, correct to the nearest penny, how much will be received by a creditor to whom he owes £176, 15*s.*

10. A bankrupt, whose total debts amount to £4678, has assets valued at £1860. If his rent, taxes, and other preferential claims amount to £451, how much do his creditors receive in the £?

11. A bankrupt has £5600 and owes £13,440. Find (1) what dividend he can pay; (2) what a person loses to whom he owes £780, 10*s.*

12. A bankrupt besides having £379, 19*s.* 3*d.* in cash has bad debts for £160, 3*s.* 4*d.*, £90, 0*s.* 6*d.*, and £70, 1*s.* 8*d.*, which are estimated to yield 2*s.*, 3*s.* 4*d.*, and 4*s.* per £ respectively. What are his liabilities if he can pay 6*s.* 4½*d.* in the £?

13. A bankrupt owing £8000 declares a dividend of 6*s.* 3*d.* in the £. In addition to his stock he has good debts valued at £500, and bad debts amounting to £800, £450, £1320, 4*s.* 2*d.*, and £960, on which he calculates to receive 2*s.*, 6*s.* 8*d.*, 4*s.*, and 11*s.* 8*d.* in the £ respectively. Find the value of his stock.

14. A bankrupt's debts amount to £3798, 6*s.* 8*d.* After paying preferential claims amounting to £156, and legal expenses of £45, 10*s.* 6*d.*, he can pay 7*s.* 3*d.* in the £ as dividend. Find his total assets.

15. The assets of an insolvent firm amount to £477, 6*s.* 8*d.*, and the liabilities to £1924, of which latter sum £120, 6*s.* 8*d.* is due for rent and wages. If a creditor whose bill amounts to £18, 13*s.* 4*d.* withdraws all claim on the estate, what dividend can be declared?

16. A merchant fails for £2850, and pays a first dividend of 10*s.* 8*d.* in the £, and afterwards a second dividend of 7*s.* 6*d.* on what was then due. How much did he pay altogether in the £?

17. A bankrupt's debts amount to £1087, 10*s.* He pays a first dividend of 12*s.* 4*d.* and a further dividend of 2*s.* 4*d.* in the £. How much is lost by a creditor to whom he owed £336, 7*s.* 6*d.*?

18. A merchant owes £6000. He has good debts to the amount of £1600 and bad debts to the amount of £1200. If he receives an average of 50 per cent. of his bad debts, find how much he can pay his creditors in the £.

19. A bankrupt's assets are £675, out of which he pays 15*s.* in the £ on one half of his debts and 12*s.* on the other half. Find the amount of his debt.

20. A bankrupt can pay 6*s.* 8*d.* in the £. If his assets were £500 more, he would pay 7*s.* 4*d.* Find his debts and assets.

21. A bankrupt's debts amount to £3241, 13s. 4d., and his assets to £1013, 0s. 5d. Find (a) what dividend he can pay; (b) what a creditor will lose to whom he owes £428, 10s.

22. After paying £128, 13s. 10d. for preferential claims, a bankrupt declares a dividend of 4s. 8d. in the £. If his debts were £4076, 1s. 4d., find his assets

23. A bankrupt's debts amount to £7450 and his assets to £1210. After paying his preferential claims in full and legal expenses of £60 he declares a dividend of 2s. 6d. in the £. Find the amount of the preferential claims.

24. A bankrupt's liabilities amount to £657, and his dividend is reduced from 2s. 6d. to 1s 9d. in the £ by the admission of a claim to preferential rank What was the amount of this claim?

25. A bankrupt's debts amount to £3675, and the dividend is reduced from 3s. to 2s 6d. by the admission of a claim to preferential rank. Find the amount of the claim.

XXVIII.—PROFIT AND LOSS.

199. In business a man may sell articles at the price he originally paid for them, i.e. at **Cost Price**, or prime cost as it is sometimes termed. He may, however, obtain a larger sum than this, in which case the articles are said to be sold at a profit; or should the selling price be smaller than the prime cost, the articles are sold at a loss. The price at which an article is originally bought is termed its Cost Price, and that at which it is sold its Selling Price. The difference between these two prices will give the **Profit or Loss.**

200.
Throughout the examples C.P. is used as a contraction for Cost Price
and S.P. „ „ „ Selling Price.

Examples—

(i.) *Sugar is bought at 15s. per cwt., and sold at 2½d. per lb. What profit is there on a sale of 3 cwts.?*

S.P. of 3 cwts. @ 2½d. per lb.	=	£3	10	0
C.P. of 3 cwts. @ 15s. per cwt.	=	2	5	0
∴ Profit on 3 cwts.	=	£1	5	0

(ii.) *600 eggs were bought, half at 3 for 2d., and the rest at a penny each. They were mixed and sold at 1s. 2d. per dozen. Find the total profit.*

S.P. of 600 eggs @ 1s. 2d. per doz.	=£2	18	4
C.P. of 300 eggs @ 3 for 2d. = 16s. 8d.			
C.P. of 300 eggs @ 1d. each = 25s.			
Total C.P. =	2	1	8
Profit =	£0	16	8

(iii.) *100 barrels of apples were bought for £50. 20 barrels were damaged and had to be sold at 6s. 8d. a barrel. At what price per barrel must the remaining 80 be sold so as to gain £16, 13s. 4d. on the whole transaction?*

C.P. of 100 barrels of apples . . . . =	£50	0	0
Amount to be gained . . . . =	16	13	4
∴ S.P. of 100 barrels . . . . =	66	13	4
S.P. of 20 ,, @ 6s. 8d per barrel =	6	13	4
∴ S.P. of 80 ,, ... =	60	0	0
∴ S.P. of each barrel = £60 ÷ 80 . . . =	15s.		

(iv.) *A man bought an equal number of articles at 3 for 2d. and at 4 for 3d. By selling them at 7 for 6d. he gained 16s. 8d. Find how many he bought.*

Select the smallest number that can be dealt with in threes, fours, and sevens, that is the L.C.M. of 3, 4, 7 = 84.

C.P. of 84 @ 3 for 2d. . . . = 56d.
C.P. of 84 @ 4 for 3d. . . . = 63d.

∴ C.P. of 168 . . . . = 119d.
S.P. of 168 @ 7 for 6d. . . . = 144d.

Gain on 168 . . . . . = 25d.
∴ Gain on 1344 . . . . . . = 200d., or 16s. 8d.

Answer 672 of each kind.

Examples 65.

1. A grocer bought 2 tons 10 cwts. 3 qrs of sugar for £59, 4s. 2d., and paid 6s. 8d. per ton for expenses. At what price per cwt. must he sell it to have a clear profit of £10, 19s. 11d.?

2. A grocer buys 183 lbs. of sugar for £2, 18s. 4d.; he puts aside 33 lbs. of it for his own use. Find what price per lb. he must charge for the remainder so as to gain one-half of his outlay.

3. A merchant sold 150 quarters of wheat at 46s. per quarter; the purchaser sold it at a profit of £15. Find his selling price per quarter.

4. A grocer bought 11 cwts. 3 qrs. of cheese for £18, 16s.; he sold one quarter of his stock at £2 per cwt., and the rest at 30s. per cwt. What was his total gain?

5. A grocer sells 2 cwts. 1 qr. of sugar at 2*d.* per lb.; 3 cwts. at $2\frac{1}{2}$*d.* per lb.; and 4 cwts. 3 qrs. at 3*d.* per lb. How much does he gain if the average price of sugar is a guinea a cwt.?

6. A coffee merchant bought 48 lbs. of coffee at a certain price per lb., and mixed it with 12 lbs. of chicory at $2\frac{3}{4}$*d* per lb.; he sold the mixture at $13\frac{1}{2}$*d.* per lb., and made a clear profit of £1, 4*s.* 9*d.* Find the price of coffee per lb.

7. A grocer supplies 16 lbs. of tea at 2*s.* 3*d.* per lb.; 28 lbs. at 2*s.* 4*d.* per lb., and half a cwt. at 3*s.* per lb. If the average cost of the tea is £14 per cwt., find the average gain per lb.

8. A and B buy oranges at 10 for 1*s.* A sells them at 9 for 1*s.* and B at seventeenpence a dozen. If they sell the same number of oranges, and A's gain is 4*s.*, what is B's gain?

9. Oranges are bought at the rate of six for 5*d.*, and sold at five for 6*d.* Find the gain on a sale of 50 dozen.

10. 45 lbs. of tea at one price are mixed with 15 lbs. at a dearer price. By selling at 4*s.* per lb., £1, 12*s.* 6*d.* is gained. Find the price of each kind of tea, the difference being 1*s.* 10*d.* per lb.

11. Coffee at £5, 12*s.* 6*d.* per cwt. and chicory at £2, 5*s.* 5*d.* per cwt. are mixed in the proportion of 2 of chicory to 5 of coffee. The mixture is retailed at 1*s* 3*d* per lb. What is the gain per cwt.?

12. A certain number of eggs was bought at 2 a penny, and the same number at 3 a penny. By selling them at 5 for 2*d.* there was a loss of 1*s.* 6*d.* How many were sold?

13. A bookseller buys books to the nominal value of £182, but he gets $3\frac{1}{2}$*d.* in the shilling discount. How much does he expend? If in addition to the discount named, the publisher sends him 13 copies at the price charged for one dozen, and the bookseller sells the books at a discount of 3*d* in the shilling, what is the amount of his profit in the transaction?

14. A quantity of apples was bought at the rate of 3 for a penny, and an equal number at 4 a penny. The lot was sold at 7 for 2*d.* How many apples were sold if the total loss was a penny?

15. A man spends equal sums in buying articles at 5 for 6*d.* and 11 for 9*d.* By selling them at 1*s.* 7*d.* per dozen he gains £3, 7*s.* 9*d.* Find the number of each kind he bought.

16. A dealer bought 80 tons of coals; by selling the coals at 1*s.* 6*d.* a bag he gains £4. Had he sold the coals at 1*s.* 4*d.* a bag he would have lost £6. Find the weight of a bag of coals.

17. If I bought 8 chests of tea, each 144 lbs., at 5*s.* a lb., and $\frac{1}{3}$ of it turning out not so good as expected, was disposed of at 3*s.* $1\frac{1}{2}$*d.* per lb, what must I charge for the remainder per lb., so as to gain £2, 8*s.* on the whole transaction?

18. A person sold $3\frac{1}{4}$ cwts. of sugar at $5\frac{1}{2}$*d.* per lb., thereby gaining £1, 4*s.* 6*d.* What would have been the gain or loss had it been sold at $4\frac{1}{4}$*d.* per lb?

19. A merchant mixes 3 qrs. 5 bush. of barley, at 32*s.* per qr., with

15 bush. of another sort at 26*s*. per qr. What did he gain or lose in all by selling the mixture at 31*s*. 6*d*. per quarter?

20. A grocer bought 16 cwts. 1 qr. 11 lbs. of sugar for £28, 12*s*. 2¼*d*., and sold it so as to gain ⅗ of what it had cost him. At what price per lb. did he sell it?

201. Profit or Loss is usually calculated as a percentage amount and, *unless otherwise stated*, must be computed on the Prime Cost of the Article. As such, Profit and Loss questions resolve themselves into easy percentage questions.

Examples—

(i.) *An article was bought for* 15*s*. *and sold at a gain of* 5%. *Find the Selling Price.*

C.P. . . . . . . = 15*s*.

Gain 5% $= \frac{5}{100}$ of 15*s* = ·75

∴ S.P. . . . . . = 15·75*s*. = 15*s*. 9*d*.

(ii.) *Articles were bought @ 5s. per dozen and sold at a loss of* 20%. *For how much each were they sold?*

C.P. of 1 dozen . . . = 5*s*.

Loss 20% $= \frac{20}{100}$ of 5*s*. = 1*s*.

∴ S P. of 1 dozen = 4*s*.

S.P. of each . . . . . = 4*d*.

(iii.) *An article costing* 37*s*. 6*d*. *was sold for* £2, 0*s*. 7½*d*. *Find the gain per cent.*

S.P. = £2 0 7½
C P. = 1 17 6
Net gain = £0 3 1½ on £1, 17*s*. 6*d*.

Hence gain per cent. $= \frac{3s.\ 1\frac{1}{2}d.}{£1,\ 17s.\ 6d.}$ of 100 = 8⅓ (see § 168).

(iv.) *Three chests of tea, one containing* 60 *lbs. at* 3*s*. 6*d*. *per lb., another* 58 *lbs at* 2*s*. 6*d*., *and the third* 340 *lbs. at* 2*s*., *are mixed. If the mixture is sold at* 2*s*. 8*d*. *per lb., find the gain or loss per cent. on the whole.*

	£	s.	d.
S.P. of 60 + 58 + 340 = 458 lbs. @ 2*s*. 8*d* per lb. =	£61	1	4
C.P. of 60 lbs. @ 3*s*. 6*d*. per lb. =	£10	10	0
58 lbs. @ 2*s*. 6*d*. ,, =	7	5	0
340 lbs. @ 2*s* ,, =	34	0	0
Total C.P. =	51	15	0
Total gain =	£9	6	4

∴ Gain per cent. $= \frac{£9\ 6\ 4}{£51\ 15\ 0} \times 100 = 18+$

Example—

(v.) *By selling an article for 7s. 6d. a person gained $12\frac{1}{2}$ per cent. What did it cost?*

S P. includes the gain, and is therefore equal to

$$\text{C.P.} + 12\tfrac{1}{2}\% \text{ of C.P.} = \frac{112\frac{1}{2}}{100} \text{ of C.P.}$$

$$\text{Hence } \frac{100}{112\frac{1}{2}} \text{ of S.P.} = \text{C.P.} \quad (\S\ 135.)$$

$$\textit{i.e. } \frac{100}{112\frac{1}{2}} \text{ of } 7s.\ 6d. = \text{C.P.}$$

$$\therefore \frac{\overset{8}{\cancel{200}}}{\underset{9}{\cancel{225}}} \text{ of } \overset{10}{\cancel{90}}d. = 80d. = 6s.\ 8d. = \text{C.P.}$$

202. The difficulty attached to this last type of question, and the tendency shown by many students to deduct the $12\frac{1}{2}\%$ from the S.P., have induced us to recommend the following simple plan of dealing with all profit and loss questions.

In many examples, not only of profit and loss questions, but also of other problems, students experience difficulty in setting down the relevant points of the question concisely before them. The following method, with profit and loss questions, should make that step easy.

After reading the question through, set down the main points in two columns, the left being for percentage prices, the right for actual prices, the question mark indicating the unknown. The percentage C.P. will always be 100. We will now work out the above examples by this method.

Examples—

(i.) would read— C.P. = 100 C.P. = 15s.
S P. = 105 S.P. = ?

Beginning with the complete column, write the question out in the ordinary proportion form, thus :—

If a C P. of 100 gives an S.P. of 105
then „ 15 „ „ ?

$$= \frac{105 \times 15}{100} = 15s.\ 9d.$$

(ii.) would read— C.P. = 100 C P. = 5s.
S.P. = 80 S.P. = ?

If a C.P. of 100 gives an S.P. of 80
then „ 5s. „ „ ?

$$= \frac{80 \times 5}{100} = 4s. \text{ per doz}$$

$\therefore$ S.P. of each = 4d.

Examples—

(iii.) would read— C.P. = 100 C.P. = 37*s.* 6*d.* = $37\frac{1}{2}$.
S.P. = ? S.P. = £2, 0*s.* $7\frac{1}{2}$*d.* = $40\frac{5}{8}$.

If C.P. of $37\frac{1}{2}$ gives an S.P. of $40\frac{5}{8}$
,, 100 ,, ,, ?

$$\frac{325 \times 100 \times 2}{8 \times 75} = 108\frac{1}{3}$$

Hence gain per cent. = $108\frac{1}{3} - 100 = 8\frac{1}{3}$.

(iv.) (after finding totals) would read—
C.P. = 100 C.P. = £51 15 0
S.P. = ? S.P. = £61 1 4

If a C.P. of £51·75 gives an S.P. of £61·0$\dot{6}$
then C.P. of 100 ,, ,, ?

$$\frac{61{\cdot}0\dot{6} \times 100}{51{\cdot}75} = 118 +$$

∴ Gain ≑ 18%.

(v.) would read— C.P. = 100 C.P. = ?
S.P. = $112\frac{1}{2}$ S.P. = 7*s.* 6*d.*

If a C.P. of 100 gives an S.P. of $112\frac{1}{2}$
then C.P. ? ,, ,, $7\frac{1}{2}$

$$\frac{100 \times 15 \times 2}{2 \times 225} = 6\frac{2}{3} = 6s.\ 8d.$$

It will be seen from this that the various types may be worked out by the same method.

Miscellaneous Examples—

(i.) *A dealer professing to sell at cost price uses a pound weight, which weighs only 15 ounces. Find his gain per cent.*

This question will be more easily understood if we remember that a person gains on what he really sells, not on what he pretends to sell; hence he gains the price of 1 ounce on 15 ounces, not on 1 lb. Or supposing the article cost the dealer 1*d.* per ounce, he sells for 16 pence that which cost him 15 pence.

Hence C.P. 100 C.P. 15,
S.P. ? S.P. 16

If C.P. 15 has S.P. 16
then C.P. 100 has ?

$$\frac{16 \times 100}{15} = 106\frac{2}{3}. \quad \therefore 6\frac{2}{3}\ \%\ \text{gain.}$$

(ii.) *A man sells 4 articles for what he paid for 5. Find his gain per cent.*

Note.—He sells 4 articles, hence the C.P. at say 1*d.* each would be 4*d.*, while the S.P. would be 5*d.* (what he paid for 5).

Hence C.P.	100	C.P.	4*d.*
S.P.	?	S.P.	5*d.*

Answer **25 per cent.**

When an article passes through various hands between producer and consumer the question may be worked by reduction, as explained in § 88.

(iii.) *A sells to B at a gain of* 10 %, *B to C at a gain of* $12\frac{1}{2}$ %, *C to D at a loss of* 20 %. *If D paid £9, 18s. for the article, find what it cost A to produce it.*

Here we have D's price to find A's price.

TABLE.

D		C		B		
80	=	100				
		112·5	=	100		
				110	=	100

D's price of £9, 18*s.* $= £9{\cdot}9 \times \frac{100}{80} \times \frac{100}{112{\cdot}5} \times \frac{100}{110}$.

= £10 is cost to A.

Examples 66.

1. If I buy 30 dozen of wine, 6 bottles containing a gallon, for £75, at what rate must I sell it per quart so as to gain 15 % on my outlay?

2. A man sells 10 cwts. of sugar at $2\frac{3}{4}d.$ per lb., thereby gaining 11*s.* 8*d.* What is his profit per cent.?

3. 100 tons of coal were bought for £80, and three-fourths of it were sold at an advance of 25 per cent., while the remainder was disposed of at 1*s.* per ton under cost price. Calculate (1) the actual profit and (2) the gain per cent.

4. A grocer buys 3 cwts. of sago at 5*d.* per lb. and 7 cwts. at $6\frac{1}{4}d.$ per lb.; he sells $5\frac{1}{2}$ cwts. at $5\frac{1}{2}d.$ per lb. At what rate per bag of 12 lbs. must he sell the rest to gain 25 per cent. on the whole?

5. What profit per cent. is made when £7, 4*s.* 11*d.* is gained by an outlay of £123, 6*s.* 8*d.*?

6. A man bought a 75-gallon cask of brandy at £1, 9*s*. 6*d*. a gallon; 3 gallons leaked out. At what rate per gallon must he sell the remainder to gain a profit of 20 per cent. on his outlay?

7. A merchant buys oranges at the rate of 2 for three-halfpence. How many should he sell for £1, 19*s*. that he may gain 30 per cent.?

8. A corn factor buys 110 quarters of corn at 6*s*. 3*d*. per bushel; 45 per cent. is damaged and sold at a loss of 6 per cent.; the rest is sold at a profit of 25 per cent. Find the gain on the whole transaction.

9. A builder sold a house for £945, thereby gaining 8 per cent. on his outlay. What did it cost him to build it? If the purchaser lets the house at £70 a year, find how much per cent. per annum he makes on the purchase-money.

10. If the wholesale and retail profits are 20 and 25 per cent. respectively, what is the manufacturer's price for an article which the retail dealer sold to a customer for £10?

11. A grocer buys $\frac{3}{4}$ cwts. of tea for £35 and sells it at the rate of 2*s*. 3*d*. per lb. Find the gain per cent. on his outlay.

12. If the wholesale and retail profits on an article are 10 and 50 per cent. respectively, what portion of the price paid by the consumer is profit? (See also § 88.)

13. A person bought 300 sheep, of which he sold 60 at £2, 3*s*. each, and found he was gaining $7\frac{1}{2}$ per cent. At what price should he sell the rest so as to gain 10 per cent. on his whole outlay?

14. I sold goods at a profit of 10 per cent.; the money thus realised I invest in goods which I sell at a loss of $12\frac{1}{2}$ per cent. How much do I gain or lose per cent. by the two transactions?

15. A man who has been selling tea at 3*s*. 4*d*. per lb. at a profit of 25 per cent., lowers his price so as to gain only 2*d*. per lb. In what ratio must his weekly sales increase so that his actual gain may be doubled?

16. A market woman bought a certain number of ducks at 2*s*. each, and $\frac{2}{3}$ of that number at 3*s*. each; by selling them at 5*s*. a pair she gained £4, 10*s*. Find (1) the number of ducks bought at each price; (2) the gain per cent.

17. A fruiterer purchases equal quantities of oranges at 3 for a penny and at 5 for 2*d*. How much per cent. does he gain or lose by selling the whole at 2 for a penny?

18. 1600 yds. of cloth were bought at 2*s*. 6*d*. per yard and sold at a profit of £35. If one-fourth were sold at cost price and one-half at a gain of 25 per cent., what percentage of profit was gained on the rest?

19. A merchant sells goods to a grocer at a profit of 60 per cent. The grocer fails, and pays 3*s*. 8*d*. in the £. How much per cent. does the merchant gain or lose?

20. A man buys 2 horses, A and B. A costs £25 more than B. He sells A at a profit of 15 per cent. and B at a profit of 8 per cent; his total gain is £37, 2*s*. What was the original price of each horse?

21. A manufacturer buys his raw material at the rate of £50 per ton. The cost of manufacture amounts to £2, 3*s.* 4*d.* per cwt. He sells the manufactured article at 11*d.* per lb. Find his gain per cent.

22. The cost of producing 5000 copies of a book is estimated to be £475. If 60 presentation copies are given away, and on the rest the author is to receive a royalty of 2*d.* per copy, at what price per dozen (giving 13 copies in the dozen) must the publisher sell the remaining copies so as to make a profit of 15 per cent.?

23. A grocer bought 10 cwts. 2 qrs. 24 lbs. of sugar for £5. Five per cent. of it was damaged in transit. What percentage on his outlay did he gain by selling the remainder at $1\frac{1}{2}$*d.* per lb.?

24. A grocer bought a cask of sugar containing $2\frac{3}{4}$ cwts. at $3\frac{1}{4}$*d.* per lb. Allowing $2\frac{1}{2}$ per cent. for waste, what must he sell it at per cwt. so as to gain 20 per cent. on his outlay?

25. A sold a house to B at a profit of 6 per cent. B sold it to C for £393, 6*s.* 1*d.*, gaining $8\frac{1}{4}$ per cent. Had A sold direct to C for £376, 15*s.*, what would have been his profit per cent.?

26. A tea dealer mixed 45 lbs. of tea with 30 lbs. of a better quality. By selling the mixture at 4*s.* per lb., he gained 20 per cent. on his outlay. Find the cost of each kind of tea, the difference in price being 1*s.* 8*d.* per lb.

27. A grocer sells coffee containing 25 per cent. of chicory at 1*s.* 4*d.* per lb. What does he gain or lose per cent. if the cost price of the pure coffee be 1*s.* and that of chicory 4*d.* per lb.?

28. Books are bought at 11 for 1*s.* and sold at 14*d.* a dozen. What is the gain per cent., and how many books must be sold to realise a profit of 10*d.*?

29. An egg merchant buys eggs at 9*d.* per doz. and sells them at 15 for 1*s.* How much per cent. on his outlay does he gain or lose?

30. A grocer bought 10 doz. of eggs at 1*s.* 3*d.* per doz. He sold one half at 1*s.* 5*d.* per doz. and the rest at 2*s.* per doz. Find his gain per cent.

31. A sold a house to B at a profit of $12\frac{1}{2}$ per cent. B sold it again to C, at a profit of 16 per cent., for £1827. Find what A paid for the house.

32. One bookseller allows his customers 2*d.* in the shilling discount and, in addition, 5 per cent. on the remaining price. Another simply allows 20 per cent. on the published price of the books. Find which terms are the better for a customer buying books, the published price of which is £6.

33. A man paid £27 for a watch, including a duty of 8 per cent. What was the net price of the watch?

34. If a merchant sells goods to a retail dealer at a profit of 40 per cent., and the retail dealer, becoming bankrupt, pays only 12*s.* 6*d.* in the £, what per cent. does the merchant lose?

35. A dealer buys one kind of tea at £10 per cwt. and another at £14, 5*s.* per cwt. and mixes them in the proportion of 2 of the first to 3

of the second. Find to two decimals his gain or loss per cent. by selling the mixture at 2*s*. 6*d*. per lb.

36. A man buys equal quantities of oranges at 4 for $2\frac{1}{2}d$. and at 3 for 2*d*. What per cent. (to two decimals) does he gain or lose by selling them at 7 for $4\frac{1}{2}d$.?

37. A wholesale grocer buys 567 cwts. of sugar at £1, 19*s*. $10\frac{1}{2}d$. per cwt. and mixes it with 1161 cwts. of a different quality worth £2, 2*s*. $6\frac{1}{2}d$. per cwt. Find what price per lb. he must charge for the mixture to gain 12 per cent.

38. If a dishonest dealer uses a weight of 15·03 oz. for 1 lb. (Avoir.) and professes to sell his goods at cost price, what does he gain per cent.?

39. A merchant bought 52 cwts. of potatoes. By selling them at 10*d*. a stone (as he thought) he gained £3, 1*s*. 4*d*., whereas had they been sold at 8*d*. a stone (using the same weights) he would have lost 13*s*. 4*d*. Find (*a*) what was wrong with his weights; (*b*) the original cost of the potatoes.

40. A dishonest fruit vendor professes to sell his goods at cost price: but he uses a weight of 14·75 oz. for the lb. weight. Find his gain per cent.

41. A piece of cloth was bought at a cheap sale, where the merchant professed to be selling at cost price. It was supposed to measure 14 yds. 2 ft., but the yard measure used was $\frac{3}{8}$ of an inch short. Find his real gain per cent. to two decimals.

42. If a draper marks his goods so as to gain 5 per cent. on the cost price, and then uses a yard stick which measures $35\frac{1}{2}$ ins., what is his real gain per cent.?

43. A dealer professes to lose 5 per cent. on a certain tea, but uses a weight equal to 15·04 oz. instead of 1 lb. Find his real loss or gain per cent.

44. Three different parcels of tea, viz., 60 lbs. at 3*s*. 6*d*. per lb. original cost, 58 lbs. at 2*s*. 6*d*., 340 lbs. at 2*s*., are mixed. At what price per lb. must the mixture be sold so as to realise a gain of 6% of the original cost?

45. A merchant sells 8 articles for what he paid for 9. Find his gain or loss per cent. Had he sold 9 for what he paid for 8, what would have been the result?

46. A wine merchant mixed 17 gallons of spirits at 10*s*. 6*d*. per gallon with 8 gallons at a different price. By selling the mixture at 13*s*. per gallon he gained 20 per cent. Find how much his two kinds of spirits differed in price per gallon.

47. A grocer buys coffee at £8, 10*s*. per cwt. and chicory at £2, 10*s*. per cwt. He mixes them in the proportion of 5 parts of chicory to 7 parts of coffee. Find what price per cwt. he must charge for the mixture to gain $16\frac{2}{3}$ per cent.

48. A grocer mixes 5 cwts. of tea which cost 12 guineas per cwt. with 2 cwts. at 14 guineas per cwt. What is his gain or loss per cent. if he sell the mixture at 2*s*. 6*d*. per lb.?

49. A grocer buys coffee at 8 guineas per cwt. and chicory at £2, 16*s.* per cwt. At what price per lb. must he sell a mixture of 5 parts of chicory to 7 of coffee to gain 25 per cent. on his outlay?

50. A merchant buys teas at 1*s.* 6*d.*, 1*s.* 8*d.*, and 1*s.* 10*d.* per lb. and mixes them in the proportion of 5, 4, and 1. At what price per lb. must he sell the mixture so as to make a profit of 25 per cent.?

203. It will not infrequently happen that a workable selling price gives a fractional percentage gain or loss (as in example 4) or *vice versâ*. It is therefore often necessary to find a workable selling price which will produce a certain desired percentage at least.

Examples—

(vi.) *Articles costing 8s. 4d a dozen are sold so as to gain* 10% *at least. Find the price at which each must be sold.*

In this case C.P. = 100   C.P. = 8*s.* 4*d.*
S P. = 110   S P = ?

If a C P. of 100 gives an S.P of 110
C.P. of 8*s.* 4*d* gives an S.P. of ?

$$\frac{110 \times 8s.\ 4d.}{100} = 9s.\ 2d.$$

S P. = 9*s* 2*d.* per dozen = $9\frac{1}{6}d.$ each

Hence S P. must be $9\frac{1}{4}d$ each to obtain a gain of 10% at least.

Note that the desired price must be taken above $9\frac{1}{6}d.$, as anything less than that would produce *less* than 10% gain.

(vii) *A consignment of apples costing* £50, 5*s.* *is slightly damaged. For how many shillings must it be sold so that the loss will not be more than* 4%?

C.P = 100   C.P. = £50·25
S P. = 96   S.P. = ?

If a C P. of 100 has an S.P. of 96
then a C.P of 50 25 has an S.P. of ?

$$\frac{96 \times 50\ 25}{100} = 48\ 24$$
$$20$$
$$964{\cdot}80$$

∴ It must be sold for **965*s*.**

204. Many business men calculate their gain or loss on the selling price instead of the cost price. Thus, if an article costing £30 be sold for £40, the gain is reckoned as £10 gain on £40, hence 25% gain, instead of £10 on £30, $33\frac{1}{3}$% gain. Care must be taken to distinguish which method of calculation is to be adopted (see § 201).

Since, in such cases, the selling price is substituted for the cost price in calculating the percentage rate of gain or loss, the same must be done in mapping out the question according to § 202, and in the percentage column, S.P., instead of C.P., will be 100.

Examples—

(viii.) *An article cost £1, 12s. 4d. For what must it be sold so that $33\frac{1}{3}$% of the returns may be profit?*

Here C.P. $= 66\frac{2}{3}$ C.P. = £1, 12s. 4d.
S.P. = 100 S.P. = ?

If a C.P. of $66\frac{2}{3}$ gives an S.P. of 100
then a C.P. of £1, 12s. 4d. gives an S.P. of ?

$$\frac{100 \times £1,\ 12s.\ 4d. \times 3}{200} = 48s.\ 6d. = £2,\ 8s.\ 6d.$$

(ix.) *A man sold a horse for £50, and found on calculating that his loss amounted to 5% of the S.P. Find the C.P.*

C.P. = 105 C.P. = ?
S.P. = 100 S.P. = 50

If a C.P. of 105 gives an S.P. of 100
then a C.P. of ? gives an S.P. of 50

$$\therefore \text{C.P.} = \frac{105 \times 50}{100} = 52\tfrac{1}{2} = £52,\ 10s.$$

or S.P. = £50
loss = 5% of S.P. = £2, 10s.
∴ C.P. = £52, 10s.

Examples 67.

1. If eggs are bought at 7d. per dozen, how many must be sold for a shilling to make as nearly as possible a profit of 20 per cent.?

2. A woman bought a certain number of ducks at 2s. 6d. each, and $\frac{4}{11}$ of that number at 3s. 4d. each. By selling at 7s. a pair she gained £3, 11s. Find (1) the number of ducks bought at each price; (2) lowest price (in exact number of pence) each must be sold at to gain 25 per cent. at least.

3. A grocer bought 64 lbs. of tea at 3s. $4\frac{1}{2}d.$ per lb., but 26 lbs. being damaged he had to dispose of this at 2s. 6d. per lb. Find, to the nearest penny, what he must sell the rest at to gain at least 8 per cent. on the whole quantity bought.

4. A trader buys 100 articles for £12, 13s. How much must he charge for each so that 8 per cent. of the amount received may be profit?

Note.—C.P. must be *less* than S.P. to give a profit.

5. A merchant buys cloth in Paris at 10 francs per metre. At what price per yard must he sell it in London to gain at least $12\frac{1}{2}$ per cent. on his outlay? (£1 = 25·2 fr.; 1 metre = 39·37 in.)

6. What percentage of profit must a trader add to the prime cost of an article in order to secure a profit of 15 per cent. on the return? What would be the selling price of an article that cost 8*s.* 6*d.* to secure this gain?

7. A merchant buys 1 cwt. of tea for £13, 6*s.* What must be charged for it per lb. so that 5 per cent. of the amount *received* may be profit?

8. If the wholesale and retail profits calculated on the return are respectively 5 per cent. and 40 per cent., what part of the price paid by the consumer is profit?

9. A merchant bought sugar at £3, 15*s.* per cwt. At what price per lb. did he sell it so as to gain 20 per cent. at least?

10. A merchant bought cloth at 3*s.* 6*d.* per yard. At what price per yard must he sell it so that $\frac{3}{8}$ of the amount received may be profit?

11. A grocer mixes 40 lbs. of tea at £14 per cwt. with 60 lbs. at £15, 8*s.* per cwt. At what rate per lb. must he sell it so that 10% at least of the returns may be profit?

12. What is the lowest quotation for soap per cwt. that a merchant can make so as not to lose more than 14 per cent. on the cost of manufacture, calculated at $5\frac{1}{2}d.$ per lb.?

205. In most firms where ready cash is paid discount is allowed. This, when calculated as a percentage, is always computed on the Marked Price of the article sold, and must be taken into account when calculating the rate of profit.

Let us consider the following example:—

A man marks his goods so as to gain 5%. On one occasion rather than lose a customer he offers 5% discount. Is he right in supposing that he loses nothing?

Suppose the article costs £100
then C.P. = 100
Gain 5% of 100 = 5
∴ Marked Price (M.P.) = 105
(This gives 5% profit.)
Then Marked Price = £105
Discount 5% of 105 = £5, 5*s.*
∴ S.P. = £99, 15*s.*

Since C.P. = £100 and S.P. = £99, 15*s.*, it is evident that there is a loss of 5*s.*

From the above example it will be seen that a man who marks his goods 10% above cost price and allows 4% discount does not gain 6%.

206. Questions involving profit or loss and discount should be worked out in two parts. The actual selling price is usually calculated first.

Rule.—Work from C.P. to M.P. by way of S.P., or from M.P. to C.P. by way of S.P.

C.P. ⇄ S.P. ⇄ M.P.

Examples—

(i.) *At what price must an article costing £1, 5s. be marked so that a profit of 14% may be realised and a discount of 5% allowed?*

First, to find S.P.— C.P. 100   C.P. £1, 5s.
S.P. 114   S.P. ?

If a C.P. of 100 gives an S.P. of 114
a C.P. of $1\frac{1}{4}$ „ „ ?

$\frac{114 \times 5}{100 \times 4}$ (leave in this form).

To find M.P.—Since answer to first part gives the S.P., then M.P. takes the place of C.P.

M.P. 100   M.P. ?
S.P. 95   S.P. $\frac{114 \times 5}{100 \times 4}$

if an M.P. of 100 has an S.P. of 95
an M.P. of ? „ „ $\frac{114 \times 5}{100 \times 4}$

$$\text{M.P.} = \frac{100 \times 114 \times 5}{95 \times 100 \times 4} = £1,\ 10s.$$

Hence the article must be marked at £1, 10s.

Proof:—5% of £1, 10s. = 1s. 6d. ∴ S.P. = £1, 8s. 6d.
Since the C.P. is £1, 5s. this gives a gain of 3s. 6d.

$$\text{and gain } \% = \frac{3s.\ 6d.}{25s.} \times 100 = 14.$$

(ii.) *A man marked an article at £1 and sold it, allowing 10% discount He obtained 20% profit. What did the article cost?*

To find S.P.— M.P. = 100   M.P. = 1
S.P. = 90   S.P. = ?

If an M.P. of 100 has an S.P. of 90
„ „ 1 „ „ ?

$$\text{S.P.} = \frac{90 \times 1}{100}.$$

To find C.P.— C.P. = 100   C.P. = ?
S.P. = 120   S.P. = $\frac{90 \times 1}{100}$

If a C.P. of 100 has an S.P. of 120
then a C.P. ? „ „ $\frac{90 \times 1}{100}$

$$\therefore \frac{100 \times 90 \times 1}{120 \times 100} = 15s. = \textbf{Cost Price.}$$

Examples—

(iii.) *A man buys a piano for £30. At what price must he mark it for sale so as to allow 10% discount and yet obtain at least 15% profit?*

To find S.P.— C.P. 100 C.P. 30
S.P. 115 S.P. ?

If a C.P. of 100 has an S.P. of 115
then C.P. of 30 ,, ,, ?

$$\therefore \text{S.P.} = \frac{115 \times 30}{100}$$

To find M.P.— M.P. 100 M.P. ?

S.P. 90 S.P. $\frac{115 \times 30}{100}$

If an M.P. of 100 gives an S.P. of 90
then an M.P. of ? ,, ,, $\frac{115 \times 30}{100}$

$$\frac{100 \times 115 \times 30}{90 \times 100} = \frac{115}{3} = £38,\ 6s.\ 8d.$$

Should the price be wanted in an exact number of shillings, take the next higher shillings, £38, 7*s*.

(iv.) *A merchant observed that if he were to allow a discount of 2½% he would lose 2%. What would be the gain or loss per cent. if he sold the articles at the price marked?*

Suppose the articles were marked at £100
Then M.P. = 100
and discount = 2½% = 2½
∴ S.P. = 97½

To find C.P.— C.P. 100 C.P. ?
S.P. 98 S.P. 97½
If a C.P. of 100 gives S.P. of 98
,, C.P. of ? gives S.P. of 97·5

$$\therefore \text{C.P.} = \frac{100 \times 97{\cdot}5}{98}$$

To find gain or loss per cent.—

C.P. = 100 C.P. = $\frac{100 \times 97{\cdot}5}{98}$

S.P. = ? S.P. = 100

If a C.P. of $\frac{100 \times 97{\cdot}5}{98}$ gives S.P. of 100
C.P. of 100 gives S.P. of ?

$$= \frac{100 \times 100 \times 98}{97{\cdot}5 \times 100} = 100{\cdot}51$$

∴ Gain per cent. = 100·51 − 100 = ·51.

Examples—

(v.) *A merchant marks his goods* 25 *per cent. above cost and allows* 10 *per cent. discount for cash. Find the selling price of an article which costs* £2.

To find M.P.

C.P.	100	C.P.	2
M.P.	125	M.P.	?

If a C.P. of 100 has M.P. 125
then C.P. of 2 ,, ?

$$\therefore \text{M.P.} = \frac{125 \times 2}{100}.$$

To find S.P.

M.P.	100	M.P.	$\frac{125 \times 2}{100}$
S.P.	90	S.P.	?

If an M.P. of 100 gives S.P. 90
then M.P. of $\frac{125 \times 2}{100}$,, ?

$$\frac{90 \times 125 \times 2}{100 \times 100} = £2,\ 5s.$$

206a. All the questions in § 206 may be worked by the principle of Reduction (§ 88).

Example (i.) may be stated thus. Express a C.P. of £1, 5s. as its corresponding M.P., given that C.P. 100 has an equivalent S.P. = 114, while M.P. 100 has equivalent S.P. 95

The table then is

C.P.		S.P.		M.P.
100	=	114		
		95	=	100.

$$\text{C.P. of } £1,\ 5s. = 1{\cdot}25 \times \frac{114}{100} \times \frac{100}{95} = £1,\ 10s.$$

(ii.) Table.

M.P.		S.P.		C.P.
100	=	90		
		120	=	100

$$\text{M.P. of } £1 = 1 \times \frac{90}{100} \times \frac{100}{120} = 15s.$$

(iii.) Table.

C.P.		S.P.		M.P.
100	=	115		
		90	=	100

$$\text{C.P. of } £30 = 30 \times \frac{115}{100} \times \frac{100}{90} = £38,\ 6s.\ 8d.$$

(iv.) Table.

M.P.		S.P.		C.P.
100	=	$97\frac{1}{2}$		
		98	=	100

$$\text{M.P. of } 100 = 100 \times \frac{97\frac{1}{2}}{100} \times \frac{100}{98}.$$ Finish as before.

(v.) Table.

C.P.		M.P.		S.P.
100	=	125		
		100	=	90

$$\text{C.P. of } £2 = 2 \times \frac{125}{100} \times \frac{90}{100} = £2\ 5s.$$

Examples 68.

1. A draper bought cloth at 3*s*. 6*d*. per yard. Find what price he must make it per yard so that he may allow a discount of 5 per cent. and still make a profit of 20 per cent.

2. If I import sugar at £15 per ton, at what price per cwt. must I quote it in order to make 20 per cent. profit and allow, besides, to my customers 10 per cent. discount on their bills?

3. A farmer bought a horse for £47, 10*s*. and sold it so as to realise 25 per cent. profit, after allowing the purchaser a reduction of 5 per cent. What price does he charge?

4. A grocer sells sugar at 4*d*. per lb. and takes off 5 per cent. for cash payment. Find what it costs him per cwt. in order that he may make a profit of 60 per cent.

5. A merchant bought cloth at 12*s*. per yard. He wishes to sell it so as to be able to allow a discount of 4 per cent. and yet make a profit of 16 per cent. Find what price per yard he must charge for it.

6. A tradesman's prices are $12\frac{1}{2}$ per cent. above cost price. If he allows a discount of £1, 2*s*. 6*d*. on a cash purchase of £22, 10*s*., what is his net profit per cent.?

7. A merchant marks on an article a price which was 50 per cent. above cost price, but to purchasers he gave 12 per cent. discount on the marked price, thereby gaining 15*s*. What was the cost price?

8. For what must an article be marked which cost £6, 13*s*. 10*d*., so that 5 per cent. of the marked price may remain profit after a discount of $3\frac{1}{3}$ per cent. has been deducted?

9. If sugar be bought at £3, 14*s*. per cwt., how must it be sold per lb. to gain $12\frac{1}{2}$ per cent. on the outlay, after deducting $7\frac{1}{2}$ per cent. discount?

10. An article cost 3 guineas. What must it be marked at to enable the merchant to allow a discount of 10 per cent., and yet to make a profit of $8\frac{1}{3}$ per cent. on his outlay?

11. A merchant bought cloth at 5*s*. 4*d*. a yard. At what price must he mark it so as to allow 4 per cent. discount and still gain $12\frac{1}{2}$ per cent. on his outlay?

12. Find the cost price of an article which, if marked at 5*s*. and sold at a discount of 10 per cent., will still return 8 per cent. profit on the cost price.

13. A merchant marks his goods so as to gain 5 per cent. profit. What is the greatest workable discount per cent. he can allow so as still to gain 2 per cent.?

14. A merchant marks his goods at 10 per cent. above cost price. What is the utmost workable discount he can allow so that he may not lose anything?

15. A draper marks his goods so as to gain as profit 5 per cent of the selling price. What gain per cent. would this be if calculated on the cost price?

16. One dealer makes his prices 10 per cent. above cost, while another arranges them so that he shall have a profit of $12\frac{1}{2}$ per cent. on the return. If the manufacturer's price for an article is £6, 10*s.*, find the difference between the dealers' respective selling prices.

17. What percentage added to the cost price of an article will allow of 5 per cent. discount being given and yet return 14 per cent. profit?

207. Frequently merchants have to readjust their price lists, and so alter their rates of profit. The following examples illustrate questions arising out of such alterations :—

Example—

(i.) *By selling an article for £2, 16s. 3d. a man gained* $12\frac{1}{2}\%$. *At what price ought he to have sold it to have gained* 20% ?

1st. To find C.P.—C.P. 100   C.P. ?
S.P. $112\frac{1}{2}$ S.P. £2, 16*s.* 3*d.*

If a C.P. of 100 gives S.P. of $112\frac{1}{2}$
then a C.P. of ? gives S.P. of £2, 16*s.* 3*d.*

$$\frac{100 \times 2\cdot8125}{112\cdot5} = \frac{2812\cdot5}{1125} = £2\cdot5.$$

2nd. To find new S.P.—C.P. = 100   C.P. = £2·5
S.P. = 120   S.P. = ?

If a C.P. of 100 gives S.P. of 120
then a C.P. of 2·5 gives S.P. of ?

$$\frac{120 \times 2\cdot5}{100} = £3.$$

The above question may be worked in one step by taking percentage selling price.

Thus :—If a percentage S.P. of $112\frac{1}{2}$ gives a corresponding actual S.P. of £2, 16*s.* 3*d.*, find the actual S.P. corresponding to a percentage S.P. of 120.

i.e. If percentage S.P. of $112\frac{1}{2}$ gives an actual S.P. of £2, 16*s.* 3*d.*
then percentage S.P. of 120 gives an actual S.P. of ?

$$\frac{\overset{\cdot025}{2\cdot8125} \times 120}{112\cdot5} = £3$$

Example—

(ii.) ***By selling an article for 12s. I would lose 4%. What would be the gain or loss % by selling it at 13s. 9d.?***

To find C.P.— C.P. 100 C.P. ?
S.P. 96 S.P. 12s.

If a C.P. of 100 gives an S.P. of 96
then a C.P. of ? gives an S.P. of 12s.

$$\frac{100 \times 12}{96} = 12s.\ 6d.$$
8

To find new gain or loss—C.P. = 100 C.P. 12s. 6d.
S.P. = ? S.P. 13s. 9d.

If a C.P. of 12s. 6d. gives an S.P. of 13s. 9d.
then a C.P. of 100 gives an S.P. of ?

11
$$\frac{13{\cdot}75 \times 100}{12{\cdot}5} = 110$$

Hence gain per cent. = 110 − 100 = 10

Or, as one :—If a percentage S.P. of 96 gives a corresponding actual S.P. of 12s., what percentage S.P. will give a corresponding S.P. of 13s. 9d.?

8
$$\frac{96 \times 13{\cdot}75}{12} = 110$$

$\therefore$ gain = 110 − 100 = 10%

Note.—Care must be taken not to work with the percentage gain or loss alone, but with the percentage S.P., as the proportions exist between all the selling prices, and not between the percentage gain or loss and the actual Selling Price.

Examples 69.

1. If a dealer loses 5 per cent. by selling a horse for £116, 17s., find how much he will lose or gain per cent. by selling the horse for £132, 4s. 6d.

2. By selling an article for £111, 0s. 10d., a man would gain 2½ per cent. What would be the gain or loss per cent. by selling at £105, 12s. 6d.?

3. By selling a carriage for £117, 6s. 6d., a tradesman would lose 5 per cent. What per cent. will be gained if he sell it for £138, 18s. 9d.?

4. A horse dealer sold a horse for £30, 12s., losing 15 per cent. on what he paid for him, whereas he ought to have gained 25 per cent. Find how much he sold the horse under the price he had fixed on.

5. A house was sold at £1188, at a loss of 4 per cent. At what ought it to have been sold so as to gain 4 per cent.?

6. A man sold a field for £138, 12s., and by so doing lost 5½ per cent. of what it had cost him. Find what per cent. he would have gained had he sold it for £184, 5s.

7. If $\frac{1}{7}$ of the cost price of an article be gained by selling it for 5s. 5d., what percentage of profit would be gained by selling it for 5s. $8\frac{1}{4}$d.?

8. A man bought a horse and sold it at a loss of 10 per cent. If he had received £9 more, he would have gained $12\frac{1}{2}$ per cent. What did the horse cost?

9. If by selling sugar at 3d. per lb. a profit of 10 per cent. on the outlay be gained, what will be the gain or loss per cent. by selling it at 26s. per cwt.?

10. A man sells an article for 15s., and loses 25 per cent. What will be the loss or gain per cent. if he sells it for 17s. 6d.? Find also the prime cost.

XXIX.—MIXTURES.

208. Students are frequently asked to determine the proportions in which certain articles of given price should be mixed to produce a mixture valued at another given price. For example, "How should wines at 45s. and 49s. per gallon be mixed to produce a mixture worth 48s. per gallon?" In actual business people seldom blend wines or teas, &c., according to their prices, but rather according to experience in working with different qualities, and testing the results in flavour, &c., of mixing them. They then determine the price of the mixture from the materials mixed and not *vice versâ*. As, however, these questions are still met with in Examination Papers, and prove somewhat interesting of themselves, we shall briefly explain how they may be worked.

Examples—

(i.) *Taking the example above, set down the working as follows:—*

Method.—Set down the net price of the mixture and arrange below it in two columns, A and B, the difference between the price of each quality mixed and the price of the mixture already written down. When the price is above that of the mixture, place the difference below A, when below that price place the difference below B. [Under A if above, under B if below.]

Net price of mixture,
48s.
A B
1s. ←——→ 3s.

In this example 1s. goes below A and 3s. below B, since 49s. is 1s. above, and 45s. is 3s. below, 48s. Now every time 1 gallon of B is sold at the price of the mixture (48s.) 3s. is gained, while on every gallon of A so sold, 1s. is lost. The dealer can therefore afford to lose 1s. *three* times for every *one* time he gains 3s. Hence he should mix his wines in the proportion of 3 of A to one of B (as shown by the arrow heads), *i.e.* 3 gallons @ 49s. should be mixed with 1 gallon @ 45s.

Examples—

(ii.) *How should teas @ 2s. 4d., 2s. 5d., 2s. 10d. per lb be mixed so as to produce a mixture which if sold at 3s per lb. will give a profit of 20%?*

First obtain the net price of the mixture, as the profit is calculated after the mixture is made up, *i e.* the cost of the mixture is calculated and 20% is added on to give the S.P.

Method.—Arranging the differences as in Ex. (i), we get under A 4*d.* and under B 2*d* and 1*d.* Fill up any gaps in a column with figures already in that column.

Calculate the mixture formed by the first opposite pairs, *i.e* 2 lbs. of A (2*s.* 10*d*) to 4 of B (2*s.* 4*d.*), which is the same proportion as 1 lb. of A (2*s.* 10*d.*) to 2 of B (2*s.* 4*d*), (obtained by cancelling). Next get the mixture formed by the second pair, *i.e.* 1 lb. of A (2*s.* 10*d.*) to 4 lbs. of B (2*s.* 5*d*), and so on for any further pairs. (The necessity for filling up the gaps will now be seen)

Net price of mixture,
2*s.* 6*d.* (§ 201).

A		B
2*d.*		1*d.*
4*d.*	⟷	2*d.*
4*d.*	⟷	1*d.*

Since each mixture so formed may be sold at the desired price, the mixture formed by putting them all together will cost the same price per lb.

From the above we have—

1st { 1 lb. @ 2*s.* 10*d.*
      2 lbs. @ 2*s* 4*d.*
2nd { 1 lb. @ 2*s* 10*d.*
      4 lbs. @ 2*s.* 5*d.*

The total mixture is therefore made up in the proportion of 2 lbs. @ 2*s.* 10*d.* to 2 lbs. @ 2*s* 4*d* to 4 lbs @ 2*s* 5*d* , or halving these quantities, **1 lb. @ 2*s.* 10*d* , 1 lb. @ 2*s* 4*d* , and 2 lbs. @ 2*s.* 5*d***

209. When 4 or more different qualities are given to be mixed, there may be several different solutions, varying according to the different arrangements of the figures in the different columns, but all equally correct.

(iii.) *How must different qualities of flour @ 30s , 33s. 6d., 28s. and 37s. per quarter be mixed so as to be sold @ 36s. per quarter and bring in a gain of 12½%?*

This may be arranged thus—

Net price of mixture. 32*s.*				Net price of mixture, 32*s.*		
A		B	OR	A		B
3 * sixpences	⟷	4 sixpences		5*s.*	⟷	2*s.*
5*s*	⟷	4*s.*		3 sixpences	⟷	8 sixpences

both of which result in a different but equally correct mixture. The greater the number of different qualities, the greater the variation in the solutions possible

* Differences must always be expressed in terms of the same unit to obtain the proportions.

210. When the total amount of the mixture wanted is given, or the total amount of one or more of the mixed kinds, first find the *proportion* of articles mixed, then calculate the necessary amounts to correspond with the given totals. Should water be one of the ingredients of the mixture, work with it as costing nothing.

Example—

(iv.) *How many gallons of water must be mixed with 48 gallons of wine at 35s. a gallon so that the mixture may be sold at 35s. a gallon and a profit of 16⅔% gained.*

Net price of mixture,
30*s.*

A   B
0*s.* 1 ⟵⟶ 35*s.* 6

Proportion of mixture is 1 of water to 6 of wine,
i.e. 8 of water to 48 of wine.

Had the total 56 been given, instead of 48 galls. of wine,
Then, as above, 1 of water to 6 of wine.
∴ water = 1 out of 7 galls. of mixture
$= \frac{1}{7}$ of 56 = 8 galls. (Sect. XXVI.)
wine = 6 out of 7 galls. of mixture
$= \frac{6}{7}$ of 56 = 48 galls.

Examples 70.

1. A merchant mixes teas at 3s. a lb. and at 1s. 6*d.* a lb. By selling the mixture at 2s. 6*d.* per lb. he gains 25 per cent. In what proportion does he mix the teas?

2. A merchant mixes teas at 1s 6*d.* and 1s. 9*d.* per lb. respectively, and sells the mixture at 1s. 10*d.* per lb., gaining 10 per cent. In what proportion are they mixed?

3. A grocer sells his best quality of tea at 3s. 6*d.* per lb., and thereby gains a profit of 5 per cent.; he sells his second quality at 2s. 6*d.* per lb., gaining a profit of 8 per cent. In what proportion does he mix the two qualities, if by selling the mixture at 2s. 9*d.* per lb. he gains 10 per cent.?

4. How must wines at 36s., 37s., 39s., and 40s. per gallon respectively be mixed so as to be sold at 41s. 3*d.* per gallon, and gain 10 per cent.?

5. A wine merchant purchases 90 gallons of whisky at 13s. per gallon. How many gallons of water must he add to enable him to retail it at 16s. a gallon, and obtain a profit of 60 per cent. on his outlay?

6. How much coffee at 1s. 8*d.* per lb. must be mixed with 45 lbs. of chicory at 7*d.* per lb., so that by selling the mixture at 1s. 8*d.* per lb. 33⅓ per cent. on the outlay may be gained?

7. A cask capable of holding 90 gallons is filled with a mixture of two kinds of wine at 15*s.* and 16*s.* 6*d.* per gallon respectively; the whole is worth £71, 5*s.* Find the number of gallons of each kind.

8. A hosier sold 90 pairs of stockings and gloves for £12, 10*s.*, the stockings being sold at 3*s.* per pair and the gloves at 2*s.* 6*d.* per pair. Find how many pairs of each were sold.

9. A cask contains 60 gallons of spirits worth 12*s.* 6*d.* per gallon. Find how many gallons of water must be added to reduce the price to 8*s.* per gallon.

10. How much confectionery at 6*d.* and 9*d.* per lb. must be mixed with 10 lbs. at 8*d.* in order to sell the lot at $8\frac{1}{2}d.$ per lb.?

11. How many lbs. of tea at 4*s.* 3*d.*, 4*s.* 6*d.*, 5*s.* 4*d.* and at 5*s.* 6*d.* per lb. may be mixed together so that the mixture may be worth 5*s.* per lb.?

12. How many gallons of rum, at 16*s.* and at 19*s.* per gallon, and of water, calculated at costing nothing, must be mixed with 4 gallons of rum at 15*s.*, so that the whole mixture may be sold at 17*s.* per gallon?

XXX.—SIMPLE INTEREST.

211. When money is out on loan the individual or bank pays the owner a sum of money for its use. This is termed **Interest**, and may be either **Simple** or **Compound**. The money lent is called the **Principal**.

212. Simple Interest is usually calculated at a **Rate per cent. per annum.** Thus 3% Simp. Int. indicates that £3 is paid every year as interest on each £100 lent for the year.

213. While all questions in Simple Interest may be worked out as proportion questions, we prefer here to make use of the well-known formulæ.

Using **I** to indicate the **Simple Interest** obtained.
" **P** " **Principal.**
" **R** " **Rate per cent.**
" **Y** " **Years** (always express the time in years and fractions).
and **A** " **Amount** due at end of time $=P+I$,

we have the following $I=\frac{PRY}{100}$ *

The reason for the formula can be easily seen, the Interest being equal to R per cent. of P for each of Y years.

$$\text{Hence} = \left(\frac{R}{100} \text{ of } P\right) \times Y.$$

* Easily remembered from similarity of sound between I and PRY.

214. In finding Interest by means of the formula, it is an advantage to write the letter at the side, and insert the values there, then transfer these to their respective positions in the formula.

Examples—

(i.) *Find the Simple Interest on £560 for 4 years @ 5% Simple Interest.*

I = ?
P = 560
R = 5
Y = 4

$$I=\frac{PRY}{100}=\frac{560\times5\times4}{100}=£112.$$

(ii.) *Find the Simple Interest on £650 for $2\frac{1}{2}$ years @ $3\frac{1}{4}$% Simple Interest.*

I = ?
P = 650
R = $3\frac{1}{4}$
Y = $2\frac{1}{2}$

$$I=\frac{PRY}{100}=\frac{650\times13\times5}{100\times4\times2}=\frac{422\cdot5}{8}=52\cdot8125$$

52·8125
20
16·2500
12
3·00

= £52, 16s. 3d.

(iii.) *Find the Simple Interest on £250, 10s. for 9 mths. @ 2% Simple Interest.*

I = ?
P = 250·5
R = 2
Y = $\frac{9}{12}=\frac{3}{4}$

$$I=\frac{PRY}{100}=\frac{250\cdot5\times2\times3}{100\times4}=\frac{7\cdot515}{2}=3\cdot7575$$

3·7575
20
15·1500
12
1·80

= £3, 15s. 2d.

(iv.) *Find the Simple Interest on £520 for 146 days @ $4\frac{1}{2}$% Simple Interest.*

I = ?
P = 520
R = $4\frac{1}{2}$
Y = $\frac{146}{365}=\frac{2}{5}$

$$I=\frac{PRY}{100}=\frac{520\times9\times2}{100\times2\times5^*}=£9\cdot36=£9,\ 7s.\ 2\cdot4d.$$

215. From the formula $I=\frac{PRY}{100}$ we can get (Section XVII.)—

$$(1)\quad \frac{I\times100}{RY}=P.$$

$$(2)\quad \frac{I\times100}{PY}=R.$$

$$(3)\quad \frac{I\times100}{PR}=Y.$$

which are used to find principal, rate, and time respectively, given the other values. The same method is adopted as in finding Interest.

* Observe that the 2 and 5 should not be cancelled, as their product forms an easy divisor (see § 25).

Examples—

(i.) ***What principal will gain £50 in 3 years at 4 per cent. per annum, simple interest?***

I = 50
P = ?
R = 4
Y = 3

$$\frac{I \times 100}{RY} = P \therefore \frac{50 \times 100}{4 \times 3} = 416.\dot{6} = P.$$

i.e. Principal is £416, 13*s*. 4*d*.

(ii.) *In what time will £450 amount to £483, 15s., at $3\frac{3}{4}$ per cent. per annum, simple interest?*

I = A − P. (§ 213) = £483, 15s. − £450 = 33·75
P = 450
R = 3·75
Y = ?

$$\frac{I \times 100}{PR} = Y.$$

$$\frac{33 \cdot 75 \times 100}{450 \times 3 \cdot 75} = Y = 2 \text{ years.}$$

(iii.) ***At what rate per cent. per annum, simple interest, will £850 amount to £956, 5s. in 4 years?***

I = 106·25
P = 850
R = ?
Y = 4

$$\frac{I \times 100}{PY} = R.$$

$$\frac{106 \cdot 25 \times 100}{850 \times 4} = R = 3 \cdot 125 = 3\tfrac{1}{8}\%.$$

216 From the formulæ $I = \frac{PRY}{100}$ and $A = P + I$

$$\text{we get } A = P + \frac{PRY}{100} = P\left(1 + \frac{RY}{100}\right)$$

by means of which we can calculate the principal, when the amount, rate, and time are given.*

Example—***What sum will amount to £520 in 4 years 2 months at 2 per cent. per annum, simple interest?***

This type of question is very frequently incorrectly done, students mistaking principal for amount, and *vice versâ*. On filling in the values as before, the student will find in such cases that there are two blanks instead of the customary one, and this itself should suggest the necessary alteration in formula.

I = ? } A = 520.
P = ? }
R = 2
Y = $4\frac{1}{6}$

$$A = P\left(1 + \frac{RY}{100}\right) \qquad 520 = P\left(1 + \frac{2 \times 4\frac{1}{6}}{100}\right)$$

$$= P\left(\frac{100 + 8\frac{1}{3}}{100}\right) \qquad = P\left(\frac{108\frac{1}{3}}{100}\right)$$

$$\therefore \frac{520 \times 100}{108\frac{1}{3}} = \frac{520 \times 100 \times 3}{325} = P = £480.$$

* If we represent the interest on £1 for 1 year by i, then

$$i = \frac{R}{100};\ A = P(1 + iY);\ \text{and } P = \frac{A}{1 + iY}.$$

These formulæ are sometimes used instead of those given above.

217. Another method in common use for such questions is to find the amount of £100 at the given rate and in the given time, and then, working by proportion, to find the principal, thus:—

£100 in $4\frac{1}{6}$ years at 2 per cent. would amount to £$108\frac{1}{3}$, then

If £100 amounts to £$108\frac{1}{3}$

? amounts to £520

$$= \frac{100 \times 520}{108\frac{1}{3}} = £480.$$

Students, however, are very apt to write this out as follows:—

If £100 amounts to £102 in 1 year at 2 per cent.

? amounts to £520 in $4\frac{1}{6}$ years at 2 per cent.

$$\frac{100 \times 520}{102 \times 4\frac{1}{6}}$$ which is obviously incorrect;

hence we advise the use of the formula method. It also preserves uniformity in working interest questions.

Examples 71.

Find the simple interest on—

1. £845, 10s. 0d. for 3 years @ 2 per cent. per annum.
2. £615, 12s. 6d. for 5 years @ $3\frac{1}{2}$ „ „
3. £1250, 15s. 0d. for 4 years @ $1\frac{1}{4}$ „ „

Find the amount of—

4. £350 put out to simple interest for 2 years @ $3\frac{3}{4}$% per annum.
5. £655 „ „ „ 4 years @ $2\frac{1}{2}$% „
6. £960 „ „ „ $3\frac{1}{2}$ years @ $1\frac{1}{2}$% „

What principal will gain—

7. £35, 1s. 6d. as interest in 2 years @ $6\frac{1}{4}$ per cent. per annum?
8. £68, 5s. 0d. „ 3 years @ $3\frac{1}{2}$ „ „
9. £123, 15s. 0d. „ $4\frac{1}{2}$ years @ 5 „

At what rate per cent. per annum, simple interest, will—

10. £840, 10s. 0d. gain £63, 0s. 9d. in 3 years?
11. £958, 6s. 8d. gain £115, 0s. 0d. in 4 years?
12. £725, 10s. 0d. amount to £816, 3s. 9d. in $2\frac{1}{2}$ years?
13. £826, 2s. 6d. amount to £922, 10s. $1\frac{1}{2}$d. in 3 years 4 months?

In what time will—

14. £650, 10s. gain £54, 4s. 2d. @ 3 per cent. simple interest?
15. £484, 15s. gain £24, 4s. 9d. @ $1\frac{1}{4}$ „ „
16. £387, 10s. amount to £410, 15s. @ 2 „ „
17. £126, 5s. amount to £136, 7s. @ $1\frac{1}{3}$ „ „
18. Find the simple interest on £845, 10s. 6d. for $3\frac{1}{4}$ years at 3 per cent. per annum.

19. On March 1st, 1886, a person borrowed £438 at 4 per cent. per annum, simple interest, promising to return it as soon as it amounted to £450. On what day did the loan expire?

20. A sum of £78 was deposited in a bank on the 18th March, and on the 11th August it had gained £2, 12*s.* of interest. What was the rate per cent. per annum? (See § 219.)

21. To what sum will £477, 3*s.* 4*d.* amount in $4\frac{1}{2}$ years at 4 per cent. per annum, simple interest?

22. In what time will £183, 6*s.* 8*d.* amount to £230, 1*s.* 8*d.* at $4\frac{1}{2}$ per cent. per annum, simple interest?

23. The interest on £210 for one year is £6, 6*s.* What is the rate per cent., and in how many years will the amount be £241, 10*s.*?

24. A man could get £7, 10*s.* every year as interest at 3 per cent. What total interest will he have if he invest his money for 4 years at $2\frac{1}{2}$ per cent. per annum? (Simple interest in both cases.)

25. In what time will £416, 13*s.* 4*d.* put out at $4\frac{1}{2}$ per cent. per annum, simple interest, amount to £522, 18*s.* 4*d.*?

26. The interest on a sum of money at the end of $6\frac{1}{4}$ years is three-eighths of the sum itself. What per cent. per annum was charged?

27. The sum of £116, 13*s.* 4*d.* is lent on March 11th, 1892, and on the 16th October following the simple interest is found to be 3 guineas. What is the rate of interest?

28. A sum of £345 is lent at $2\frac{1}{2}$ per cent. per annum, simple interest, on condition of its being returned when it amounts to £697, 8*s.* $1\frac{1}{2}$*d.* How long will it be on loan?

29. At what rate per cent. per annum will the simple interest on £236, 6*s.* 8*d.* amount to £17, 14*s.* 6*d.* in $2\frac{1}{2}$ years?

30. A freehold house brings in a clear £50 per annum, and is sold for 23 years' purchase, the vendor allowing £700 of the purchase-money to remain on mortgage at 4%. Find what rate per cent. per annum the investment will pay the purchaser.

31. If in $4\frac{1}{2}$ years, at simple interest, £1 becomes £1, 4*s.* 6*d.*, what is the rate per cent. per annum at which it was lent?

32. At what rate per cent. will the simple interest on £1250 for 5 years amount to £265, 12*s.* 6*d.*?

33. A freehold house is let for £66 per annum. It was sold for 27 years' purchase, but £620 of the purchase-money had to be borrowed at 5 per cent. per annum. What rate of interest does the purchaser get for his money?

34. A and B are twins, aged 26 years. A determines to invest £25 on his twenty-seventh and each successive birthday at 4 per cent. per annum, simple interest. B, at the same time, insures his life for a certain sum, paying an annual premium of £2, being $1\frac{1}{4}$ per cent. on the sum assured. Compare the amounts which A and B respectively will have to bequeath, dying at the commencement of their thirty-third year, after A has made his sixth investment.

35. Find, to the nearest penny, the simple interest on £243, 18*s.* 9*d.* for $2\frac{1}{2}$ years at $3\frac{3}{4}$ per cent. per annum.

36. A man invests two equal sums of money. The one bears interest at the rate of $2\frac{1}{2}$ per cent. per annum, the other at the rate of 3 per cent. per annum. At the end of $5\frac{1}{2}$ years he has received £18, 7*s.* $1\frac{1}{2}$*d.* more from the latter than from the former. Find the sums invested.

37. Divide £3300 into two parts, such that the simple interest on the one at 3 per cent. for $4\frac{1}{2}$ years would be equal to the simple interest on the other at $2\frac{1}{4}$ per cent. per annum for 5 years.

38. If in $3\frac{1}{4}$ years at simple interest £1 becomes £1, 2*s.* 2*d.*, what is the rate per cent. per annum at which it is lent?

Examples 72.

1. What sum will amount to £1079, 10*s.* in 6 years at $4\frac{1}{2}$ per cent. per annum, simple interest?

2. If £267, 3*s.* 9*d.* is the amount of a certain sum for 3 years 9 months at 5 per cent. per annum, simple interest, how much of it is interest?

3. If I lend a friend a sum of money at $2\frac{1}{2}$ per cent. per annum, simple interest, and he returns me £935, 11*s.* at the end of 4 years, what is the amount of the loan?

4. Find what sum of money must be invested at $3\frac{3}{4}$ per cent. per annum, simple interest, in order to amount to £24[illegible], 18*s.* 9*d.* in $2\frac{1}{2}$ years.

5. What sum of money will amount to £725, 2*s.* 4·38*d.* if put out at simple interest for 3 years at $3\frac{1}{4}$ per cent. per annum?

6. What principal would amount to £1320, 6*s.* in 7 years at 5 per cent. per annum, simple interest?

7. A certain sum of money put out at $4\frac{1}{2}$ per cent., simple interest, for 10 years amounted to £1383 after income tax was deducted from the interest at the rate of 5*d.* in the £. What was the sum?

8. If I pay a man £5717, 16*s.* 8*d.* in return for a sum of money lent me 6 months ago at 3 per cent. per annum, simple interest, calculate the sum lent.

9. What sum is that which if put out to interest at $2\frac{1}{4}$ per cent. per annum for 3 years 9 months would amount to £1735?

10. Find what sum of money I must lend at 2 per cent. per annum, simple interest, so that at the end of 2 years 6 months I may obtain £403, 14*s.* 6*d.* in full pavment of loan and interest.

218. Method of Approximation.

Third, tenth and tenth rule.

It has already been pointed out that it is not always an advantage to cancel out factors (§ 25). Sometimes a solution may be obtained more readily by increasing the numerator and denominator of the fraction, as in the following method of approximation used in calculating interest for days (when the days do not form an easy fraction of a year). This method is known as the **Third, tenth and tenth rule.**

Example—

(i.) *Find the Simple Interest on £845 for 227 days, at $1\frac{1}{2}$ per cent. per annum.*

I = ?
P = 845
R = $1\frac{1}{2}$ = 1·5
Y = $\frac{227}{365}$

$$I = \frac{PRY}{100} = \frac{845 \times 1·5 \times 227}{100 \times 365}$$

Multiplying top and bottom of fraction by 2 we get—

$$\frac{845 \times 1·5 \times 227 \times 2}{100 \times 365 \times 2} = \frac{575445}{73000}$$

$$= 575445 \times 0·0000137 \text{ nearly} \quad \left(\frac{1}{73000} = 0·0000137\right).$$

Instead of multiplying, the result may be obtained by taking 575445, adding to it $\frac{1}{3}$ of itself + $\frac{1}{10}$ of that result + $\frac{1}{10}$ of the new result, and then dividing the sum of all by 100,000, *i.e.* move point 5 places to left.

Then we have

575445			1
191815	$\frac{1}{3}$	=	·3333
19181·5	$\frac{1}{10}$ of $\frac{1}{3}$	=	·0333
1918·15	$\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{3}$	=	·0033
£7·8835965			0·00001369

The last part of the working is most conveniently put thus :—

$$\frac{845 \times 1·5 \times 227 \times 2}{100 \times 365 \times 2} = 575445$$

191815
19182 (neglecting decimals).
1918

(point 5 places to left) 7·88360 = **£7, 17s. 8d.**
20
17·67200
12
8·064

Where the interest is over £10, deduct from it $\frac{1}{10000}$ of itself; *e.g.* if, by above, the interest = £48·73654, a more exact answer would be obtained thus—

48·73654 (point 5 places to left)
487 (number 5 places to right)
48·73167
= £48. 14s. 7½d.

219. In many interest questions the time has to be calculated from given data. Care must be exercised to avoid the error of counting too many days. The following is perhaps the safest method:—

Examples—

(ii.) *Find the interest on a sum of money from 3rd August 1901 till 29th April 1902.*

From 3rd August to 31st August	=	28 days (31 – 3).
September	=	30 "
October	=	31 "
November	=	30 "
December	=	31 "
January	=	31 "
February	=	28 "
March	=	31 "
to 29th April	=	29 "
Total	=	269 days.

220. Frequently the principal put into the bank by a firm is varied during the year while the interest may remain at the same rate per cent. The interest is sometimes calculated on every complete month, dating from a fixed day of the month.

(iii.) *A certain bank pays $2\frac{3}{4}$ per cent. interest on all deposits. It is computed by months on every complete pound, reckoning from the 20th of each month, and is added to the principal every year in December. A firm deposits £150 on 31st December, and £235 on 19th February; withdraws £300 on 21st April; deposits £265, 10s. on 3rd July, £150 on 18th July, and £260, 10s. on 19th October; withdraws £400 on 1st November. Find the balance in the bank at the end of the year?*

20th Jan. till 20th Feb. the principal obtaining interest
= £150 for 1 mth. . . . . = £150 for 1 mth.
20th Feb. till 20th Apr. the principal obtaining interest
= £150 + £235 = £385 for 2 mths. . . . . = 770 " 1 "
20th April till 20th July the principal obtaining interest
= £385 – £300 = £85 for 3 mths. . . . . = 255 " 1 "
20th July till 20th Oct. the principal obtaining interest
= £85 + £265, 10s. + £150 = £500 (10s.) for 3 mths. = 1500 " 1 "
20th Oct. till 20th Dec. the principal obtaining interest
= £500, 10s. + £260, 10s. – £400 = £361 for 2 mths. = 722 " 1 "

Interest to be calculated = Interest on £3397 for 1 mth.

$$I = \frac{3397 \times 11 \times 1}{100 \times 4 \times 12} = \frac{37367}{4800} = £7{\cdot}7848$$

∴ Interest	= £	7	15	8
Principal	=	361	0	0
Balance	= £	368	15	8

221. Should the interest be calculated by days a somewhat similar method is adopted. Take the above example.

31st Dec till 19th Feb.,	£150	for 50 days = 150 × 50 =	7500	for 1	day
19th Feb. till 21st Apr.,	£385	for 61 days = 385 × 61 =	23485	,,	,,
21st Apr. till 3rd July,	£85	for 73 days = 85 × 73 =	6205	,,	,,
3rd July till 18th July,	£350 (10s.)	for 15 days = 350 × 15 =	5250	,,	,,
18th July till 19th Oct.,	£500 (10s.)	for 93 days = 500 × 93 =	46500	,,	,,
19th Oct. till 1st Nov.,	£761	for 13 days = 761 × 13 =	9893	,,	,,
1st Nov. till 31st Dec.,	£361	for 60 days = 361 × 60 =	21660	,,	,,

Interest to be calculated = Interest on £120,493 for 1 day

$$\text{Interest} = \frac{120493 \times 2{\cdot}75 \times 1 \times 2}{100 \times 365 \times 2} = \begin{array}{r} 662712 \\ 220904 \\ 22090 \\ 2209 \\ \hline £9{\cdot}07915 \end{array}$$

Interest . . =	£9	1	7
Principal . =	361	0	0
Balance in Bank	£370	1	7

222. Frequently the rate of interest is changed throughout the year. If the principal remains the same the interest may be calculated by finding the average rate (§ 172).

Example—

(iv.) *Find the Simple Interest on £350 deposited for one year, the rate of interest fluctuating as follows:—At beginning of year 3%, on 1st March fell to $2\frac{3}{4}$, on 1st May to $2\frac{1}{2}$, on 1st July rises to 3, on 1st October to $3\frac{1}{8}$, on 1st November fell to 3, and remained so till end of the year.*

Interest from	1st Jan. till 1st March	= 3% for 2 mths.	= 6%	for 1 mth.
,, ,,	1st Mar. till 1st May	= $2\frac{3}{4}$% ,, 2 ,,	= $5\frac{1}{2}$%	,, 1 ,,
,, ,,	1st May till 1st July	= $2\frac{1}{2}$% ,, 2 ,,	= 5%	,, 1 ,,
,, ,,	1st July till 1st Oct.	= 3% ,, 3 ,,	= 9%	,, 1 ,,
,, ,,	1st Oct. till 1st Nov.	= $3\frac{1}{8}$% ,, 1 ,,	= $3\frac{1}{8}$%	,, 1 ,,
,, ,,	1st Nov. till 1st Jan.	= 3% ,, 2 ,,	= 6%	,, 1 ,,

$$\text{Average rate} = \frac{34\frac{5}{8}}{12}\% \text{ per annum.}$$
$$= \frac{34{\cdot}625}{12}$$

$$\text{Interest} = \frac{\overset{17{\cdot}3125}{350 \times 34{\cdot}625}}{12 \times 100} = \frac{121{\cdot}1875}{12} = 10{\cdot}099 = £10,\ 2s.$$

$$\begin{array}{r} 20 \\ \hline 1{\cdot}980 \\ 12 \\ \hline 11{\cdot}76 \end{array}$$

Examples 73.

1. Find the simple interest on—

(a) £4000 for 37 days @ 3 per cent. per annum.
(b) £5000 „ 136 „ @ 4 do.
(c) £6500 „ 25 „ @ 7 do.
(d) £25, 14s. 6d. „ 41 „ @ $2\frac{1}{2}$ do.

2. Find the amount at simple interest on—

(a) £1008 for 157 days @ $2\frac{1}{2}$ do.
(b) £842, 10s. „ 216 „ @ 4 do.
(c) £560 „ 150 „ @ $7\frac{1}{2}$ do.

3. Find the simple interest on—

(a) £684 from June 3 to Oct. 30 @ 3 do.
(b) £984, 17s. 6d. „ Mar. 6 to Nov. 18 @ 6 do.
(c) £1850, 10s. „ Jan. 27 to July 30 @ $2\frac{1}{2}$ do.
(d) £690, 12s. 6d. „ Mar. 12 to Dec. 31 @ $3\frac{1}{2}$ do.

4. Find the amount at simple interest of—

(a) £483, 5s. 6d. from Mar. 18 to Nov. 26 @ 8 do.
(b) £750 „ April 4 to Aug. 27 @ 3 do.
(c) £76, 15s. „ July 31 to May 8 @ 4 do.

5. The sum of £5000 is deposited with the bank on January 1st, 1890, and withdrawn on May 27th of the same year. Find the interest gained at 3 per cent. per annum.

6. Find the interest on a loan of £750 from 4th January to 30th April at 6 per cent. per annum, after deducting income tax at the rate of 1s. 2d. in the £.

7. I deposited £420 on May 5th in a bank paying interest at the rate of 3 per cent. per annum. On June 4th the interest fell to $2\frac{1}{2}$ per cent., and on September 10th it rose to $2\frac{3}{4}$ per cent. Find, to the nearest penny, the interest due on November 12th in the same year.

8. On 1st January 1890 a sum of money was deposited in a bank to receive interest at 3 per cent. per annum. On 4th July the rate of interest was lowered to $2\frac{1}{2}$, but on 14th August it was raised to $3\frac{1}{2}$ and continued at that rate for the rest of the year. Find the average rate of interest for the year.

9. A man deposited £500 in the bank on March 9th when interest was at 4 per cent. On May 9th the bank rate fell to $3\frac{1}{2}$ and on July 9th to 3. On 9th August it rose to $3\frac{1}{4}$. Find, to the nearest penny, the interest due on December 9th of the same year.

10. £8400 was paid into the bank on February 1st, 1900, and withdrawn on July 12th of the same year. To what had it amounted, if interest was paid at the rate of $3\frac{1}{2}$ per cent. per annum?

11. A man deposited £750, 10s. on March 18th in a bank paying interest at $3\frac{1}{2}$ per cent. per annum. On May 16th the interest rose to $3\frac{3}{4}$ per cent., and on August 1st it fell to $3\frac{1}{4}$ per cent. Find, to the nearest penny, the interest due on 25th November.

12. The bank rate on 1st January was 3 per cent.; on February 16th it fell to $2\frac{1}{2}$, and on April 9th to $2\frac{3}{8}$; it rose to $2\frac{3}{4}$ on July 4th, and on September 16th again reached 3, at which it remained till the

end of the year. Calculate the average rate and the interest due on £840 for the whole year.

13. The following transactions appear in a merchant's bank-book:—Balance as on 31st December 1895, £640; 23rd February, deposited £125; 13th April, withdrawn £350; May 3rd, deposited £360; July 19th, deposited £120; August 7th, withdrawn £427; October 30th, deposited £135; December 3rd, withdrawn £290. Calculate to the nearest penny the balance at the end of the year, interest being paid at the rate of 4 per cent. per annum.

14. On 1st July 1901 a person contracted to buy a house for £300, to be paid as follows:—10 per cent. deposit down, and the remainder by 6 equal half-yearly instalments commencing 1st January 1902; and together with each instalment, further to pay half a year's interest at 5 per cent. on the unpaid balance of the purchase-money. Find the average of the six half-yearly payments.

15. The National Debt was £684,070,959, upon which 3 per cent. was annually paid for interest and expenses. If £25,000,000 were annually set aside for the purpose and to reduce the debt, find to the nearest pound the amount of the debt at the end of 3 years.

XXXI.—STOCKS AND SHARES.

223. When a person wishes to begin a business of any kind on his own account, he must, of necessity, have money wherewith to provide the plant to carry on the business, or the goods he desires to sell. It frequently happens that the person does not have sufficient money for the purpose, and so two or three combine and form a partnership, their total moneys forming the capital of the concern (see § 193).

224. With a large undertaking such as the construction of a railway, the capital demanded is so great that hundreds or even thousands of people must combine to raise the necessary amount of money. In such a case, a **Company**, not a partnership, is said to be formed, and the different partners are termed **Shareholders**. It is obvious that, where the number of shareholders is so large each cannot take an active share in the business; in fact, most of the shareholders do not wish to do so, and, on that account directors are appointed, who are held responsible for the proper management of the affairs of the Company. The profits are divided at intervals among the shareholders, according to the shares held by each.

225. When a new Company is being floated, it is customary to ask the public to purchase the shares at a stated value. Application for shares is then made by those desirous of becoming shareholders, the shares are allotted by the promoters of the

Company, and the money **paid up**. For example, if a Company is being floated, and a capital of £50,000 is wanted, shares may be offered at £10 each. Each individual who desires to join the Company may pay up one or more £10's. On receiving his share receipt he is entitled to benefit from the profits in proportion to his money invested. So far, this is much the same as lending his money or putting it in the bank; but money on loan can generally be recalled at pleasure or after due notice has been given, whereas, when invested in a Company, this is impossible. Take the case of a new Railway Company. When the shareholders pay over their money, much of it is used to purchase land, rails, engines, waggons, carriages, put up stations, depôts, &c., and consequently is not available for paying back to people desirous of withdrawing from the Company. Hence, the individual must remain a shareholder, or get some person to buy his share, *i.e.* to purchase his share receipt just as if it were a piece of ordinary merchandise. When such a person is found the shares are **transferred**. This transfer can take place only through a **Stockbroker** in the **Stock Exchange**—a place specially suited for the sale and purchase of shares in Companies.

226. If money is deposited in a bank or put out on loan, the interest generally obtainable ranges from about 1 to 4 per cent. Some Companies, however, make such large profits that they can declare a **dividend** of from 5 to 10 per cent., and in some cases even as much as 40 per cent. has been paid as interest to the shareholders. It is evident that, in the case of such a Company, a great many people will desire to have shares in it; so that, if any person has shares to sell, many will wish to buy, and the ordinary rules of supply and demand will regulate the price at which the shares will be sold. He may then obtain £120 for a £100 share. This is termed its **Selling Price**. Again, a Company may pay a poor dividend, and few people care to invest in it, while many, who already have bought shares, are desirous of obtaining cash for them. In this case they may sell a £100 share for as little as £60.

227. When a person sells his shares at exactly what was originally paid for them, he is said to dispose of them "**at par**"; if he gets more than that, they are sold "**at a premium**"; if less, "**at a discount**."

228. The selling prices of the shares of the chief Companies are quoted in the newspapers. Usually two prices are given,

e.g. $94\frac{1}{2}$; $94\frac{5}{8}$. These inform us that the brokers sold to buyers at the higher price and bought from sellers at the lower price.

229. When a dividend is declared, the amount each shareholder gets is proportional to the Stock he holds, independent altogether of the price he has paid for it. Thus, a man who buys £100 Stock and pays £120 for it, on a dividend of 4 per cent. being declared, obtains £4 only, *i.e.* 4 per cent of £100, not 4 per cent. of £120. In this case he gets less than 4 per cent on his money invested in the Company, as he had to spend £120 to obtain £4 interest.

230. Should a Company fail, the shareholders must make up the loss. In **Limited Liability Companies**, the creditors may draw upon the shareholders' money to the full extent of their shares only; *i.e.* shareholders lose all the money they have invested. If the Company is not limited, as was the case of the City of Glasgow Bank in 1878, the shareholders are held liable, and must pay off the complete debts, just as an ordinary bankrupt.

231. When, therefore, a person wishes to invest in a Company, he should consider at least three things—

First. The prospects of the Company.

Second. The price of the Stock * or share.

Third. The real interest paid on money invested, calculated from the dividend declared (§ 248).

232. Closely connected with Stock questions are questions on Government loans.

The money required by different Governments to carry on their affairs is generally raised by taxation, but, in times of special strain, as when the country is at war, the income from ordinary taxation would never meet the expenditure. Again, it would obviously be unfair to make the one set of people bear the whole burden of the expense. Posterity is likely to benefit from the progress of the country, and therefore ought to share in the expense necessary to secure its freedom. This belief led to a system of **loans** being introduced, by which the people in a country could loan the Government money for which they received an annual interest. Money so lent is really **invested**, as a person

* When a person may invest any odd amount of money in a company, he is said to purchase **Stock**, as distinguished from **shares**, of which he must buy an exact number. *E.g.* a person might invest £1568 in Stock, but could not with this sum buy shares of £10 each at par. He might, of course, buy 156 shares and keep the £8 To convert shares into Stock multiply the nominal value of each share by the number of shares.

cannot recall the loan at pleasure, but must, as with Stock, find a purchaser for his share. When a person buys this Government Stock he really buys the right to receive a perpetual annuity calculated on the amount of Stock purchased, not on the price paid. The rate of interest is fixed by the Government, and generally ranges between $2\frac{1}{2}$ and $3\frac{3}{4}$ per cent. This being known, the price of the Stock will fluctuate according to the credit of the Government and the probability of its being able to pay the yearly interest. Occasionally a Government redeems part of this debt at par. The date of such a transaction is advised long beforehand, so that investors may be prepared for such a contingency. Government Stocks are variously called "**Funds**," "**Consols**," &c.

232a. Three kinds of Stock are commonly met with:—

Ordinary Stock, § 224.

Preferred Stock, to which a fixed dividend is paid before allocating anything to the Ordinary Stock.

Debenture Stock, in reality a loan raised by the company on the security of its effects, and obtaining a fixed dividend as part of the ordinary expenses of the company.

233. It has already been shown that all transactions in Stocks must be made through a **broker**, who charges for his labour usually $\frac{1}{8}$ per cent on the Stock bought or sold. Thus, if a Stock is quoted at 94, a person must pay a broker £94 to obtain £100 of Stock, and in addition he must pay him £$\frac{1}{8}$ as his commission, so that £100 Stock really costs £$94\frac{1}{8}$. When a broker sells Stock quoted at 98, he receives £98 for the £100 Stock sold, but from this he deducts his £$\frac{1}{8}$ commission, and hands over £$97\frac{7}{8}$ as the net proceeds of the sale.

Frequently the quoted prices include the broker's charge, and thus represent net prices. In the examples set the prices are net unless where otherwise stated.

---

234. The chief difficulty in dealing with "Stocks" questions arise from the somewhat ambiguous wording employed. At the outset the student should, therefore, extract from the

question the essential parts with the proper reading. A clear distinction should be made between Cash and Stock by always writing Stock after the sum representing Stock, and leaving Cash in the usual form. Thus "£1000" means £1000 cash, whereas "£1000 Stock" means an amount of Stock which originally cost £1000, but whose value is now regulated by the price of the Stock.

235 For the sake of brevity many Stocks are named by means of the interest they pay. Thus the "Four per Cents." refers to some Company's Stock which pays a dividend of £4 on every £100 Stock.

236. Consider now the expression "The Four per Cents. at 97." This refers to a certain Company's Stock which pays a dividend of £4 on every £100 Stock, and further states that each £100 Stock can be purchased for £97. A person A, who possesses £97, has as much wealth as B, who has in his possession a share receipt for £100 Stock in this particular Company. They could therefore exchange without gain or loss to either. A can then claim a right to the £4 interest paid on the £100 Stock when the next dividend is declared, and we see that to obtain this £4 he has to part with £97. Hence he receives as interest £4 on £97. The "Four per Cents at £97" has thus a double reading.

If read as 4 per cent. it can only be, and must always be, completed as £4 per £100 Stock.

If cash is being dealt with, omit the term "per cent." altogether, and read as £4 on £97.

Such a reading as £4 per cent. on £97 is meaningless, and leads to the serious error of calculating what interest will be paid on £97 if £4 is paid on £100.

Lastly, if no question of interest is involved, omit all reference to the 4 per cents., as in such cases the expression is a mere name, and its place might be supplied by any other name, as "Railway Stock," which has the merit of being less confusing.

236a Since 5s. per cent. may be written $\frac{5s.}{£100}$, and 3d. per dozen $\frac{3d.}{dozen}$ or $\frac{3d.}{12}$, so $\frac{£90}{£100\ Stock}$ reads £90 per £100 Stock and $\frac{£100\ Stock}{£103\frac{1}{2}}$ as £100 Stock per (or for) £103½. This method may be used to abbreviate many calculations in Stocks.

Examples—

(i.) *A person wishes to buy £6000 3 per cent. Stock when it is quoted at 105. What must he pay for it?*

(This question involves no calculation of interest, so omit 3 per cent. and work the rest by proportion.)

If £100 Stock * costs £105
then £6000 ,, ,, ?

$$= £\frac{105 \times 6000 \text{ Stock}}{100 \text{ Stock}} = £6300.$$

Cost is **£6300.**

Short Method—Cost $= \frac{£105}{£100 \text{ Stock}}$ of £6000 Stock * = **£6300.**

(ii.) *A person owning £5600 Stock in the 5 per cents. @ £95 wishes to sell out. How much money will the Stock produce?*

This is similar to Example (i.)

If £100 Stock * produce £95
then £5600 ,, ,, ?

$$£\frac{95 \times 5600 \text{ Stock}}{100 \text{ Stock}} = £5320$$

The Stock produces **£5320**

Short Method—Stock produces $\frac{£95}{£100 \text{ Stock}}$ of £5600 Stock = **£5320.**

(iii.) *A person invests £9680 in the 6 per cents. @ £88. How much Stock does he acquire?*

If £100 Stock * cost £88
then ? ,, ,, £9680

$$£\frac{100 \text{ Stock} \times 9680}{88} = £11,000 \text{ Stock}$$

Amount of Stock = £11,000 Stock

Short Method—Stock = $\frac{£100 \text{ Stock}}{£88}$ of £9680 = £11,000 Stock

* By adopting this method of writing Stock where Stock [illegible] observing [illegible] so common [illegible] plified as we proceed.

(iv.) *A merchant who has £7000 3½ per cent. Stock wishes to discharge a debt of £4590 when the Stock is quoted at 102. How much of his Stock must he sell for this purpose?*

If £100 Stock * produces £102
then ? „ „ £4590

$$£\frac{100 \text{ Stock} \times 4590}{102} = £4500 \text{ Stock}.$$

Amount of Stock sold = **£4500 Stock.**

Short Method—Stock $\frac{£100 \text{ Stock}}{£102}$ of £4590 = **£4500 Stock**

Note—The necessity of cancelling the names in the above working is a safeguard against inaccuracy Thus, in Exampl (i.), if a student inadvertently had set down, $\frac{£100 \text{ Stock}}{£105}$ of £6000 Stock, the fact that no cancelling can take place and that the answer must contain "square stock" will be sufficient to reveal the error So in Example (iii) $\frac{£88}{£100 \text{ Stock}}$ of £9680 would not give an amount of Stock as required by the question, but a number of "square pounds" per so much Stock.

Examples 74.

1. Find the cost of £8250 Stock in the 3 per cents. at 96.

2. Calculate the cost of £5320 Indian Stock quoted at 2 above par.

3. I want to buy 600 £10 shares in a company. The shares are quoted at 8¾. What shall I pay for them, and what amount of Stock shall I then possess?

4. When a certain Stock is selling at 25 discount, what money will a person require to buy £5375 of the Stock?

5. Find how much a man gets by selling £627, 10s. of a 4 per cent Stock at 80.

6. When Stock is at 47½ for a £50 share, what does a man realise who sells 350 shares?

7. A man owns £7000 Stock. If he sells a quarter of it at 95¾ and the remainder at 96¼, find his total loss supposing he bought it at par.

8. How much 3 per cent. Stock at 84⅝ can I get for £8872, 10s.?

9. When the 4 per cents. are quoted at 30 premium, how much Stock will a man obtain for £1696, 10s.?

10. How many £12 shares quoted at 15⅛ can a man purchase with £1769, 12s. 6d.?

11. A man owes £575, 17s. 6d. How much Stock must he sell out of the 3½ per cents. at 92⅛ to get enough money to discharge the debt?

12. A person holds 4000 £1 shares in a concern when they are

* See note, p. 193.

quoted at 32*s*. 6*d*. How many of these must he sell out, at least, to realise £3500?

13. A person owes £6000. He offers £300 in cash and a number of £10 shares in a company quoted at $7\frac{1}{2}$. Calculate the requisite number of shares.

14. If by investing £7500 I can procure £8000 Stock, how much below par is the Stock selling at?

15. Find the price of the 3 per cents. when £950 Stock realises £831, 5*s*.

16.* Give the quotation for the 5 per cent. Colonial Stock at which I bought £7250 for £7431, 5*s*.

17. A man had £10,000 Stock, half of which he obtained in the 3 per cents. when they were at $99\frac{1}{2}$, and the remainder when they were quoted at $\frac{1}{8}$ premium.* He sells out all his Stock when the quotation is $99\frac{1}{4}\frac{5}{8}$. How much does he gain or lose?

18. A man invests £7620 in the 3 per cents. at $95\frac{1}{4}$; he sells out $\frac{1}{4}$ of his Stock when the funds have fallen to $93\frac{1}{2}$, £3600 Stock when they have risen to 96, and the remainder at par. Find what he has gained or lost.

237. Questions on Income :—

Examples—

(v.) *A person owns £6300 4% Stock quoted at 84. Find the income he derives from this.*

The £6300 in the question is Stock, hence to obtain income read 4% as £4 per £100 Stock.

If £4 is the income on £100 Stock
then ? ,, ,, £6300 ,,

$$£\frac{4 \times 6300 \text{ Stock}}{100 \text{ Stock}} = \textbf{£252}.$$

$$\textit{Short Method}\text{—Income} = \frac{£4}{£100 \text{ Stock}} \text{ of } £6300 \text{ Stock} = \textbf{£252}.$$

* Questions similar to (v.), (vi.), and (xvii) are continually confused. Attention to the above method makes mistakes impossible, as the question checks itself; *e.g.* suppose a student started Ex. vi. "If £4 is the income on £100," he must write Stock after the £100 (see note, p. 193). Hence in writing the second line he has no place to put the £6300, since cash cannot appear in the same column with Stock. Similarly with the corresponding mistake in Ex. v. (See also note after Ex. iv., p. 194.)

(vi.) *A person invests £6300 in the 4 per cents. at 84. **Find his income.***

The £6300 in this case is cash (for a man cannot invest Stock). Hence the reading for income is £4 income on £84.

If £4 is the income on £84 *
then ? ,, ,, £6300

$$£\frac{4 \times \overset{300}{\cancel{6300}}}{\underset{4}{\cancel{84}}} = \mathbf{£300.}$$

Short Method—Income $= \frac{£4}{£84}$ of £6300 = **£300.**

Sometimes the dividend declared is not free of income tax. In such cases deduct the tax from the percentage dividend and proceed as before.

(vi.a) *How much must I invest in $4\frac{1}{2}$ per cents. at 104 to secure an income of £85, 10s. after paying income tax of 1s. in the £?*

Here the dividend of $4\frac{1}{2}$ per cent. is reduced by 4s. 6d. of income tax, and so the real dividend is £4, 5s. 6d. per cent.

If £4·275 is income on £104
then £85·5 ,, ?

$$\frac{£104 \times £85·5}{£4·275} = \mathbf{£2080.}$$

Short Method—Amount invested $= \frac{£104}{£4·275}$ of £85·5 = **£2080.**

---

* See note. p. 195.

Examples 75.

1. What income do I derive from 400 £10 shares bought at $9\frac{3}{8}$ when a dividend of $4\frac{1}{2}$ per cent. is declared?

2. What income will a man obtain by investing £712, 10s. in the $3\frac{1}{4}$ per cents. at 95?

3. Find the dividend on £8325 three per cent. Stock.

4. When the five per cents. are at 135, what income would be derived from investing £3388, 10s. in them?

5. If I invest £3570 in Stock at 2 premium which pays a dividend of 7 per cent., what income do I derive from it?

6. I bought 40 shares of £60 each at 65. After receiving a dividend amounting to 10 per cent. on my outlay I sold at 70. What was the total gain?

7. Find the income from investing £4560, half in the $3\frac{3}{4}$ per cents. at 76 and half in the $12\frac{1}{2}$ per cents. at $142\frac{1}{2}$.

8. What sum should be invested in the $3\frac{1}{2}$ per cents. at 98 to yield a net annual income of £413, after paying an income tax of 4d. in the £?

9. How much 3 per cent. Stock does a man hold whose annual income is £321, 15s., after paying a tax of 6d. per £?

10. A person bought £3000 four per cent. Stock when they were quoted at 5 premium. On receiving his dividend he sold out at such a price that he found that he had gained nothing at all by the whole transaction. At what price did he sell?

11. How much should I invest in the three per cents. at $93\frac{1}{4}$ to secure an income of £70 a year?

12. What sum must be invested in the purchase of 4 per cent. railway debentures at $108\frac{3}{4}$ so as to produce an income of £100 a year after paying an income tax of 8*d*. in the £?

13. A certain sum invested in the three per cents. yields an annual income of £75. The same sum invested in a $5\frac{1}{4}$ per cent. Stock at par would yield an annual income of £126, 13*s*. $1\frac{1}{2}$*d*. Find (1) the sum invested; (2) the price of the three per cents.

14. What income does a man obtain by investing £3220 in a $3\frac{1}{2}$ per cent. stock at $80\frac{1}{2}$?

15. A person sells out of the 3 per cents. at 96 and invests the proceeds in railway 5 per cent. Stock at par. How much per cent. is his income increased?

16. A company, whose head office is in Edinburgh, carries on trade in Mexico, and the profits remitted in one year amount to 182.441 dollars. The capital of the company is £250,000 ordinary Stock, and it has also £100,000 of preference Stock. If the rate of exchange for a dollar be 23·45 pence, find to the nearest pound how much of the profits remain after paying a dividend of 4 per cent. on the preference and 5 per cent. on the ordinary Stock

238. Questions involving the Transfer of Stocks:—

Examples—

(vii.) *A person possessing* £12,000 5 *per cent. Stock at* 98, *wishes to sell out and invest the proceeds in* $7\frac{1}{2}$ *per cents. at* 108. *Find how much of the latter Stock he can obtain.*

This question may be worked in two parts.

First sell out the 5 per cent. Stock at 98 (Ex. ii.) and then invest the proceeds in the $7\frac{1}{2}$ per cent. Stock at 108 (Ex. iii.).

It may be seen, however, that if on selling his £12,000 Stock at 98 he obtained £x cash, and immediately reinvested this at 98, he would just get back his £12,000 Stock; but if he invested the £x at 108 he would get less.

Hence in one question:—

If by reinvesting at 98 he would get £12,000 Stock
then " " 108 " " ?

$$= £\frac{12000 \text{ Stock} \times 98}{108}$$

$$= £10,888\tfrac{8}{9} \text{ Stock.}$$

The latter method is recommended as the shorter one.

239. Frequently when large amounts of Stock are bought or sold, the transaction has to be completed in several deals, and in most cases each deal affects the subsequent price of the Stock.

(viii.) *A person having £35,000 4 per cent. Stock at 103, sells out in lots of £5000. The effect of each sale lowers the price of the Stock by* $\frac{1}{4}$%. *How much does he obtain for all the Stock?*

First the "Average Price" of the Stock must be obtained. He sold in 7 lots, @ 103, $102\frac{3}{4}$, $102\frac{1}{2}$, $102\frac{1}{4}$, 102, $101\frac{3}{4}$, $101\frac{1}{2}$; the average of these is $102\frac{1}{4}$ (in this case the middle term).

If £100 Stock produces on the average $102\frac{1}{4}$

then £35,000 "

$$= \frac{£102 \cdot 25 \times £35,000 \text{ Stock}}{£100 \text{ Stock}} = £35,787 \cdot 5$$

Stock produces **£35,787, 10s.**

Short Method—Stock produces $= \frac{£102\frac{1}{4}}{£100 \text{ Stock}}$ of £35,000 Stock

= **£35,787, 10s.**

(ix.) *If with the proceeds of Ex.* (viii.) *he purchased £36,000 Stock in the 3 per cents. at 95 in lots of £6000, the effect of each purchase being to raise the price* $\frac{1}{8}$%, *how much money will he have left?*

Average price $= \frac{1}{6}$ of $(95 + 95\frac{1}{8} + 95\frac{1}{4} + 95\frac{3}{8} + 95\frac{1}{2} + 95\frac{5}{8}) = 95\frac{5}{16}$.

If £100 Stock costs on the average $95\frac{5}{16}$

then £36,000 " $\frac{£95 \cdot 3125 \times £36,000 \text{ Stock}}{£100 \text{ Stock}}$

Cost = **£34,312, 10s.**

and money left = £35,787, 10s. − £34,312, 10s. = **£1475.**

Short Method—Cost $= \frac{£95 \cdot 3125}{£100 \text{ Stock}}$ of £36,000 Stock = **£34,312, 10s.**

Examples 76.

1. I held £10,000 Stock in the $3\frac{1}{2}$ per cents at 77. I sold out and bought into the $4\frac{1}{2}$ per cents at $91\frac{2}{3}$. How much Stock do I now hold, and how has my yearly income been affected?

2. A person sells £880 of Stock in the 3 per cents. at 95, and invests the proceeds in the 4 per cents at a premium of £4, 10s. How much Stock did he purchase, and what was the change in his income?

3. A man sold £1950 of 3 per cent Stock at $112\frac{1}{3}$, and invested the proceeds in the 5 per cents. at £130. Find the alteration in his income.

4. A person sells £7300 3 per cent Stock at 96, and invests his money in a 5 per cent Stock at £106, 13*s* 4*d*. How is his annual income affected?

5. A person invested £4370 in a 4 per cent. railway Stock at 115. When it rises to 125 he sells out and invests the proceeds in $2\frac{3}{4}$ per cent Consols at 95. Find the change in his income.

6. A man invests as much money in $4\frac{1}{4}$ per cent Stock as yield an income of £46, 15*s*. He sells out at 108 and invests the proceeds in the 3 per cents at 99. By how much is his income altered?

7. A man has £6000 5 per cent Stock transferred from the Stock at 125 into 3 per cent Consols, and a loss in income of £50 is incurred. Find the price of Consols.

8. £4000 3 per cent Consols at $100\frac{7}{8}$ are transferred into a 4 per cent. Railway Debenture Stock at $110\frac{5}{8}$, the expense of the operation amounting to £43, 10*s*. Find the alteration in the income produced.

9. A person invests £13,650 in a 4 per cent. Stock at 91. On the Stock falling to 75 he sells out, and investing the proceeds in an 8 per cent. Stock he finds that he thereby loses as interest £60. What is the price of the latter Stock?

10. £3000 3 per cent Consols at $101\frac{1}{4}$ are transferred into a $4\frac{1}{2}$ per cent. Railway Debenture Stock at $125\frac{1}{4}$, the expenses of the operation amounting to £31, 10*s*. Find the alteration in income produced.

11. A man has £3333, 6*s* 8*d*. Midland 4 per cent Debenture Stock. He sells out at 120 and invests the proceeds in the 3 per cent Consols losing thereby £13, 6*s*. 8*d*. in annual income. Find the price of the Consols.

12. £5151 is invested in 5 per cent Stock at 101; the Stock rising to 105, it is sold out and the proceeds invested in Stock at 102, which pays $4\frac{3}{4}$ per cent. Find the change in income.

13. I hold £6000 of $2\frac{3}{4}$ per cent. Stock at $112\frac{1}{2}$, and decide to transfer it to the $2\frac{1}{2}$ per cents. What must be the price of the latter Stock so that my income may be increased by £3, 15*s*.?

14. A person held £2950 Stock in the 3 per cents. when they stood at 83½; but when they had fallen 2¼ per cent. he sold out and invested the proceeds in a 5 per cent. Stock at 108. Find the alteration in his income.

15. A man realised £2730 by selling a 3 per cent. Stock at 113¾, and invested one-third of this sum in a 4 per cent Stock at 130, and the remainder in a 2¾ per cent. Stock at 113¾. Find how much of each kind of Stock he bought, and the difference it made in his income.

16. By selling out 3 per cent. Consols at 102¾, and investing the proceeds in a Railway Stock at 137, a man finds that he can double his income. What is the annual dividend on the Railway Stock?

17. I sell out £10,000 Great Western Railway 4 per cent. Debenture Stock at £133 and buy £8000 Stock in the 2¾ per cent. Consols at £95¾. I lend the rest of my money on mortgage at 3¾ per cent. Find the change in my income.

18. A person has £1500 in the 3 per cents. at 73, and when the funds rise 2 per cent. he sells out and invests in the 4 per cents. at 90. Calculate the difference in his income.

19. A man has £2100 Stock in the 3 per cents., which he transfers to the 3½ per cents. at 87½, thus increasing his income by £7. What is the price of the 3 per cents.?

20. A person borrowed a sum of money at 3½ per cent. per annum simple interest, and invested it in a Stock at 115, paying 4½ per cent. dividend. If he gained thereby £190 in 11½ years, how much money did he borrow?

21. If £1820 Stock which produces 5 per cent., and £3510 Stock which produces 3 per cent., be sold out when the former is at 117 and the latter at 91, and if the proceeds be invested in 6½ per cent. Stock at 169, what will be the resulting change in income?

22. A man has £2000 Stock in the 3 per cents. By selling out at 95 and reinvesting in an 8 per cent. Stock he increased his income by £20 a year. What did he pay for £100 of the new Stock?

23. If £100 of 2¾ per cent. consolidated Stock cost £97⅝, what quantity of the Stock would cost £7497, 12s., and what annual income would be derived from it?

24. A person owns £80,000 Indian Stock, which he desires to sell in lots of £5000. He disposes of the first lot at 124, but the sale causes a fall in the price of the Stock of ¼ per cent. Each succeeding sale results in a further fall of ¼ per cent. in the price of the Stock. If the expense of the sale amount to £100, calculate the net proceeds of the entire sale.

25. A person has £5000 which he instructs an agent to invest in 3 per cent. Colonial Stock standing at 102. The agent buys in lots of £500 Stock, and as each deal is completed the price of the Stock rises $\frac{1}{16}$ per cent. The agent buys 9 lots, when he is instructed to cease buying. If his charge amounts to £10, how much money should he return to his employer?

240. Questions involving Brokerage :—

As has been already explained (§ 233) Stock must be bought and sold only through a Stockbroker, who charges for his labour usually $\frac{1}{8}$ per cent. It must be observed that he always charges on the amount of Stock bought or sold, and not on the amount of money obtained in the transaction. His charge, therefore, affects the *price* of the Stock.

241. Suppose a man desires to buy Stock through a broker when it is quoted at 94. To obtain the £100 Stock he must give the broker £94, and, in addition, must pay the broker £$\frac{1}{8}$ as commission (not $\frac{1}{8}$% on £94, since the charge is made on amount of Stock). Hence, he really parts with £$94\frac{1}{8}$ to obtain £100 Stock, *i e.* its price is $94\frac{1}{8}$.

Similarly, if a man sells £100 Stock, quoted at 94, through a broker, the broker will hand over £94 as the money obtained by the sale and then receive back £$\frac{1}{8}$ as his commission, so that the original owner of the Stock gets only $93\frac{7}{8}$, or its price, so far as he is concerned, is $93\frac{7}{8}$.

242. Often the price as quoted includes brokerage, but when it does not, allowance must be made for the broker's charge *at the very outset*, and the price of the Stock modified accordingly.

Examples—

(x.) *What must be paid for* £3000 4 *per cents. at* 102, *brokerage* $\frac{1}{8}$%?

In this case the broker's charge alters the price of the Stock to $102\frac{1}{8}$.

Hence—

If £100 Stock cost £$102\frac{1}{8}$
then £3000 ,, ?

$$= \frac{£102 \cdot 125 \times £3000 \text{ Stock}}{£100 \text{ Stock}} = \mathbf{£3063,\ 15s.}$$

Short Method—Cost $= \frac{£102 \cdot 125}{£100 \text{ Stock}}$ of £3000 Stock $=$ **£3063, 15s.**

(xi.) *A person invested* £8762, 10*s. in the* 4 *per cents. at* $87\frac{1}{2}$, *brokerage* $\frac{1}{8}$%. *Find his income.*

The broker's charge alters the price of the Stock to $87\frac{5}{8}$.

If £4 is the income on £$87\frac{5}{8}$
then ? ,, ,, £$8762\frac{1}{2}$

$$\frac{£4 \times £8762 \cdot 5}{£87 \cdot 625} = \mathbf{£400\ income.}$$

Short Method—Income $= \frac{£4}{£87 \cdot 625}$ of £8762·5 $=$ **£400.**

Examples—

(xii.) *How much Stock, quoted at 104¼, brokerage ⅛%, must be sold to realise £8330? Find also the broker's charge.*

The broker's charge alters the price of Stock to 104⅛.

If £100 Stock realises £104·125
? ,, ,, £8330

$$\frac{£100 \text{ Stock} \times £8330}{£104 \cdot 125} = £8000 \text{ Stock.}$$

Amount sold = **£8000 Stock.**

Broker's charge = ⅛% of £8000 = **£10.**

Short Method—Stock $= \dfrac{£100 \text{ Stock}}{£104 \cdot 125}$ of £8330 = **£8000 Stock.**

(xiii.) *A person transfers £8100 Stock from the 3 per cents. at 94½ to the 4½ per cents. at 101⅛, brokerage in each case being ⅛%. Find the alteration in his income.*

The first Stock would be sold at 94⅜; the second would be bought at 101¼.

(See Ex. vii.) If by reinvesting at 94⅜ he would obtain £8100 Stock
then ,, ,, 101¼ ,, ,, ?

$$= £8100 \text{ Stock} \times \frac{755}{8} \times \frac{4}{405}$$

(with cancellation: 8100 → 100, 755 → 151, 8 → 2, 405 → 81)

= £7550 Stock.

Income from 1st Stock = 3 % of £8100 = £243.
,, ,, 2nd ,, = 4½% of £7550 = £339·75

Difference = **£96. 15s. Gain.**

(xiv.) *By selling out £4170 4 per cents. at 103 I can purchase with the proceeds £4115 5 per cent. Stock. Find the quoted price of the latter Stock, brokerage in both cases being ⅛%.*

Stock is sold at 102⅞.

If by reinvesting at 102⅞ I obtain £4170 Stock
then ,, ,, ? ,, £4115 ,,

$$= £\frac{823 \times 4170 \text{ Stock}}{8 \times 4115 \text{ Stock}} = £104\tfrac{1}{4}$$

(with cancellation: 4115 → 5)

104¼ is the price including brokerage,

∴ quoted price is **104⅛.**

The last two examples could have been worked by obtaining the cash values of the Stock sold and then from that calculating the income or price as required.

Examples 77.

1. A man invested £797 in the three per cent. Consols at $99\frac{1}{2}$, and sold out when they rose to £$102\frac{1}{4}$. How much did he gain after paying $\frac{1}{8}$ per cent. brokerage on each transaction?

2. A person who owns £21,700 Stock in the $2\frac{3}{4}$ per cent. Consols sells it at $95\frac{1}{4}$, paying $\frac{1}{8}$ per cent. for brokerage. He invests the money received in N. S. Wales 4 per cents. at $108\frac{1}{2}$, paying no brokerage on the reinvestment. Find the change in his income.

3. A person sells out twenty-five Egyptian £100 bonds at $50\frac{5}{8}$ and invests the proceeds in a Railway Stock at 125; the brokerage for selling bonds is $\frac{1}{8}$ per cent. on the Stock. What amount of Railway Stock was bought?

4. How much must be invested in the $3\frac{1}{2}$ per cents. at $97\frac{7}{8}$ (brokerage $\frac{1}{8}$) to yield a dividend of 50 guineas?

5. Find the alteration in income occasioned by changing £3200 Stock from the 3 per cents. at $86\frac{3}{8}$ to 4 per cent. Stock at $114\frac{7}{8}$, the brokerage being $\frac{1}{8}$ per cent.

6. A person invests £2645 in railway shares at $114\frac{3}{4}$, the annual dividend being £5 on each share; afterwards he sells out at $125\frac{1}{4}$ and invests in 3 per cent. Consols at $93\frac{3}{8}$ (brokerage on Railway Stock $\frac{1}{4}$, on Consols $\frac{1}{8}$). Find the change in income.

7. I invest a sum of money in the 6 per cents. at $112\frac{1}{4}$, and after receiving a half-year's dividend, I sell out at $115\frac{3}{8}$, gaining thereby £103, 8*s*. in all. What sum did I invest (brokerage $\frac{1}{4}$)?

8. A person invested £1911 in 3 per cent. Stock at $79\frac{1}{2}$. He sold out at a higher price and realised £2061. If he paid $\frac{1}{8}$ per cent. commission on each transaction, at what price did he sell?

9. Find the change in income due to transferring £5670 of $4\frac{1}{2}$ per cent. Stock at $83\frac{1}{8}$ to $5\frac{1}{2}$ per cent. Stock at $94\frac{3}{8}$, a brokerage of $\frac{1}{8}$ per cent. being charged on each transaction.

10. A man derives an income of £126, 13*s*. 4*d*. from money invested in the 3 per cent. Consols. He sells out when they are at 93, and buys $3\frac{1}{2}$ per cents. at 95. How much is his income increased by the transfer?

11. I invest £2139 in the 5 per cents. at $114\frac{7}{8}$, and afterwards sell at $135\frac{1}{8}$ and invest in the 4 per cents. at $92\frac{7}{8}$. Find the change in income (brokerage $\frac{1}{8}$).

12. I invest £9075 in the 3 per cents. at $90\frac{5}{8}$, and when they have risen to $91\frac{1}{8}$, I sell out and invest in the $3\frac{1}{2}$ per cents. at $97\frac{3}{8}$. What is the change in my income (brokerage $\frac{1}{8}$ on each transaction)?

13. A person has £10,000 in the 3 per cents., which he sells and reinvests in the $3\frac{1}{2}$ per cents. at $87\frac{3}{8}$ (brokerage $\frac{1}{8}$ on each transaction), and increases his income by £10. Find the price of the 3 per cents.

14. I wish to sell out £45,000 4 per cent. preference shares at 105 and to purchase £45,000 5 per cent. debentures at 131. It is calculated that the result of each sale of £5000 will be to lower the price of the

Stock which I am selling by $\frac{1}{16}$ per cent. (*i.e.* the first £5000 will be sold at 105, the next at $104\frac{15}{16}$, &c.), and that each purchase of £5000 will raise the price of that which I am buying by $\frac{1}{8}$ per cent. If my broker's charges for brokerage for the whole transaction amount to £50, what will be the total amount of his bill?

15. A broker charges $\frac{1}{8}$ per cent. commission on *money* invested and charges the highest quotation of the Stock for the day ($92\frac{5}{8}$), having bought at the lowest ($92\frac{1}{2}$). What is his profit on the investment of £3087, 10*s.*?

16. A man sells out £110,000 Stock at 106 in lots of £10,000 Stock. The effect of the sale of each lot is to lower the price $\frac{1}{8}$ per cent. With the proceeds he purchases £100,000 5 per cent. Stock at 110 in lots of £20,000 Stock, each purchase sending the price of the 5 per cents. up $\frac{1}{4}$ per cent. If his broker charges £50 in all, what cash has he left out of the transaction?

17. A broker is employed to invest a sum of money in the Funds, and receives as brokerage £45, being $\frac{1}{8}$ per cent. on the purchase-money or $\frac{1}{10}$ per cent. on the nominal value of the Stock. What was the amount of the Stock purchased, and at what price per cent. was it standing?

18. A capitalist sells out £1250 3 per cent. Stock at 88 and invests the proceeds in Shipping Company shares at $137\frac{1}{2}$, and thereby increases his income by one-third. What dividend does the Shipping Company pay?

19. How much must a man invest in the 3 per cents. at 114 to obtain an income of £256, 6*s.*, after paying an income tax of 7*d.* in the £, brokerage being charged at the rate of $\frac{1}{4}$ per cent.?

20. A person has £9500, half of which he invests in the 3 per cents. at $94\frac{5}{8}$, and half in the 5 per cents. at $118\frac{5}{8}$. On receiving the first dividend from each, he sells out of the 3 per cents. now at $96\frac{1}{8}$ and invests the total proceeds in the 5 per cents. now at $128\frac{5}{8}$. On receiving his next dividend he sells out all his Stock, which is then at $120\frac{1}{8}$. If there is a charge of $\frac{1}{8}$ per cent. brokerage on every transaction, find the ultimate total gain or loss.

243. When a new Company is formed it is often unnecessary to call up all the Capital at the very outset. It is important to observe in such cases that the dividend declared is paid only on Paid-up Capital.

Example—

(xv.) *The Share Capital of a Company is made up of 50,000 £10 shares. When the first dividend is declared (3%), the shareholders have paid up only £6 per share. Find the dividend due one who has 600 shares in the Company.*

The money actually invested = £6 × 600 = £3600

∴ Dividend = 3% of 3600 = £108.

244. When investing in a Company one must be careful to ascertain whether the shares are fully paid up or not, as the persons in possession of the shares are held liable for all Capital not yet called up.

Examples—

(xvi.) *The £50 shares of a Company are quoted at 34 when they are only half paid up. Find the cost of 100 of these shares, and the interest obtained on them from a $2\frac{1}{2}$% dividend.*

Cost of shares as they stand $= £34 \times 100 = £3400$
Amount still to be called up $= £25 \times 100 = 2500$
Real cost $= £5900$

(Of course the £2500 may not be called up for a considerable time, yet the investor must be prepared to pay it when called upon.)

Amount paid up $= £25 \times 100 = £2500$
Dividend $= 2\frac{1}{2}$% of paid-up Capital $=$ **£62, 10s.**

245. To find the rate of interest which a Stock pays. By **Rate of Interest** is always to be understood the amount of interest obtained by investing £100 in the concern, and must be distinguished from the dividend paid on £100 Stock.

(xvii.) *Find the rate of interest paid by the 4 per cents at 95.*

Interest of £4 is obtained by investing £95
,, ? ,, ,, £100

$$\frac{4 \times \overset{20}{100}}{\underset{19}{95}} = \frac{80}{19} = £4\frac{4}{19} = \textbf{£4, 4s. } 2\tfrac{1}{2}d.$$

Short Method—Rate of Interest $= \frac{£4}{£95}$ of £100 $=$ **£4, 4s. $2\frac{1}{2}$d.**

246. From this we obtain a method of comparing Stocks.

(xviii.) *Which is the better investment, the 5 per cents at 103 or the $3\frac{1}{2}$ per cents at 85? Compare the rates of interest.*

1st Case—If £5 is interest on investing £103
? ,, ,, £100

$$\frac{£5 \times £100}{£103} = £4.8.$$

2nd Case—If £3.5 is interest on investing £85
? ,, ,, £100

$$\frac{£3.5 \times £100}{£85} = £4.1.$$

Short Method—

1st Interest $= \frac{£5}{£103}$ of £100 $=$ **£4.8.**

2nd Interest $= \frac{£3.5}{£85}$ of £100 $=$ **£4.1.**

The first is seen to pay the higher rate of interest, hence, *cæteris paribus* (see § 231), it is the better Stock. Observe that by expressing the rates in decimal form, the comparison is more quickly made.

247. A shorter method of comparing is to see what interest would be obtained by investing the amount of the price of one of the Stocks in the other, and compare the result with the dividend obtained in its own Stock.

Thus, by investing £85, price of second Stock (Ex. xviii.), in the 5 per cents. at 103, we obtain as interest–

If £5 is interest on £103
? „ „ £85

$$\frac{£5 \times £85}{£103} = £\frac{425}{103} = £4{\cdot}1$$

Short Method—

$$\text{Interest} = \frac{£5}{£103} \text{ of } £85 = £4{\cdot}1,$$

whereas, in its own Stock, the £85 produces only 3·5. Hence, first Stock is preferable.

248. It should be noted that the dividend paid by a Stock is not always a safe guide as to its value. Several concerns paying large dividends have collapsed suddenly, during recent years. One ought to study the whole "life" of the Company and its prospects before concluding that it is "safe." Of course the amount of Capital paid up must also be taken into account, as a high percentage dividend may dwindle down considerably when issued on Capital paid up. *E.g.* 8% dividend on £10 shares (£3 paid up) quoted at £5 falls to the much lower figure of 4 8%, and includes the risk of a further payment of £7 per share at any time, the future dividend on which may be much lower than 8%.

Examples 78.

1. What is the rate per cent. of the interest that a man gets on money invested in a $3\frac{1}{2}$ per cent. Stock, the price of which is $109\frac{3}{8}$ (including brokerage)? What income would he get on £1500 so invested?

2. What must the price of the 5 per cents. Stocks be to be as good an investment as the 3 per cents. at $92\frac{1}{8}$?

3. A person invests £2100 in the $2\frac{1}{2}$ per cents. at $104\frac{7}{8}$. What amount of interest will he receive (charge for brokerage being $\frac{1}{8}$ per cent.)? Find also at what price he must buy $4\frac{1}{2}$ per cent. Stock in order to obtain the same rate of interest.

4. A man spent £1100 in purchasing $3\frac{1}{2}$ per cent. Stock at $82\frac{1}{2}$. What income did he obtain, and what per cent. did he get on the money he invested?

5. On a certain day the $3\frac{1}{4}$ and the $3\frac{3}{4}$ per cent. Stocks were respectively $91\frac{1}{8}$ and $94\frac{1}{2}$, including brokerage. Find which is the better Stock, and calculate to the nearest penny the difference in income from investing £20,000 in each.

6. What must be the market value of the 3 per cent Consols in order that after deducting an income tax of 7*d.* in the pound they may yield 4 per cent. interest?

7. Which would yield the better return, Great Western Stock at $136\frac{1}{2}$ or Caledonian Stock at 125, the former paying $5\frac{1}{2}$ per cent. dividend and the latter 5 per cent.? If a man sold £1000 Stock in the less profitable Stock and invested the proceeds in the other, what additional income would he receive, assuming that he paid $\frac{1}{8}$ per cent. brokerage on the Stock sold?

8. What must be the market value of 3 per cent. Stock in order that, after deducting an income tax of 10*d.* in the £, it may yield $3\frac{1}{2}$ per cent. interest?

9. Find what amount must be invested in 3 per cent. Stock at $87\frac{5}{8}$ (brokerage $\frac{1}{8}$) so that the yearly income derived therefrom (after deducting income tax at 6*d.* in the £) may be £300.

10. A man invests £840 in the $2\frac{3}{4}$ per cents. at 99 and £1470 in the 4 per cents. at 126. Find the average rate of interest per cent. which he obtains.

11. If the three per cents. are at $87\frac{1}{8}$, and railway debentures paying $4\frac{1}{2}$ per cent. are at a premium of 25 per cent., by which investment can the larger income be secured; and by how much will it exceed the smaller if the sums invested be £2613, 15s.?

12. If a man receives 4 per cent. interest on his capital by investing in $3\frac{1}{2}$ per cents, find (1) the price of the Stock; (2) how much Stock can be bought for £1400.

13. A person invests £28,227 in a 3 per cent. Stock at 97. On the Stock rising to par he sells out, and, investing the proceeds in a new Stock at 291, he finds that he thereby gains in interest £77. What is the *rate of interest* of the latter Stock?

14. Two persons have equal amounts invested, the one in the three per cents. and the other in the $3\frac{1}{2}$ per cents., and both receive equal amounts of interest. If the $3\frac{1}{2}$ per cents. are at $87\frac{1}{2}$, find the price of the 3 per cents.

15. A person sells Stock A, paying $6\frac{1}{2}$ per cent., at $128\frac{1}{2}$ and invests in Stock B, paying 3 per cent., at $72\frac{1}{2}$. By how much per cent. will the interest of his investment be altered?

16. A man has £2000, part of which he invests in the 3 per cents. at 90 and the remainder in the 5 per cents. at 125. What sum has he invested in the respective Stocks if he makes $3\frac{1}{2}$ per cent. on the whole? (See § 208.)

*17. The difference between the income derived from investing a certain sum in the 5 per cents. at 125 and in the $6\frac{1}{4}$ per cents. at 135 is 11 guineas. How much was invested?

18. A person finds that if he invests his money in the 6 per cents. at 126 his income will be £22, 10s. greater than if he invests it in the 9 per cents. at 210. Find the sum invested.

19. A person invests £1200 in the two and a half per cents. at 96.

Find how much he must further invest in 4 per cent. preference Stock at 130 so as to secure an all round 3 per cent. return.

20. £500 $4\frac{1}{2}$ per cent. preference Stock is bought at $111\frac{1}{2}$. The stamp and fee cost £2, 10*s.*, and commission is charged at the rate of $\frac{1}{8}$ per cent. (per £100 Stock). What is the actual interest produced by the money invested?

21. A person purchased 500 £20 shares in a concern. They are quoted at £15, but are only half paid up. Calculate (1) what he will receive when a 4 per cent. dividend is declared; (2) the full cost of the shares supposing they are fully called up while he has them.

22. I bought 25 shares of a certain Company for £135 each, the paid-up capital for each share being £100. During the year I got a dividend of $5\frac{1}{2}$ per cent. on the paid-up capital, and a bonus of £35 on my 25 shares. What rate of interest did my investment bring me?

23. Which is the better investment—a purchase of £25 shares in a company (£20 paid up) quoted at $22\frac{1}{2}$ paying a dividend of 8 per cent., or thread shares of £1 (fully paid up) selling at 35*s.* 6*d.* and earning a dividend of $12\frac{1}{2}$ per cent.?

24. A person has shares in a company valued at £1 each, but of this only 12*s.* 6*d.* has been called up. If during a certain year a dividend of 4 per cent. is declared on the paid-up capital and the remaining capital called up but no dividend paid on it, what rate per cent. of interest does the investor get for his money?

249. Frequently the price of a Stock is quoted **cum div.** or **ex. div.** This means that the particular company is about to declare a dividend, and the price quoted includes (cum) or excludes (ex.) the right to receive the dividend. *E.g.* if a person buys Stock at 94 "cum div." he is entitled to share in the dividend just about to be declared, to the extent he would had he held the Stock for a considerable time previous.* If he buys at 94 "ex. div." this means that the dividend when declared shall be paid to the person who sold the Stock and not to the new purchaser.

Given the price "cum div." and the dividend, we can calculate what ought to be the price "ex. div." and *vice versâ*.

Example—*A concern is about to pay a dividend which is expected to be 8 per cent. If the Stock is quoted at 105 "ex. div.," what ought to be the price "cum div."?*

* Suppose a person A is selling the Stock and B buying:—If A sells at 105 "ex. div." he obtains 105 for the £100 Stock and then gets £8 of dividend. He therefore gets altogether £113. This must be the price B would require to pay A for the Stock if A were to sell "cum div.," as B would get the £8 interest on each £100 Stock bought, and hence his real payment for the £100 Stock would be 105, the price "ex. div."

Examples 79.

1. I invest £26,180 in the 3 per cents at $93\frac{1}{2}$, but shortly after sell out a half of my Stock at $92\frac{1}{4}$ and with the proceeds buy in the 4 per cents at 97. Find, correct to the nearest penny, the difference in my income.

2. What rate of interest does a man get for money invested in the $4\frac{1}{2}$ per cents. at 95 if he has to pay an income tax of 6*d.* in the £ on his gross income from the Stock?

3. A man sells out of the 3 per cent. Consols at 90 and invests in shares paying 5 per cent. interest. If he makes an increase of £336 on his previous income of £864, at what price did he buy in?

4. The total capital of a concern is £50,000, and the profits for a certain year amount, when untaxed, to £2000. A man holds £5000 of the Stock. Calculate the difference in his income under the following conditions:—(1) The company pays income tax of 1*s* in the £ on the total profit and divides the residue among the shareholders, who pay no further tax. (2) The company pays no tax, but each shareholder is taxed on his dividend, those not exceeding £400 being allowed an abatement of £160. If under the second condition his net income represented a return of 4 per cent on his money invested, at what price did he buy?

5. Two Stocks are standing, the $4\frac{1}{2}$ per cents. at 120, and the 4 per cents. at 80. How ought a capital of £1000 to be invested between the two so as to bring in a yearly income of £45? (See § 208.)

6. A man has £2160, part of which he invests in the 3 per cent. Stock at 84 and the remainder in 5 per cent. Stock at 112? How must he divide the money so as to get the same interest on each investment?

7. I sell out £40,000 Stock from the 3 per cents. at 95 and invest the proceeds, half in the 4 per cents. at 104 and half in 4 per cents. at $108\frac{1}{2}$ Find, to the nearest penny, the difference in my income.

8. A man sells out £5833, 6*s.* 8*d.* $2\frac{1}{2}$ per cent. Consols at 96 and invests the proceeds partly in $4\frac{1}{2}$ per cent. Railway Debentures at 135 and partly in 3 per cent. Colonial Stock at 96. His income is increased by £35, 18*s.* 8*d.* Find how much of each kind of Stock he bought on reinvestment.

9. What must a person have invested in the 3 per cents. at 87 if the transfer of three-fourths of his capital to the 4 per cents. at 120 would diminish his income by £4 (neglect brokerage)?

10. At what must a man have purchased the 3 per cents. if they yield him an interest of $4\frac{1}{8}$ per cent. after paying an income tax of 6*d.* in the £ on his dividend, brokerage being $\frac{1}{8}$ per cent.?

11. In a certain company the preference Stock pays a dividend of 5 per cent. annually in October. If this Stock is quoted at 114 "ex. div.," what ought to be its quotation "cum div."? What interest does the Stock pay

12. A person has an income derived from £3360, which was originally invested for him in the 4 per cents. at 96. If he now sells out at 94 and invests one-half in Railway Stock at $82\frac{1}{4}$, which pays a dividend of 3 per cent., and the other in Bank Stock at $164\frac{1}{2}$, paying $8\frac{1}{2}$ per cent. dividend, what difference will he find in his income?

13. If a Stock, of which the estimated dividend due is 4 per cent., be quoted at 94 "ex. div.," what difference would it make to a man who has £23,030 to invest, whether he buys "ex. div." or "cum div.," the dividend when declared being $4\frac{1}{8}$% and the Stock then sold out?

14. What must be invested in the 4 per cents. at $84\frac{1}{2}$ (brokerage $\frac{1}{8}$ per cent.) so that the yearly income derived therefrom, after deducting a tax of 1s. in the £ (on all save £160), may be £54?

15. Explain the following cutting from the money column of a morning newspaper:—"Amongst Foreign Rails, Mexican Rails were strong, the Ordinary rose $\frac{1}{2}$, First Preferred 2, and Second Preferred 1. Canadian Pacific fell $\frac{3}{4}$ to $48\frac{3}{4}$."

16. How much money must I lay out in the $3\frac{1}{2}$ per cents., at a discount of $12\frac{1}{2}$, to obtain an income of £140, brokerage being at the rate of $\frac{1}{8}$ per cent.?

17. A person invests £1500 in the $4\frac{1}{8}$ per cents. at 75. How much must he invest in the $5\frac{1}{4}$ per cents. at 150 so as to secure an all round 4 per cent. dividend?

18. A man sells out £5000 Stock from the 3 per cents. at 94, brokerage $\frac{1}{8}$ per cent., and invests the half of his money in the $2\frac{3}{4}$ per cents. at $93\frac{3}{4}$, brokerage $\frac{1}{8}$, and loans the rest at 4 per cent. Calculate the difference this makes in the amount of his income.

19. A person invests two sums of £780, one in the 3 per cents. at 80 (price including brokerage), and the other in the 4 per cents. at $97\frac{3}{8}$ (brokerage extra, $\frac{1}{8}$ per cent.). On receiving his first year's dividend, he sells out of both at par (the expenses of the sales amounting to £3, 10*s.*), and loans the total money on mortgage at 5 per cent. per annum. Calculate his first return from the mortgage.

20. A French Company pays a dividend of 4 per cent. Calculate in English money, to the nearest penny, the dividend an investor gets who purchases 3000 shares in it at 32·5 francs per share, the original value of which was 30 francs (£1 = 25·2 francs).

21. A person has money which he invests as follows:—£5200 in the $3\frac{1}{2}$ per cents. at 104, £2010 in the $4\frac{1}{2}$ per cents. at 134, and £1764 on loan at $3\frac{1}{2}$ per cent. Find his average interest per cent.

22. Find which Stock yields the better return—French 20 franc shares quoted at 25, paying $3\frac{1}{2}$ per cent., or English Railway Stock at 90, paying $3\frac{1}{4}$ per cent., if the rate of exchange is £1 = 25 francs. Find the difference in income from investing £1000 in each. Neglect brokerage.

XXXII.—BANK CHEQUES AND BILLS.

250. Money transactions are frequently made by means of cheques instead of by cash.

A cheque is a written order on a banker requiring him to pay the sum stated on it, either on demand, or to the order of, or through the banking account of the person or firm mentioned on it.

The following illustrates the appearance of a cheque.—

No. 597. GLASGOW, 21*st Oct.* 1901.

COMMERCIAL BANK OF SCOTLAND, LTD.

Pay Mr. ALFRED SMITH or bearer Fifty-seven pounds ten shillings and sixpence.

£57 : 10 : 6 EDWARD BRYSON.

Embossed Penny Stamp

251. Any one presenting the above form of cheque at the bank will receive the money at once, it being deducted from Mr. Bryson's account in the Commercial Bank of Scotland. Such a cheque is termed **payable to bearer**, and the bearer usually signs his name on the back in token of receipt of the money.

When, however, it is stated on the cheque that it is **payable to order**, then Mr. Smith must endorse it, *i.e.* sign his name on the back of it, after which the money will be paid to any one who presents the cheque. A cheque payable to bearer need not be endorsed. If, however, the money is paid on the cheque being presented by a person who has no account in that bank, the banker usually asks for an endorsement as a receipt. It will be seen that a cheque payable to order is more secure than one payable to bearer; but, as an endorsed cheque may be lost, many firms are in the habit of **crossing their cheques**. This consists in writing the words *& Co.* between two parallel lines drawn across the face of the cheque thus—

/ & Co. /

When such a cheque is presented at the bank the banker merely transfers the money from Mr. Bryson's account to Mr. Smith's. Should Mr. Smith not have an account at the Commercial Bank he presents it at his own bank, the officials of which obtain the necessary transfer through the Bankers' Clearing-House, to which reference will be made later. If Mr. Smith should have no bank account he must get the money paid to him through some one who has.

252. Cheques are thus a great convenience, lessening as they do the actual handling of money in settling accounts. Frequently,

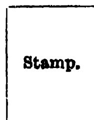
the same cheque passes through many hands before being presented at the bank. Thus, in the example before us :—Mr. Smith might use the cheque to pay some other account, say to Mr. Brown, who in turn might use it to pay some other person. Each individual who uses it in this way must endorse it, and becomes responsible for its amount should the bank refuse payment, *i.e.* if Mr. Bryson has not sufficient money in the bank when it is presented there.

Cheque forms are usually supplied by the banks in "books." The cheques are numbered consecutively, and there is a counterfoil to each. The counterfoil should be filled up similar to the body of the cheque and kept for reference.

253. The person who writes out the cheque is called the **drawer**, the banker is called the **drawee**, and the person to whom it is payable is called the **payee.**

254. Frequently a person does not desire the money to be paid over at once, in which case a **Promissory Note**, **Bank Draft**, **Inland Bill or Acceptance**, or a **Bill of Exchange** is made out.

255. A **Promissory Note** is a written promise made by one person to pay a certain sum of money to another person, or his order, or to bearer, on a certain date mentioned in the note. The following illustrates the appearance of a promissory note :—



£60.

GLASGOW, 20*th Sept.* 1901.

Three months after date I promise to pay to the order of WALTER SINCLAIR the sum of Sixty pounds sterling for value received.

GEORGE BARNETT.

Walter Sinclair may use this note in much the same way as if it were a cheque, and pass it on to others after endorsing it, or present it to the bank or to a bill broker, and obtain money for it ; provided of course that George Barnett is known to be likely to pay or honour the note when due. As, however, all who negotiate this note know that the actual payment will not be made till the specified date, they make a deduction from its face value on account of their having to wait for their money. The deduction is termed **Discount**, and bears a relation to the current rate of interest at the bank

256. A Bank Draft or **Bill of Exchange**, whether inland or foreign, differs essentially from a promissory note. Thus in the case under consideration, where Walter Sinclair has sold goods to George Barnett, while George Barnett would write out a promissory note, if a bill were used Walter Sinclair would write it out thus:—

GLASGOW, 20*th Sept.* 1901.

Stamp.

£60.

Three months after date pay to me or my order the sum of Sixty pounds sterling for value received.

George Barnett.

Accepted, George Barnett.

WALTER SINCLAIR.

GEORGE BARNETT.

MR. GEORGE BARNETT,
3 Maxwell Square, Glasgow.

Accepted, GEORGE BARNETT.

Walter Sinclair would then send the bill to George Barnett, who would **accept it**, *i.e.* would write across the face of it, in any of the ways indicated above, the word **accepted**, with the date, his signature, and the name of the bank, if any, where payment would be made. The bill is then returned to Walter Sinclair, who can use it in the same way as a cheque or promissory note. In this case also, if the bill is negotiated either through other firms or through the bank, or through a bill broker, discount is taken off to recompense for the time that will elapse before the bill is paid up.

257. Foreign Bills of Exchange are much the same as inland, but for safety are usually written in sets of three, thus:—

Stamp.

PAYABLE IN GLASGOW.

No. 639. £60.

CALCUTTA, 30*th Sept.* 1901.

Six months after sight of this First of Exchange (second and third of same tenor being unpaid) pay to me or my order the sum of Sixty pounds sterling for value received as advised.

WALTER SINCLAIR.

To MR. GEORGE BARNETT,
3 Maxwell Square, Glasgow

On presenting the above as it is to the bank at Calcutta, along with the bills of lading (a note of the goods actually shipped signed by the captain) as a guarantee that the debt is genuine,

Walter Sinclair would receive immediate payment The bank would then forward the bill and other papers to Glasgow, and as soon as George Barnett honours the bill he receives the necessary bills of lading entitling him to unload his goods. When such a course is impossible, Walter Sinclair must send on the bill to George Barnett, who endorses and returns it, as in the case of an Inland Bill.

The other two are mere copies of the first, written out in case the first is lost in transit. They are sent on at different times, but two of the set become useless whenever one of the three is paid.

258. It will be seen that George Barnett does not require to send any money abroad, he merely pays it in Glasgow. Nor does the banker here require to send money, as some firms in Calcutta will be owing money in Glasgow, and pay it in Calcutta. There will, however, be some difference in the totals of these, thus necessitating the transmission of some money between countries, but little in comparison with what there would otherwise be. The bankers and bill-brokers take all this into account when discounting the bills, and charge accordingly. If there should be a great demand in Glasgow for payment of goods bought by Calcutta merchants, the bankers in Glasgow, fearing the necessity of having to transmit money from Calcutta to Glasgow, will charge more for the accommodation. This extra charge must be borne by the Calcutta merchants, hence they will consider the advisability of sending the cash themselves as being cheaper, and, accordingly, the demand for bills will decline. The bankers thus fix what is termed the **rate of Exchange.**

259. Take another example with French money.

The British sovereign contains gold to the value of 25·225 francs, and ought therefore to be exchanged for that number. This is termed their **par of exchange** (§ 141); but as it costs money to bring francs from Paris to London, or to send sovereigns to Paris, this cost must be considered in estimating the value of the coins. Even should merchants choose to transmit **bullion** (solid bars of metal) instead of **specie** (coins), the cost of transit still comes in. Bullion merchants and various agents undertake the transmission of money in either form, and hence are in competition with the banks and bill-brokers. When, therefore, a person A, in London, owes money to B, in Paris, he may ask B to draw a bill on London; or A may purchase in London some bills already drawn on Paris and transmit them to B, who would either

wait till they were due, and thus receive his payment from a merchant, C, in Paris; or, as is more usual, B would sell it to a broker in Paris, who would collect it when due. If there happened to be in London a great demand for bills on Paris, the brokers would charge accordingly. A London merchant might, under such circumstances, get in exchange for a sovereign a bill on Paris for 25 francs. This would be the **Course of Exchange.** Bills on Paris would be at a premium, for London merchants are paying more than the value for them. At the same time in Paris bills on London would be at a discount The course of exchange cannot, however, differ much from the par of exchange, for then people would begin to transmit money through the bullion merchants as agents. Again, the competition between the banks and bill-brokers keeps the course of exchange from fluctuating very much.

Per Pro.—When one person signs for two or more he may add the abbreviation *per pro* This signifies "per procuration," and intimates that the agent has but a limited authority to sign, and the principal is only bound by such signature if the agent in so signing was acting within the actual limits of his authority. The proper method of using the abbreviation is *per pro. John Jones & Co., Thomas Smith.* The letters p p. are also used as a further abbreviation.

260. It will be seen from the foregoing that many cheques and bills, &c., pass through a number of hands and go far from the original parties interested before they become useless owing to their being paid. The **Bankers' Clearing-House** is the place where all may be cleared up, the cheques and bills transferred to the right banks, &c., the necessary payments being made, just as in the Railway Clearing-House the various through tickets, &c., are valued and exchanged.

XXXIII.—DISCOUNT.

261. It has been shown (§ 255) that when bankers, bill-brokers, and others pay over money for a bill, or, as it is termed, discount the bill, they deduct a certain sum as payment for the loan they are giving. This discount is calculated exactly as simple interest on the full amount of the bill, counting from the day the bill is discounted till the day that it becomes legally due. It has to be observed that Bills are not legally due till three days after they are nominally due. These three days are termed days of grace.

262. Banker's discount is calculated as follows:—

Example—

(i.) *A bill for £800 is drawn on 1st October for 6 months, and discounted on 3rd January at $2\frac{1}{2}$%. Find the discount deducted by the banker.*

Bill is legally due on 4th April, so that the time upon which the banker calculates his discount is from 3rd January to 4th April = 91 days.

$$I = \frac{PRY}{100} = \frac{800 \times 2{\cdot}5 \times 91 \times 2}{365 \times 100 \times 2}$$

```
  364000
  121333
   12133
    1213
4·98680 = £4, 19s. 9d.
     20
 19·736
     12
    8·8
```

263. The sum discounted being £800, and the discount £4, 19s. 9d., the owner of the bill will receive the difference, £795, 0s. 3d. If this sum, called the **Present Value** of the bill, were lodged in the bank on 3rd January and drawn on 4th April, and interest obtained at the rate of $2\frac{1}{2}$ per cent. per annum, an easy calculation will show that it would not amount to £800, the original amount of the bill. Now, theoretically, the discount should have equalled the interest obtainable from reinvesting the money at the same rate of interest. This theoretical discount is termed **True discount** (a discount seldom used) to distinguish it from the discount in daily use, which would seem to be more deserving of the name "true." Obviously the banker by charging discount, as he does, obtains a greater interest for his money than the rate charged for discounting. For, take a bill of £100 due 1 year hence, rate of discount being 5 per cent.; the banker deducts £5 as discount and hands over £95. At the end of the year the banker obtains £100 (the total amount of the bill), thus getting £5 as interest for the loan of £95 for 1 year, which works out at $5\frac{5}{19}$ per cent. of interest.

264. As some calculations still necessitate a knowledge of the so-called True Discount, we shall illustrate the method of working these. The following contractions are used throughout the examples:—

B.	**stands for**	**Bill.**
P.V.	"	**Present Value.**
T.D.	"	**True Discount.**
B.D.	"	**Banker's Discount.**

265. From what has been already said it will be seen that the True Discount on any bill when deducted from it will produce a Present Value such as, if put out to interest from the day of discounting till the day the bill is due, would produce as interest a sum equal to the True Discount deducted from the bill. The bill would, on being discounted, be reduced to a Present Value, and this P.V. would again produce the original bill.

266. The following diagram shows the relations existing between the various terms used.

Given:—rate per cent. = $2\frac{1}{2}$; time till bill is due = 2 years.

		£	
Interest. →	Bill	= 105	
	Present value	= 100	Interest.
Banker's £5, 5s.	Discount	= 5	True

From this it is seen B. − P.V. = T.D.
B. − T.D. = P.V.
B. = P.V. + T.D.

Also B.D. = Interest on B. = Interest on (P.V. + T.D.)
and T.D. = Interest on P.V.
difference = Interest on T.D.

So T.D. is the present value of the corresponding B.D.
Interest on B. = B.D.
B. − B.D. = Banker's P.V.

267. To calculate T.D., or true P.V., state the main points of the question in two columns, that on the left being the calculation where the P.V. is supposed to be £100, the right-hand column being for the values in the question.

Example—

(i.) *Find the Present Value of a bill for £357 due in 6 months' time at 4 per cent. per annum Simple Interest.*

Bill	102	Bill	357
P.V.	100	P.V.	?
T.D.	2	T.D.	?

By proportion:—If a Bill of 102 has a P.V. of 100
then „ 357 „ ?

$$\frac{\overset{50}{\cancel{100}} \times \overset{7}{\cancel{357}}}{\underset{2}{\cancel{102}}} = £350 = \text{P.V.}$$

By filling in P.V. = 350 in right-hand column, the T.D. can be calculated by subtraction.

Example—

(ii.) *A gentleman buys a house for £610 on condition that he does not pay it until the expiry of 10 months. At the end of 2 months, however, he wishes to pay it. How much should be deducted from the price so that the original owner may lose nothing, bank interest being at the rate of 2½ per cent. What is here wanted is the T.D. on £610 due in 8 months.*

Bill	101⅔	Bill £610
P.V.	100	P.V. ?
T.D.	1⅔	T.D. ?

By proportion :—If a Bill of 101⅔ has a T.D. of 1⅔,
then „ 610 „ ?

$$=\frac{5}{3}\times\frac{610\times 3}{305}=£10.$$

268. It has been already pointed out (§ 261) that ordinary discount is calculated merely as simple interest. Hence, if the difference between the True and the Banker's Discount be desired, calculate both of these discounts separately, and then subtract to get the difference between them. Note that the Banker's Discount is always the greater.

Example—*A bill of £720 is due in 6 months. Find the difference in the amount of discount according as Banker's or True Discount is calculated, the rate being 4 per cent.*

$$\text{Banker's discount}=I=\frac{BRY}{100}=\frac{720\times 4\times 1}{2\times 100}=£14{\cdot}4$$

True discount :—		
B = 102		B = 720
P.V = 100		P.V. = ?
T.D. = 2		T.D. = ?

If a B of 102 gives a T.D. of 2
then „ 720 „ ?

$$\frac{2\times 720}{102}=14{\cdot}1176$$

Difference = £14·4 − £14·1176 = £0·2823 = 5s. 8d.

269. The table on page 217 shows the relation that exists between the True Discount and the Banker's Discount to be such that the difference between the two is equal to the interest on the

True Discount for the time and rate calculated in discounting the bill. A further examination of this relation will show that the True Discount is the Present Value of a Bill, equal in amount to the Banker's discount. Should then the difference between the two discounts be given and also the rate per cent. and time, we may calculate therefrom the exact value of both discounts and the amount of the original bill.

Example—*If the difference between the True and the Banker's Discount on a certain bill due 3 months hence is 5s., the rate being 4 per cent., find (1) True Discount, (2) Banker's Discount, and (3) amount of bill discounted.*

(1) To find True Discount—

5s. is the interest on the T D for 3 months @ 4%.

$$I=\frac{PRY}{100} \quad \therefore \frac{I\times 100}{RY}=P$$

$$\frac{\cdot 25\times 100}{4\times \cdot 25}=£25 \text{ (the True Discount).}$$

(2) To find Banker's Discount—

£25 + 5s (the difference)

= £25, 5s. (the Banker's Discount).

(3) To find amount of Bill—

Since B.D. is the interest on the Bill,

$$\text{and } \frac{I\times 100}{RY}=P$$

$$\text{we have } \frac{25\cdot 25\times 100}{4\times \cdot 25}=£2525 \text{ (Amount of Bill).}$$

The Bill might also have been calculated from the T.D.

thus:		
	B = 101	B ?
	P.V. = 100	P.V.
	T.D. = 1	T.D. = £25

If a B of 101 has a T.D. of 1

then " ? " 25

$$=\frac{101\times 25}{1}=£2525.$$

270. From what has been already shown as to the relation between the two discounts, it will be seen that, should the interest on any sum be equal to the true discount on a certain Bill (time and rate in both calculations being the same), the given sum must be the Present Value of the given Bill.

The T.D. on a Bill is equal to the interest on its P.V. (§ 266).

E.g. If the interest on £700 for 6 months at 5 per cent. is equal to the True Discount on £717, 10*s.* for the same time and at the same rate, then £700 is the P.V. of £717, 10*s.*, and so the T.D. is equal to £17, 10*s.*, for (§ 266)

T.D. on → B. = £717, 10*s.*
P.V. = £700, 0*s.*
Discount = £17, 10*s.*

From this we can easily calculate, by simple interest rules (§ 215), the rate of interest *or* the time the Bill has to run, given the time *or* the rate. It may also be used to find the Banker's Discount on the larger sum should the time *or* rate be unknown. In the above example, suppose the rate were unknown, then we know that £17, 10*s.* is the interest on £700 for 6 months at the unknown rate, which on calculation (§ 215) is found to be 5 per cent. Then the Banker's Discount on £717, 10*s.* for 6 months at 5 per cent. could be found to be £17, 18*s.* 9*d.* Similar working is to be adopted when rate is given and time unknown.

Examples 80.

Note.—Where exact dates are given, allow "days of grace"; where periods are given, assume that the period includes the days of grace, all answers to be correct to nearest penny. Calculate banker's discount unless otherwise stated.

1. Find the true discount on a bill for £370, 2*s.* at 3 per cent. nominally due on September 1st and paid on March 18th.

2. Find the present value of £1000, due 5 years hence, at 3 per cent. simple interest, allowing true discount.

3. Find the present worth of £504, due in 3 years, at 4 per cent. simple interest, allowing true discount.

4. A man had a bill of £282 due to him at the end of 4 years. Find what sum he should take for a cash payment, interest being at the rate of 5 per cent. per annum.

5. A person has drawn a bill on 9th April at six months for £4040. What sum of money ought he to deposit in a bank, paying 5 per cent. interest, on 31st July so as to be able to draw sufficient to pay the bill when due?

6. A bill for £3019, 10*s.* was drawn on 1st March at 7 months. When ought a payment of £3000 cash to be accepted and the bill discharged, interest being at the rate of $3\frac{1}{4}$ per cent. per annum?

7. How much ready money ought a man to take in payment of a bill for £894, 15s. $3\frac{3}{4}d$. due in 2 years 3 months; interest being reckoned at $4\frac{1}{2}$ per cent. per annum?

8. The present value, allowing true discount, of £678, 8s., due 16 months hence, is £640. What is the rate per cent., simple interest?

9. Find the present worth of a bill for £644, 10s. 6d., drawn March 3rd for 9 months and discounted May 1st, at $2\frac{1}{2}$ per cent. per annum, allowing true discount.

10. The true discount on a bill of £62, 9s. 6d. at 4 per cent. is 9s. $1\frac{1}{2}d$. When is the bill due?

11. Calculate the difference between the banker's and the true discount on £1287, 10s. for 8 months at $4\frac{1}{2}$ per cent. per annum.

12. The true discount on a certain sum for 9 months at 4 per cent. per annum is £12, 16s. 6d. What is the sum?

13. What is the present value of £965, 10s., due 9 months hence, allowing true discount at the rate of $5\frac{1}{2}$ per cent. per annum, simple interest?

14. Find the difference between the two kinds of discount on £127, 2s., for half a year, at 5 per cent. per annum.

15. The true discount on a bill of £187, 8s. 6d. at 4 per cent. per annum is £1, 9s. 9d. When is the bill due?

16. If the banker's discount on £5600 at $3\frac{1}{2}$ per cent. per annum be equal to the true discount on £5747 for the same time at the same rate, when are the sums due?

17. Find the difference between the banker's discount and the true discount on a bill for £400, due in 9 months, when interest is at 4 per cent. per annum.

18. Show that a bill-broker, who deducts as discount 5 per cent. of the amount of a bill, due 12 months hence, gets $5\frac{5}{19}$ per cent. for his money.

19. The rate of interest is 5 per cent. per annum. What was lost on 288 yards sold for £182, 5s., if the goods were bought 6 months ago for 12s. 6d. per yard?

20. Find the difference between the two kinds of discount on a bill of £620, due in 10 months, at 4 per cent. per annum.

21. Find the difference between the true and banker's discount on a bill for £218, 15s., drawn 15th April for 3 months and discounted 6th May at 4 per cent. per annum?

22. What is the present worth of £10,673, due 7 months hence, the rate of interest being $4\frac{1}{2}$ per cent. per annum, allowing true discount?

23. Find the present value of £1500, due 3 years hence, at 5 per cent. per annum, simple interest; calculating (1) banker's discount, (2) true discount.

24. What rate per cent. does a bill-broker really get who discounts a bill, due a year hence, at 4 per cent.?

25. Find the present value of a bill for £614, 14s. 2d., drawn May 8th, at seven months, and discounted September 17th at $4\frac{1}{2}$ per cent. per annum, allowing true discount.

26. What sum of money paid just now would discharge a debt of £206, 8s. 7d. due 8 months hence, at 4½ per cent. per annum?

27. What does a banker gain by discounting a bill for £200, due 3 years hence, at 4 per cent. per annum?

28. How much does a banker gain on discounting a bill for £650, due in 3 months, money being worth 6 per cent.?

29. The ordinary and the true discounts on a sum of money, due 4 years hence, amount respectively to £9, 19s. 6d. and £8, 15s. Find the sum of money.

30. A banker gains £1, 0s. 8d. by deducting interest instead of discount on a bill, due 10 months hence, at 5 per cent. per annum. What is the amount of the bill?

31. What rate per cent. does a banker really charge who discounts a bill due in 5 months at 6 per cent. per annum?

32. A man draws a bill for £750 for 4 months on 11th May and has it discounted on 3rd July. Find its value on that day, taking (1) banker's discount, (2) true discount, interest in both cases being at the rate of 2½ per cent. per annum.

33. Allowing true discount, find the present value at the date of the first payment of an annuity of £1,15, 10s. per annum for 3 years, reckoning simple interest at 5 per cent. per annum.

34. A owes £1515 due in 3 months. If he pays £1000 down just now, how much ought he to pay at the end of the 6 months, money being worth 4 per cent.?

35. If the difference between the true discount and the banker's discount on a sum, due in 4 months at 3 per cent., is £4, find the bill.

36. A bill is discounted at 3 per cent. per annum. If banker's discount be calculated, at what rate of interest per cent. (correct to the nearest sixteenth) must the proceeds be invested, so that nothing may be lost? Had true discount been calculated, what would the rate have been?

37. Find the present value of a bursary of £80 per annum, payable at beginning of each year, for 3 years, simple interest being reckoned at 4 per cent. per annum.

38. What rate per cent. does a banker obtain for his money when he discounts a bill drawn on 4th October for 3 months and discounted on 15th October at 4¾ per cent. per annum?

39. A person wishes to invest such a sum as will enable him to pay a bursary of £50 a year, payable at end of each year, for 5 years. Find the sum, to the nearest pound, simple interest being at 4 per cent. per annum.

Examples 81.

1. A bill for £560 is drawn on 3rd January for 3 months and is discounted on 20th February at 4 per cent. per annum. What is the amount of the banker's discount?

2. A has a bill for £350, 10*s.* drawn on 15th August for 4 months. What is its value to him on 3rd September, interest being at the rate of 5 per cent.?

3. A bill for £30 drawn on 4th February 1901 for 2 months was discounted on 1st March. What was its value on that day, interest being $2\frac{1}{2}$ per cent.?

4. A bill drawn 27th March for £500 at six months is discounted on 4th June, interest being at 3 per cent. What sum does the owner of the bill receive?

5. Find the banker's discount on a bill for £25, 14*s.* 6*d.* drawn on 3rd May for 6 months and discounted on 26th September at $2\frac{1}{2}$ per cent. per annum, simple interest.

6. A buys goods from B valued at £120. B offers him a discount of 3 per cent. for cash, or payment in full after 6 months. If A accepts the latter and gives a bill for the amount, which B discounts immediately, what will B gain or lose, the bank rate of discount being 4 per cent. per annum?

7. A person on presenting a bill for three months at the bank got 14*s.* $7\frac{1}{2}$*d.* deducted as discount. If the rate was 5 per cent., what money did he receive for the bill?

8. A bill for £1812 drawn July 13th at 5 months is discounted on October 4th at $3\frac{1}{4}$ per cent. Find the true discount, and also the banker's discount.

9. A bill drawn February 25th for £875, 5*s.* 8*d.* at 7 months is discounted on June 4th, interest being at the rate of 5 per cent. What sum does the owner of the bill receive?

10. A bill for £860, 10*s.* 6*d.* is due on October 18th. Find its value on 13th July, interest being 8 per cent. per annum.

11. A owes B £300 and wishes to delay payment for 6 months. B requests him to draw out a bill at 6 months for an amount that, if discounted immediately at 4 per cent. per annum, will yield exactly £300. For how much must the bill be drawn?

12. A merchant gets a bill discounted by a banker at $3\frac{1}{2}$ per cent. per annum. What is the lowest rate of interest per cent. per annum correct to 2 decimals that he must invest the proceeds at so as not to lose anything?

13. A owes B £650, due in one year. If B instead of waiting gets the bill discounted at the bank at once at 2 per cent., and invests the proceeds in a concern paying $2\frac{1}{2}$ per cent. interest, what would be his ultimate gain or loss?

14. If the exchange in Glasgow on Paris be 25·25, and the rate of banker's discount in Glasgow be 4 per cent. per annum, what debt can be discharged in Paris by a person in Glasgow who holds a three months' bill in Glasgow for £1000?

15. A London merchant buys goods in Paris valued at 20,000 francs, for which he writes out a 3 months' bill. At the same time he sells goods to a French firm for £500, to be paid in 6 months. If he

surrenders this latter bill to a firm of discounters in London, how much additional English money must he give them so that they may meet the 3 months' bill when due, discount in both countries being calculated at 4 per cent. per annum? (Exchange in Paris and London being at 25·0.)

16. A owes B £600 and wishes to pay in 9 months. B demands a bill at 9 months' for an account that, if immediately discounted at 4 per cent., would yield the £600 due. For how much must the bill be drawn?

(For additional examples on Discount see Examples 86.)

XXXIV.—LOGARITHMS

271. Powers and Roots of Numbers.

In the course of working out examples it is often necessary to multiply a number by itself one or more times. The results of such multiplications are known as the **powers** of the number multiplied. Thus, the product obtained by multiplying a number by itself **once** is called the **second power** or **square** of the number. Multiplying this result by the number, *i.e.* multiplying the number by itself **twice**, we get the **third power** or **cube** of the number. Multiplying the number by itself **three times** we get the **fourth power** of the number, and so on.

272. The converse of such multiplication or finding of the powers of a number is the **extracting of roots**. The **second** or **square root** of a number is that number which, when squared gives the original number; the **third** or **cube root** of a number is that number which, when cubed, gives the original number; the **fourth root** of a number is that number which, when raised to the fourth power, gives the original number, and so on.

273. Indices.

It has been pointed out (§ 134) that x^2 is used to represent xx or $x \times x$. This method of writing the **powers** of numbers is extended so as to include not only integral powers but also fractional powers or **roots**. Thus—

2^5 represents $2 \times 2 \times 2 \times 2 \times 2$;
x^8 „ $x \times x \times x \times x \times x \times x \times x \times x$
and $x^{\frac{1}{2}}$ is used to represent the square root of x
$x^{\frac{1}{5}}$ „ „ „ fifth „ x

The number which is used to represent the power or root is called the index.

Fractional indices may be written in the decimal form, then we have $x^{.5}$ to represent the square root or ·5 power of x,
$x^{.2}$ „ „ fifth „ ·2 „ x, &c.

274. Logarithms, as will be explained later, are really **indices**, and obey all the **Laws of Indices**, so that a knowledge of these is necessary to the understanding of the principles which underlie all working with logarithms.

275. Laws of Indices.

1st Index Law:—

To multiply powers of the same quantity, add their indices.

Examples—

(i.)
$$2^2 \times 2^3 = 2^{2+3} = 2^5$$
$$\text{for } 4 \times 8 = 32 \quad = 2^5$$
$$\text{or, since } 2^2 = 2 \times 2 \text{ and } 2^3 = 2 \times 2 \times 2$$
$$\text{we have } 2^2 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

(ii.)
$$x^3 \times x^4 = x^{3+4} = x^7$$
$$\text{since } x^3 = x \times x \times x$$
$$\text{and } x^4 = x \times x \times x \times x$$
$$\text{we have } x^3 \times x^4 = x \times x \times x \times x \times x \times x \times x = x^7$$

276. The converse of this enables us to divide powers of the same quantity, and may be stated thus:—

To divide a power of a quantity by another power of the same quantity, subtract the index of the divisor from that of the dividend.

Examples—

(i.)
$$2^5 \div 2^3 = 2^{5-3} = 2^2$$
$$\text{for } 32 \div 8 = 4 \quad = 2^2$$
$$\text{or, since } 2^5 = 2 \times 2 \times 2 \times 2 \times 2$$
$$\text{and } 2^3 = 2 \times 2 \times 2$$
$$\text{we have (§ 84) } \frac{2^5}{2^3} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = 2 \times 2 = 2^2$$

(ii.)
$$x^6 \div x^2 = x^{6-2} = x^4$$
$$\text{since } x^6 = x \times x \times x \times x \times x \times x$$
$$\text{and } x^2 = x \times x$$
$$\text{then } x^6 \div x^2 = \frac{x^6}{x^2} = \frac{x \times x \times x \times x \times x \times x}{x \times x} = x \times x \times x \times x = x^4$$

277. In connection with this we have two very interesting and important results :—

1st. $x^0 = 1$, *i.e.* the zero power of any quantity = 1.

We shall prove this for 2^0 and x^0.

(i.) $2^4 \div 2^4 = 2^{4-4} = 2^0$ (§ 276)

but $2^4 \div 2^4 = \frac{2^4}{2^4} = 1$ (§ 34)

or $16 \div 16 = \frac{16}{16} = 1$

$\therefore 2^0 = 1.$

(ii.) Similarly $x^3 \div x^3 = x^{3-3} = x^0$

but $x^3 \div x^3 = \frac{x^3}{x^3} = 1$

$\therefore x^0 = 1.$

2nd. $x^{-2} = \frac{1}{x^2}$, *i.e.* the negative power of any quantity is a method of representing the reciprocal of the quantity raised to that power.

(i.) $2^2 \div 2^5 = 2^{2-5} = 2^{-3}$ (§ 276)

but $2^2 \div 2^5 = \frac{2^2}{2^5} = \frac{2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2^3}$ (§ 34)

for $4 \div 32 = \frac{4}{32} = \frac{1}{8} = \frac{1}{2^3}$

$\therefore 2^{-3} = \frac{1}{2^3}.$

(ii.) $x^3 \div x^7 = x^{3-7} = x^{-4}$ (§ 276)

and $x^3 \div x^7 = \frac{x^3}{x^7} = \frac{x \times x \times x}{x \times x \times x \times x \times x \times x \times x} = \frac{1}{x^4}$

$\therefore x^{-4} = \frac{1}{x^4}.$

278. 2nd Index Law.

To find a power of a power of a quantity, multiply the indices.

Examples—

(i.) The fourth power of $2^2 = 2^8$

i.e. $(2^2)^4 = 2^{2 \times 4} = 2^8$

since $4^4 = 256 = 2^8$

(ii.) $(x^3)^3 = x^{3 \times 3} = x^9$

since $(x^3)^3 = x^3 \times x^3 \times x^3$ (§ 273)

$= x \times x \times x \times x \times x \times x \times x \times x \times x = x^9$

279. These laws can be extended to include negative and fractional indices, for the proofs of which the student is referred to any standard text-book on algebra. We shall show, however, the application of the second law to a fractional index.

280. We have seen (§ 273) that $x^{\frac{1}{5}}$ or $x^{\cdot 2}$ is used to represent the fifth root of x. From the meaning of the fifth root (§ 272), it is evident that if we raise $x^{\frac{1}{5}}$ to the fifth power we should obtain x as the result.

By the second law of indices $(x^{\frac{1}{5}})^5 = x^{\frac{1}{5}\times 5} = x$.

Again, the square root of the square root of a quantity is the fourth root, and this also agrees with the second law—

$$E.g.\ (x^{\frac{1}{2}})^{\frac{1}{2}} = x^{\frac{1}{2}\times\frac{1}{2}} = x^{\frac{1}{4}} \text{ (the fourth root of } x),$$

$$\text{or } (16^{\frac{1}{2}})^{\frac{1}{2}} = 4^{\frac{1}{2}} = 2$$

$$\text{and } (16^{\frac{1}{2}})^{\frac{1}{2}} = 16^{\frac{1}{2}\times\frac{1}{2}} = 16^{\frac{1}{4}} = 2.$$

281. It is necessary that the student should understand what is meant by fractional powers such as $\frac{2}{3}$, 1 5 or $1\frac{1}{2}$ or $\frac{3}{2}$, &c.

$x^{\frac{2}{3}}$ is used to represent (1) the cube root of the square of x, or (2) the square of the cube root of x and, as such, obeys the laws of indices. If $x^{\frac{2}{3}}$ is taken to represent the cube root of the square of x, then, when raised to the third power, the result should be x^2 (§ 273), but, by the second law, we have—

$$(x^{\frac{2}{3}})^3 = x^{\frac{2}{3}\times 3} = x^2.$$

∴ with the meaning given, such an index obeys the laws. Similarly with the other reading.

Examples 82.

Find the values of—

1.	$2^3 \times 2^4$	$2^2 \times 2^4$	$3^2 \times 3^3$	$5^3 \times 5$.
2.	$2^6 \div 2^4$	$2^8 \div 2^7$	$2^{12} \div 2^9$	$3^5 \div 3^3$.
3.	$3^6 \div 3^6$	$3^4 \div 3^7$	$4^3 \div 4^5$	$5^7 \div 5^7$.
4.	2^{-2}	3^{-1}	4^{-3}	5^{-2}.
5.	$16^{\frac{1}{2}}$	$81^{\frac{1}{2}}$	$144^{\frac{1}{2}}$	$343^{\frac{1}{3}}$.
6.	$(-8)^{\frac{1}{3}}$.	$(-125)^{\frac{1}{3}}$	$(-64)^{\frac{1}{3}}$.	
7.	$4^{-\frac{1}{2}}$	$36^{-\frac{1}{2}}$	$81^{-\frac{1}{4}}$.	
8.	$16^{\cdot 5}$	$32^{\cdot 2}$	$27^{\cdot [illegible]}$.	
9.	$8^{\frac{2}{3}}$	$16^{\frac{3}{4}}$	$216^{\frac{2}{3}}$.	
10.	$9^{\frac{3}{2}}$	$8^{\frac{4}{3}}$	$16^{\frac{5}{4}}$.	
11.	$36^{1\cdot 5}$	$81^{1\cdot [illegible]}$.	$16^{\cdot [illegible]}$.	$64^{-[illegible]}$
12.	[illegible]	[illegible]	$(8^{[illegible]})^{\frac{1}{3}}$.	

282. We now proceed to show the relation between indices and logarithms. The square root of 10 may be written down $10^{\cdot5}$, and its value, correct to five significant figures, is 3·1623.

The square root of this, *i.e.* the fourth root of 10, may be written as $10^{\cdot25}$, and its value, correct to five significant figures, is 1·7783.

Continuing the extraction of square roots, we get a series of results which may be tabulated thus:—

Power.	Value.
0·5	3·1623
0·25	1·7783
0·125	1·3335
0·0625	1·1548
0·03125	1·0746
&c.	

Suppose now we wish to find the eighth power of 1·3335. We may multiply up, but since $1\cdot3335 = 10^{\cdot125}$, we have

$$(1\cdot3335)^8 = (10^{\cdot125})^8 = 10^{\cdot125\times8} = 10^1 = 10.$$

283. The above shows the use made of indices in logarithms. When we desire to multiply or divide, find a power, or extract a root of any number, instead of proceeding by the ordinary rules of arithmetic, we find what powers of 10 will produce the given numbers, and then apply the laws of indices. **Tables of logarithms** are prepared, which give, or enable us to calculate, the necessary indices; and corresponding tables—called **Tables of antilogarithms**—enable us to find a number when the index of the power of 10 is known.

Example.—*Multiply* 1·3335 *by* 1·1548.

From the above table we have

$$1\cdot3335 = 10^{\cdot125}$$
$$\text{and } 1\cdot1548 = 10^{\cdot0625}$$
$$\therefore\ 1\cdot3335 \times 1\cdot1548 = 10^{\cdot125} \times 10^{\cdot0625}$$
$$= 10^{\cdot125+\cdot0625} \text{ (1st Index Law)}$$
$$= 10^{\cdot1875}$$

Consulting a table of antilogarithms we would find

$$10^{\cdot1875} = 1\cdot5399$$
$$\therefore\ 1\cdot3335 \times 1\cdot1548 = 1\cdot5399.$$

284. The tables on pp. 358-361 do not deal with numbers above 10, and all the indices, or, as we may now call them, **logarithms** are fractional. In tables of logarithms this is always

the case, there being no necessity to supply the logarithms for other numbers, as they can be easily calculated from those supplied (see § 287). It may be remarked here that it is not necessary to take the number 10 as the number or **base** to which all the other numbers are referred. Any other number might be taken, but 10 is generally selected as the most suitable, since all modern systems of numeration are decimal systems. Logarithms to the base 10 are often referred to as **Common Logarithms**, and are always meant when the base is not stated.

285. It was shown (§ 277) that $x^0=1$, and therefore $10^0=1$, *i.e.* the logarithm of 1 is 0. Now $10^1=10$ and the logarithm of 10 is 1; the logarithm of $100(=10^2)$ is 2; of $1000(=10^3)$ is 3, &c.

Again, (§ 277) $\frac{1}{10}=10^{-1}$, therefore the logarithm of $\frac{1}{10}$ or 0·1 is -1, or $\bar{1}$ as it is usually written. Similarly, the logarithm of $\frac{1}{100}$ $\left(=\frac{1}{10^2}\text{ or }0\cdot01\right)$ is $\bar{2}$; and the logarithm of $\frac{1}{1000}=\left(\frac{1}{10^3}\text{ or }0\cdot001\right)$ is $\bar{3}$, &c.

286 From the table of § 282, we see that the logarithm of 1·7783 is 0·25, now

$$\begin{aligned}
17\cdot783&=1\cdot7783\times10\\
&=10^{\cdot25}\times10^1\\
&=10^{1\cdot25}\\
\therefore \text{log. } 17\cdot783&=1\cdot25\\
\text{Similarly, since } 177\cdot83&=1\cdot7783\times100\\
&=10^{\cdot25}\times10^2\\
&=10^{2\cdot25}\\
\therefore \text{log. } 177\cdot83&=2\cdot25\\
\text{and log. } 1778\cdot3&=3\cdot25\\
\text{log. } 17783\cdot&=4\cdot25\\
\text{Again, } 0\cdot17783&=1\cdot7783\div10\\
&=1\cdot7783\times\tfrac{1}{10}\\
&=10^{\cdot25}\times10^{-1}\\
&=10^{-1+\cdot25}\\
\therefore \text{log. } 0\cdot17783&=\bar{1}\cdot25^*\\
\text{Similarly, log. } 0\cdot017783&=\bar{2}\cdot25\\
\text{and log. } 0\cdot0017783&=\bar{3}\cdot25\\
&\text{\&c.}
\end{aligned}$$

* When the logarithm is less than 0, the fractional part is left positive and only the integral part is made negative.

287. Adopting the above results, we can draw out the following table :—

log	17783·	=	4·25
"	1778·3	=	3·25
"	177·83	=	2·25
"	17·783	=	1·25
"	1·7783	=	0·25
"	0·17783	=	$\bar{1}$·25
"	0·017783	=	$\bar{2}$·25
"	0·0017783	=	$\bar{3}$·25
	&c.		&c.

We thus see that the fractional parts of the logarithms of the above numbers are the same as the logarithm of 1·7783. The fractional part of the logarithm—**the mantissa**—is the only part supplied by the tables. The integral part—**the characteristic**—can always be obtained from the following table, which is drawn up from the above results, and is easily remembered. The table may be used for finding the characteristics of logarithms from the numbers, and the position of the decimal point in numbers, from the characteristics of their logarithms.

288.

Position of 1st Significant Figure of the Number.	Characteristic of Logarithm.
Thousands place (3 places to left of unit) . .	3
Hundreds " (2 " " ") . .	2
Tens " (1 " " ") . .	1
Units " (0 " " ") . .	0
1st Decimal " (1 place to right of unit) . .	$\bar{1}$
2nd " " (2 " " ") . .	$\bar{2}$
3rd " " (3 " " ") . .	$\bar{3}$

The table may be extended indefinitely in either direction.

289. Table of Logarithms :—Numerous tables of logarithms of from four to ten figures are issued. Tables of four-figure logarithms will be found on pp. 358–361 for use in this section, and in questions on Compound Interest (Sect. XXXV.) and Annuities

(Sect. XXXVI.). It should be noted that the number of figures in the table of logarithms indicates the number of significant figures to which results, obtained by use of the tables, are correct. If ten significant figures are required in a result, there is absolutely no use in using anything less than a table of ten-figure logarithms.

290. Use of tables :—As a table of four figure logarithms is usually sufficient for calculations in Compound Interest and Annuities, we shall explain the method of using those on pp. 358–361.

1st. **Use of logarithm tables** :—The first column gives the first two significant figures of the number. Should the number consist of two figures, or if the other figure or figures are 0's, the logarithm is given in the second column, thus:—

log	4·5	=	0·6532
„	3600	=	3·5441
„	0·00073	=	$\bar{4}$·8633

The next nine columns are headed 1 to 9, and give the logarithm of numbers of which the first two significant figures are in the first column and the third in any of these nine (in cases where the number consists of three figures, or the remaining figures are 0's), thus:—

log.	3·86	=	0·5866
„	427	=	2·6304
„	783000	=	5·8938
„	0·00652	=	$\bar{3}$·8142

If the number has a fourth significant figure, other than 0, the logarithm is found by adding to the logarithm obtained from the first three significant figures the number in the last set of columns on the same line as the first two significant figures and in the same column as the fourth significant figure, thus:—

log	6·835	=	(8344+3)	=	0·8347
„	0·09148	=	(9609+4)	=	$\bar{2}$·9613

2nd. Use of anti-logarithm tables :—The method of using these is very similar to that of the logarithm tables. The student must remember that only the mantissa of the logarithm is to be used in working with the tables.

The first two figures of the mantissa are to be found in the

first column, and if the fourth figure is a 0, the number is to be found on the same line as the first two, and in the same column as the third figure, thus:—

$0{\cdot}3400 =$ log. $2{\cdot}188$
$3{\cdot}6300 =$ „ 4266
$\bar{1}{\cdot}5400 =$ „ $0{\cdot}3467$
$0{\cdot}4560 =$ „ $2{\cdot}858$
$2{\cdot}7310 =$ „ $538{\cdot}3$
$\bar{2}{\cdot}1730 =$ „ $0{\cdot}01489$

When the fourth figure is other than zero the number is obtained by adding to the number obtained from the first three figures the number in the last set of columns on the same line as the first two and in the same column as the fourth figure of the mantissa, thus:—

$0{\cdot}4567 = (2858+5) =$ log. $2{\cdot}863$
$1{\cdot}9104 = (8128+8) =$ „ $81{\cdot}36$
$\bar{1}{\cdot}6053 = (4027+3) =$ „ $0{\cdot}4030$

Examples 83.

Find the logarithms of the following numbers:—

1. 435
2. 16·7
3. 4062
4. 81·56
5. 0·1407
6. 797·6
7. 8·236
8. 897100
9. 0·09534
10. 427000
11. 63070
12. 0·00068

Given the following logarithms, find, using the tables of antilogarithms, the numbers of which they are the logarithms.

13. $1{\cdot}8470$
14. $0{\cdot}6834$
15. $3{\cdot}3323$
16. $\bar{1}{\cdot}4500$
17. $0{\cdot}7374$
18. $\bar{2}{\cdot}7805$
19. $0{\cdot}0184$
20. $0{\cdot}0807$
21. $4{\cdot}8335$
22. $\bar{3}{\cdot}7062$
23. $8{\cdot}8359$
24. $\bar{4}{\cdot}0068$

291. Care must be taken in dealing with the negative characteristics. It should be remembered that the mantissa is always positive, and additions, &c., must be worked accordingly.

E.g. (i.) $\bar{2}\cdot5678+2\cdot4365=1\cdot0043$

the addition of the mantissae gives 1·0043

that of the characteristics $\bar{2}+2$ gives 0

$\therefore$ the sum is 1·0043

(ii.) $\bar{1}\cdot2394-2\cdot8395=\bar{4}\cdot3999$

the subtraction of the mantissae gives $-1+\cdot3999$

that of the characteristics gives -3

$\therefore\ \bar{1}\cdot2394-2\cdot8395=\bar{4}\cdot3999$

(iii.) $\bar{2}\cdot6384\times3=\bar{5}\cdot9152$

the product of ·6384 by 3 is 1·9152

" " -2 by 3 is -6

$\therefore\ \bar{2}\cdot6384\times3=\bar{6}+1\cdot9152$

$=\bar{5}\cdot9152$

(iv.) $\bar{1}\cdot5472\div4=\bar{1}\cdot8868$

The characteristic must be integral, and as in the above example it is negative, we must not say $1\cdot5\div4$. It is therefore necessary to make the characteristic exactly divisible by 4. This is done by making it $\bar{4}$ and adding the 3 thus subtracted to the mantissa.

We then get $(\bar{4}+3\cdot5472)\div4=\bar{1}\cdot8868$.

292. Multiplication by logarithms:—

Example—(i.) *Multiply* 6384 *by* 39·47.

From the tables we find

mantissa of log. 6384 = ·8051

from (§ 288) characteristic " " = 3

$\therefore$ log. 6384 = 3·8051

again mantissa of log. 39·47 = ·5963

characteristic " " = 1

$\therefore$ log. 39·47 = 1·5963

but log. (6384 × 39·47) = log. 6384 + log. 39·47 (§ 275)

= 3·8051 + 1·5963

= 5·4014

From the anti-logarithm tables mantissa ·4014 gives number 2520

$\therefore$ (§ 288) 6384 × 39·47 = 252000.

This would be written thus—

6384 × 39·47

log. 6384 = 3·8051

log. 39·47 = 1·5963

$\therefore$ log. (6384 × 39·47) = 5·4014

but log. 252000 = 5·4014

$\therefore$ 6384 × 39·47 = 252000.

Example—(ii.) *Multiply* 63·54 *by* 1·341.

log. 63 54 = 1·8031
log. 1 341 = 0·1274
∴ log. (63·54 × 1·341) = 1 9305
but log. 85 21 = 1 9305
∴ 63 54 × 1·8031 = **85·21.**

293. Division by logarithms :—

Examples—(i.) 8539 ÷ 47·34

log. 8539 = 3·9314
„ 47·34 = 1 6753
but log. (8539 ÷ 47·34) = log 8539 - log. 47·34 (§ 276)
∴ log. (8539 ÷ 47·34) = 3·9314 − 1·6753
= 2·2561
and log. 180 3 = 2 2558
∴ 8539 ÷ 47·34 = **180·3.**

(ii.) 1·735 ÷ 36·73

log. 1 735 = 0·2392
„ 36 73 = 1 5651
∴ log. (1735 ÷ 36 73) = $\bar{2}$·6741
but log. 0 04722 = $\bar{2}$ 6741
1 735 − 36 73 = **0·04722.**

294. Finding powers and roots of numbers by logarithms :—

Examples—(i.) *Find the fifth power of* 1·035.

log. $(1·035)^5$ = 5 × log. 1·035 (§ 278)
= 5 × 0 0149
= 0 0745
but log. 1·187 = 0·0745
∴ $(1·035)^5$ = **1·187.**

(ii.) *Find the fourth root of* 1 256.

log. $(1·256)^{\frac{1}{4}}$ = $\frac{1}{4}$ log. 1 256 (§ 278)
= $\frac{1}{4}$ × 0·0990
= 0·0248
but log. 1·059 = 0·0248
∴ the fourth root of 1·256 is **1·059.**

Examples 84.

Find the results in the following examples, correct to 4 significant figures, using the tables on pp. 358–361.

Multiply—

1. 6345 by 23·76.
2. 801 7 by 2·546.
3. 5·638 by 0·037
4. 86 36 by 0·00475.
5. 673 by 38·54.
6. 0·789 by 59·6.
7. 0·037 by 0·789.
8. 6·357 by 0·0734.
9. 2·106 by 3·006.
10. 34·51 by 6·7.
11. 3·73 by 8976.
12. 0·00594 by 0·00747.

Divide—

13. 412·3 by 1·54.
14. 68·95 by 3·24.
15. 341·6 by 88·15.
16. 64·14 by 6·8253.
17. 451 by 16·32.
18. 8·567 by 3·748.
19. 0·815 by 0·567.
20. 4·135 by 0·0512.
21. 0·984 by 1·884.
22. 62·39 by 475·3.
23. 1·518 by 6754.
24. 0·00372 by 8·877.

Examples 85.

Evaluate, correct to 4 significant figures—

1. $(6834)^2$.
2. $(42{\cdot}67)^4$.
3. $(3{\cdot}635)^{24}$.
4. $(1{\cdot}05)^5$.
5. $(1{\cdot}045)^{3{\cdot}5}$.
6. $(1{\cdot}022)^4$.
7. $(1{\cdot}023)^{6{\cdot}25}$.
8. $(33{\cdot}64)^2$.
9. $(0{\cdot}6347)^5$.
10. $(0{\cdot}00468)^7$.
11. $(0{\cdot}0723)^8$.
12. $(0{\cdot}0283)^3$.
13. $(4{\cdot}784)^{\frac{1}{2}}$.
14. $(653{\cdot}6)^{\frac{1}{3}}$.
15. $(3{\cdot}692)^{\frac{1}{5}}$.
16. $(8{\cdot}729)^{\frac{1}{3}}$.
17. $\sqrt{6{\cdot}38}$. $\sqrt{0{\cdot}9}$.
18. $\sqrt[3]{36{\cdot}74}$. $\sqrt[3]{8{\cdot}513}$.
19. $\sqrt[5]{435{\cdot}6}$. $\sqrt[7]{70{\cdot}59}$
20. $(63{\cdot}75)^{\frac{3}{4}}$. $(836{\cdot}9)^{\frac{2}{3}}$
21. $(1{\cdot}045)^6 \times 7328$.
22. $(1{\cdot}035)^{15} \times 6840$.
23. $(1{\cdot}06)^{12} \times (1{\cdot}03) \times 7356$.
24. $7384 = (1{\cdot}075)^{15} \times P$. Find P.
25. $9320 = (1{\cdot}05)^x \times 6000$. Find x.
26. $42560 = (a)^{-5} \times 8346$. Find a.
27. $3 = (a)^{10}$. Find a.

Find value of—

28. $\frac{1}{(1{\cdot}05)^6}$. 29. $\frac{1}{(1{\cdot}035)^{10}}$. 30. $\frac{1}{(1{\cdot}045)^{20}}$.

(For additional examples, work Examples 36, 46, and 47 by logarithms.)

XXXV.—COMPOUND INTEREST.

295. When a sum of money is deposited in the bank for a year, the interest becomes due at the end of that time, and the depositor has the right to withdraw it. Instead of doing so, he may allow the interest to be added to his original principal, so

that during the second year he gets interest on both the original principal and the interest already acquired, and so on, as each new interest is gained it is added to the increasing principal. This adding on of the interest is usually done by the bankers themselves, and the total interest calculated in this way is termed the **Compound Interest** to distinguish it from Simple Interest, where, as we have seen, the interest is allowed to accumulate without itself earning interest.

296. Compound Interest may be calculated by finding the Simple Interest on the Principal for each new year and adding it on to obtain the new principal. This is continued till the last interest has been added. Then, if the original principal is deducted, the remainder is the Compound Interest.

In the following examples, P_1, P_2, P_3, &c., stand for 1st, 2nd, 3rd, &c., Principals, and I_1, I_2, I_3 for the Interests for 1st, 2nd, 3rd, &c., periods. In calculating the interest adopt the method of § 166, and always work in decimals.

Examples—

(i.) *Find the Compound Interest on £1200 for 3 years @ $2\frac{1}{2}$ per cent. per annum.*

$P_1 = £1200$

$I_1 = \frac{2\frac{1}{2}}{100}$ of $P_1 = 30$

$P_2 = £1230$

$I_2 = \frac{2\frac{1}{2}}{100}$ of $P_2 = 30{\cdot}75$

$P_3 = £1260{\cdot}75$

$I_3 = \frac{2\frac{1}{2}}{100}$ of $P_3 = 31{\cdot}51875$

$P_4 = £1292{\cdot}26875$

$P_1 = 1200$

Compound Interest = £92·269 = £92, 5s. 5d.

(ii.) *Find the Compound Interest on £3250 for $1\frac{1}{2}$ years @ $3\frac{3}{4}$ per cent. per annum.*

$P_1 = £3250$

$I_1 = \frac{3\frac{3}{4}}{100}$ of $P_1 =$ 97·5
24·375

$P_2 = £3371·875$

$I_2 = \frac{1\frac{7}{8}}{100}$ of $P_2 =$

33 71875	(1)	$1\frac{7}{8}$
16 85938	($\frac{1}{2}$)	
8 42969	($\frac{1}{4}$)	
4 21485	($\frac{1}{8}$)	

$P_3 = £3435\ 09767$
$P_1 = 3250$

Compound Interest = £185·098
= **£185, 2s.**

(iii.) *Find the amount of £630, 12s. 6d. put out at Compound Interest for 2 years at 4 per cent. per annum, payable half yearly.*

Since interest is payable half yearly, 2 years @ 4% will represent 4 half-years @ 2%

$P_1 = £630·625$

$I_1 = \frac{2}{100}$ of $P_1 =$ 12 6125

$P_2 = £643\ 2375$

$I_2 = \frac{2}{100}$ of $P_2 =$ 12·86475

$P_3 = £656\ 10225$

$I_3 = \frac{2}{100}$ of $P_3 =$ 13 122045

$P_4 = £669·224295$

$I_4 = \frac{2}{100}$ of $P_4 =$ 13·3844859

∴ P_5, *i.e.* Amount = £682·609 = **£682, 12s. 2d.**

297. Second Method of calculating Compound Interest:—

It will be evident to the student that the above method would become very cumbrous and unsuitable if the money is put out for a large number of years, or if the rate of interest is fractional. It is only suitable for cases where the rate per cent. is a simple fraction of 100 and the number of years is small. In this second method we find the amount that £1 would become at the given rate and for the given time, and from that calculate the amount which the given principal would become. The principles underlying the method will be best understood from the explanation of the following examples:—

Examples—

(i.) *Find the Compound Interest on £1200 for 3 years @ $2\frac{1}{2}$%.*

£100 for 1 year @ $2\frac{1}{2}$% would amount to £102·5.

∴ £1 „ 1 „ @ $2\frac{1}{2}$% „ „ £1·025.

i.e. every £1 which remains in the bank for 1 year becomes £1·025 at the end of that time.

Now, every £1 of the above sum becomes £1·025 at the end of the first year, and at the end of the second year each of these £1·025 will become £1·025 × 1·025 = £$(1·025)^2$

i.e. every £1 of the original sum becomes £$(1·025)^2$ at the end of two years.

Similarly each of these £$(1·025)^2$ will become £$(1·025)^2$ × 1·025 = £$(1·025)^3$ at the end of the third year,

or, every £1 of the original sum becomes £$(1·025)^3$ at the end of three years, and so on for each year.

It is therefore evident that to find what £1 will amount to for a given number of years at a given rate per cent. per annum compound interest, we require to raise the amount obtained in one year to the power represented by the number of years (§ 271).

Applying this to the above example we have—

1·025 1·025 ——— 1·025 2050 5125 ——— 1·050625 1·025 ——— 1·05062 | 5 2101 | 2 525 | 3 ——— 1·07689 1200 ——— 1292·26800	£1 amounts to £1·025 at the end of one year. ∴ £1 „ £$(1·025)^3$ at the end of three years. or £1 „ £1·07689 „ „ „ but if £1 amounts to £1·07689 in three years then £1200 amounts to £1·07689 × 1200 in three years *i.e.* £1200 „ £1292·268 „ „ and Interest = £92·268 = **£92, 5s. 4d.** 20 ——— 5·360 12 ——— 4·32

Examples—

(ii.) *Find the amount of £630, 12s. 6d. put out at Compound Interest for 2 years at 4 per cent. per annum, interest payable half-yearly, i.e. 4 half-years at 2 per cent.*

Amount of £1 for four half-years is $£(1{\cdot}02)^4$

i.e. „ £1 „ „ £1·08243

and „ £630·625 „ „ £1·08243 × 630·625

= £682·607 = £682, 12s. 2d.

```
1·02
1·02
----
1·02
 204
------
1·0404
1·0404
------
1·0404 | 0
  4161 | 6
    41 | 6
---------
 1·08243 |
630·625
---------
 6·49458 | 0
  32471  | 9
    649  | 5
     21  | 6
      5  | 4
---------
682·607  |
```

```
    20
 12·140
    12
 ------
  1·68
```

Note—Five places of decimals will usually be found sufficient in the multiplications and three places in the last result.

If now we represent the final amount by A, and let a be the amount of £1 for one *period*, then with P for Principal and n for the number of periods we get the formula—

$$A = (a)^n \times P.$$

Example (i.) could then be set down thus—

$$A = (a)^n \times P$$
$$= (1{\cdot}025)^3 \times 1200$$
$$= 1{\cdot}07689 \times 1200$$
$$= 1292{\cdot}268$$
$$P = 1200$$

Compound Interest = £92, 5s. 4d.

(ii.) could then be set down—

$$A = (a)^n \times P$$
$$= (1{\cdot}02)^4 \times 630{\cdot}625$$
$$= 1{\cdot}08243 \times 630{\cdot}625$$
$$= £682{\cdot}607$$
$$= £682,\ 12s.\ 2d.$$

Note.—Five places decimals will usually be found sufficient in finding $(a)^n$ and three places in final multiplication.

(iii.) *Find the Compound Interest on £2500 for $4\frac{1}{2}$ years @ $3\frac{1}{2}$ per cent. per annum.*

Let a_1 be the amount of £1 for $\frac{1}{2}$ year

$$\begin{aligned} \text{then } A &= (a)^n \times a_1 \times P \\ &= (1{\cdot}035)^4 \times 1{\cdot}0175 \times 2500 \\ &= 1{\cdot}14730 \times 1{\cdot}0175 \times 2500 \\ &= 2918{\cdot}445 \\ P &= 2500 \end{aligned}$$

Compound Interest = **£418, 8s. 11d.**

(iv.) *Find what sum will amount to £1292·268 in 3 years @ $2\frac{1}{2}$ per cent. Compound Interest.*

$$A = (a)^n \times P$$

$$\frac{A}{(a)^n} = P$$

$$\frac{1292{\cdot}268}{(1{\cdot}025)^3} = P$$

∴ (see Ex. i.) 1200 = P.

(v.) *What sum will gain £418, 8s. 11d. in $4\frac{1}{2}$ years @ $3\frac{1}{2}$ per cent. Compound Interest?*

Here it is best to find the Compound Interest on £1 only.

$$\begin{aligned} A &= a^n \times a_1 \times P \\ &= (1{\cdot}035)^4 \times 1{\cdot}0175 \times 1 \\ &= 1{\cdot}16738 \\ P &= 1 \end{aligned}$$

Interest on £1 = ·16738

" ? = 418·445

$$\text{Sum} = \frac{418{\cdot}445}{0{\cdot}16738} = \text{£}2500. \quad \text{(See Ex. iii.)}$$

(vi.) *What rate per cent. per annum does a person really get who obtains 4 per cent. per annum payable quarterly?*

$$\begin{aligned} \text{Taking } P = \text{£}1 \text{ we get } A &= (a)^n \times P \\ &= (1{\cdot}01)^4 \times 1 \\ &= 1{\cdot}04060401 \\ P &= 1 \end{aligned}$$

∴ Interest on £1 = ·04060401

∴ **Rate = 4·060401.**

Note—Always take a as the amount of £1 for one *period*; in this case, *a quarter*. 4 per cent. is the **Nominal Rate**, 4·060401 is the **Effective Rate.**

298. Tables of Compound Interest are drawn up, giving the amounts of £1, *i e a*, at various rates of interest, for 1, 2, 3, &c, years. As we have already seen, the amount of any other principal can be calculated from that of £1, at the rate per cent. and for the given time, by multiplying by the principal. These tables are calculated by logarithms, and are usually given correct to seven or ten significant figures. We give example of a table at 4 per cent per annum for 1–10 years, correct to seven significant figures.

Yrs	4 per Cent	Yrs	4 per Cent
1	1 040000	6	1 265318
2	1 081600	7	1 315930
3	1 124864	8	1 368568
4	1 169858	9	1 423311
5	1 216652	10	1 480243

(Similar tables will be found on p. 254, column 2.)

Examples 86.

1. Find the compound interest on £684 for 2 years at 3 per cent. per annum.
2. Find the compound interest on £789, 10*s.* for three years at 4 per cent. per annum.
3. Find the amount of £2650 for $3\frac{1}{2}$ years at 5 per cent per annum, compound interest.
4. Find the amount of £3874, 12*s.* 6*d.* for $2\frac{1}{4}$ years, at 4 per cent. per annum, compound interest.
5. Find the compound interest on £5000 for 3 years at $2\frac{1}{2}$ per cent. per annum.
6. Find the compound interest on £839, 16*s.* for 4 years at $2\frac{3}{4}$ per cent. per annum.
7. Find, to the nearest penny, the compound interest on £745, 8*s.* 3*d.* for 1 year 9 months at $3\frac{1}{2}$ per cent. per annum.
8. Find the amount of £850 for 4 years at 6 per cent. per annum, compound interest.
9. Find the amount of £639 for 6 years at 5 per cent. per annum, compound interest.

10. Find the compound interest on £300 for 4 years at 3 per cent. per annum, payable half-yearly.

11. What is the amount of £200 for 3 years at $2\frac{1}{2}$ per cent. per annum, compound interest, payable half-yearly?

12. What sum, at compound interest, will amount to £413, 8*s.* 9*d.* in 2 years at 5 per cent. per annum?

13. What sum of money will amount to £735 in 2 years at 5 per cent. per annum, compound interest?

14. Find the capital whose compound interest for 3 years at 4 per cent. is £406, 15*s.* 4*d.*

15. Find the difference between the simple and compound interests on £1680 for 3 years at 4 per cent. per annum.

16. Find the difference between the simple and compound interests on £275 for 3 years at 5 per cent. per annum.

17. Find the difference between the simple and compound interests on £415, 10*s.* for 3 years at $3\frac{1}{2}$ per cent. per annum.

18. Find the difference between the simple and compound interests on £425 for 4 years at 4 per cent. per annum.

19. The interest on a certain sum for 1 year is £10. The compound interest on the same sum at the same rate for 2 years is £21, 5*s.* What is the sum?

20. Find, to the nearest penny, without unnecessary calculations, the amount at the end of two years of £1564 invested at 6 per cent. per annum, compound interest, payable half-yearly.

21. Find, to the nearest penny, the compound interest on £8425, 10*s.* for 5 years at 3 per cent. per annum.

22. What principal will produce £94, 11*s.* 6*d.* in 3 years at 5 per cent. per annum, compound interest?

23. Find the difference between the simple and compound interests on £1250 for 4 years at 5 per cent. per annum.

24. On what sum does a person gain £61 by lending it at compound instead of simple interest for 3 years at 5 per cent.?

25. Find the amount of £500 at 4 per cent. per annum, compound interest, for 2 years, interest being due half-yearly.

26. Find the compound interest on £7250 for $2\frac{1}{2}$ years at 6 per cent. per annum.

27. What is the compound interest on £600 at 5 per cent. per annum for $1\frac{1}{2}$ years when the interest is calculated half-yearly?

28. What rate of interest per annum does a person get who is paid at the rate of 5 per cent. compound interest payable half-yearly?

29. What rate of interest per annum does a person get who is paid at the rate of 3 per cent. compound interest payable quarterly?

30. What sum put out for $1\frac{1}{2}$ years at 4 per cent. compound interest, payable half-yearly, will amount to £6632, 11*s.*?

31. How many prizes, each worth £15, 15*s.* 3*d.*, could be paid from the interest of £800 for 3 years at 5 per cent. per annum, compound interest?

32. Find the difference between the amount of £1800 for 3 years at 4 per cent. per annum, compound interest, and the present value of the same sum, due 3 years hence, at 4 per cent. simple interest.

33. Find the true discount on £100 due two years hence at 4 per cent. per annum, compound interest.

34. A person, aged 18 is left a legacy of £30,000, payable on his attaining his majority. What is the true value of the bequest at the present time, compound interest being calculated at the rate of 3 per cent. per annum?

35. Find the present value of a legacy of £20,000, due 3 years hence, compound interest being calculated at the rate of 4 per cent. per annum.

36. A owes B £500, due in 3 years. B offers him a discount of 4 per cent. for cash, but A prefers to invest the £500 in a concern paying 4 per cent. per annum, compound interest, and to meet the bill when due. B gets the bill discounted at once at the bank at 4 per cent. and invests in the same concern as A. What does A gain more than B?

37. Two equal sums are deposited at the same time, the one at 5 per cent. per annum, compound interest, and the other at 6 per cent. per annum, simple interest. At the end of how many years will the accumulated amount of the first exceed that of the second?

38. What is the least exact number of years in which the compound interest at 3 per cent. per annum will exceed the simple interest at 4 per cent. per annum on the same sum?

39. On what sum does the difference between the simple and compound interests for 2 years at 3 per cent. per annum, amount to 25*s.*?

40. The difference between the simple and the compound interest on a certain sum, at 5 per cent. for 3 years, is £6, 2*s.* What is the sum?

299. Application of Logarithms :—

We have already seen (§ 294) that logarithms supply us with an easy method of finding the power of a number, and on this account they are especially useful in working questions in Compound Interest. Logarithms also provide a means of finding amounts for years and fractions of years, which are troublesome to work out by the second method. Logarithms also enable us to work out many reverse questions, as Examples ii., iii., iv., which are not practicable by ordinary arithmetic. The following examples are worked with the aid of four-figure logarithms (see pages 358–361). With these logarithms only three figures can be considered as correct, the fourth figure is doubtful; when seven figures are used, six figures may be taken as correct, the seventh is only approximate

Examples—

(i.) *Find the Compound Interest on £235 for $4\frac{1}{2}$ years at $2\frac{3}{4}$ per cent. per annum.*

$$A = a^n \times a_1 \times P$$
$$\log. A = n \log. a + \log. a_1 + \log. P$$
$$= 4 \log. 1{\cdot}0275 + \log. 1{\cdot}01375 + \log. 235$$

$$4 \log. 1{\cdot}0275 = 0{\cdot}0468$$
$$\log. 1{\cdot}01375 = 0{\cdot}0059$$
$$\log. 235 = 2{\cdot}3711$$

$$\therefore \log. A = 2{\cdot}4238$$
$$A = £265{\cdot}4$$
$$P = £235$$
$$I = £30,\ 8s.$$

(ii.) *A person places £150 in a bank for five years, and at the end of that time receives £178, 1s. The interest was added to the principal at the end of each year, but the rate was not constant. What uniform rate would have given the same amount?*

$$A = (a)^n \times P$$
$$\log. A = n \log. a + \log. P$$
$$5 \log. a = \log. A - \log. P$$
$$= \log. 178{\cdot}05 - \log. 150.$$

$$\log. 178{\cdot}05 = 2{\cdot}2505$$
$$\log. 150 = 2{\cdot}1761$$

$$5 \log. a = 0{\cdot}0744$$
$$\log. a = 0{\cdot}0149$$
$$a = 1{\cdot}035$$

$\therefore$ the rate per cent. per annum is $3\frac{1}{2}$.

(iii.) *At the birth of a son a man deposits £300 in a bank with the provision that the money is to be handed over on the son's twenty first birthday. If the rate of interest during the twenty one years is $2\frac{1}{2}$ per cent per annum what sum will the son receive?*

$$A=(a^n)\times P$$
$$\log A=n\log a+\log P$$
$$=21\log 1\ 025+\log 300.$$

$$21\log 1\ 025=21\times 0\ 0107$$
$$=0\ 2247$$
$$\log 300=2\ 4771$$

$$\log A=2\ 7018$$
$$A=503\ 2$$
$$=£503,\ 4s.$$

(iv.) *In what time will a sum of money double itself at the rate of 2 per cent per annum, Compound Interest?*

This requires us to find in what time £1 amounts to £2

$$A=(a)^n\times P$$
$$\log A=n\log a+\log P$$
$$\log 2=n\log 1\ 02+\log 1$$
$$n\log 1\ 02=\log 2-\log 1$$
$$n=\frac{\log 2-\log 1}{\log 1\ 02}$$
$$=\frac{0\ 3010}{0\ 0086}$$
$$=35\text{ years.}$$

Examples 87.

1. Find, to the nearest penny, the compound interest on £5938, 10s. for 4 years at $2\frac{1}{2}$ per cent. per annum, interest payable half-yearly.

2. Find, without unnecessary calculation, the amount, to the nearest penny, at the end of 2 years of £1248 invested at 4 per cent. per annum, compound interest, payable half-yearly.

3. Find the amount of £600 for 8 years at 4 per cent. per annum, compound interest.

4. Find the difference between the simple and compound interest on £850 for 10 years at 3 per cent. per annum.

5. Find the compound interest on £475 for $9\frac{3}{4}$ years at $2\frac{1}{2}$ per cent. per annum.

6. Find the difference between the simple and compound interest on £1000 for 12 years at 5 per cent. per annum.

7. What is the least number of years a sum lent at 15 per cent. per annum, compound interest, would more than double itself?

8. In how many years will £115 at 4 per cent. per annum, compound interest, just exceed £120 at 3 per cent. per annum, compound interest, and by how much will it exceed it at the end of the last year?

9. A legacy remained unclaimed for 4 years, and at the end of that time it amounted to £14,586, 1s. 6d., compound interest being reckoned at 5 per cent. per annum. What was the amount of the legacy?

10. Find the difference between the compound interest on £840 for 3 years at 5 per cent. and for 4 years at 4 per cent. (payable half-yearly in each case), in decimals of £1.

11. At what rate per cent. per annum, compound interest, will £132,651 amount to £148,877 in 3 years?

12. What principal will produce £40, 16s. as compound interest in 2 years at 4 per cent. per annum?

13. What sum will gain in 3 years at 4 per cent. per annum, simple interest, as much as £400 gains in 2 years at 5 per cent. per annum, compound interest?

14. What is the least exact number of years for which a sum of money must remain at 10 per cent. per annum, compound interest, in order that the resulting amount may be more than double the original sum? Find, to the nearest penny, the amount to which £1000 would increase in that number of years.

15. At what rate per cent. per annum, compound interest, will £2000 amount to £2163, 4s. in 2 years?

16. In what time will £230, 15s. amount to £267, 2s. 5d., at 5 per cent. per annum, compound interest?

17. In what time will £1760, 10s. amount to £1942, 4s. $9\frac{1}{2}d.$, at 4 per cent. per annum, compound interest?

18. In what time will the interest on £6500 amount to £655, 19s. 11d., at 3 per cent. per annum, compound interest?

19. In what time will £1000 amount to £1169, 17s. 2d., at 8 per cent. per annum, compound interest, payable half-yearly?

20. In what time will £1 amount to £10 at 3 per cent. per annum, compound interest?

21. At what rate per cent. per annum, compound interest, will £270 amount to £286, 8s. 10d. in 2 years?

22. At what rate per cent. per annum, compound interest, will £2080 amount to £2339, 14s. 4d. in 3 years?

23. At what rate per cent. per annum, compound interest, will £960 amount to £1200 in 13 years?

24. If a sum of money trebles itself in 32 years, what is the rate of compound interest per annum?

XXXVI.—ANNUITIES.

300. An **Annuity** is a certain sum of money which a person receives every year. The sum may be paid in one or more instalments per year.

301. Annuities are divided into two classes:—**Annuities certain and life annuities.** Annuities certain are those in which the payment is to be made for a definite period or in perpetuity. The latter are called **perpetual annuities.** These annuities do not depend upon any contingent event as in the case of life annuities, which are usually payable till the death of the person.

302. Questions referring to **perpetual annuities** are the easiest to deal with, as the yearly payment is simply the interest on the money invested. This class of annuities comes under the rules of investments, interest, &c.

303. In dealing with **Annuities** the formulæ used in connection with Simple Interest (p. 178) and Compound Interest (p. 239) are employed together with the following:—

(a) $S_{\overline{n}|}$ = amount of an annuity of £1 for n years.

The amount of an annuity is the actual value of all the payments if they were left unpaid till the last payment is due, together with interest on all the different payments from the time each becomes due till the date of the final payment.

Take an annuity of £1 for n years and, calculating back from the last payment (allowing Simple Interest), we find—

The last £1 paid gets no interest, so it amounts to £1.

The 2nd last £1 paid gets 1 year's interest, so it amounts to $1 + i$. (See page 180.)

The 3rd last £1 paid gets 2 years' interest, so it amounts to $1 + 2i$.

And so on to the 1st £1 paid, which gets $(n - 1)$ years' interest, so it amounts to $1 + (n - 1)i$.

The sum of all these $S_{\overline{n}|} = 1 + (1 + i) + (1 + 2i) + \ldots + \{1 + (n - 1)i\}$.

This is an Arithmetical Progression. (See any standard Algebra.)

Hence Formula I.—$S_{\overline{n}|}$ (at Simple Interest) $= \frac{n}{2}\{2 + (n - 1)i\}$.

(The amount for £P is obtained by multiplying the answer by £P).

When Compound Interest is allowed, calculating from the beginning of the payment of the annuity, we find—

At end of 1st year the 1st £1 paid will amount to £1 (just due—no interest).
At end of 2nd year the same £1 will amount to a (1 year's interest).
At end of 3rd year the same £1 will amount to a^2 (see Compound Interest, § 297).
At end of nth year the same £1 will amount to $a^{(n-1)}$

Similar reasoning will show that—

At end of nth year the 2nd £1 will have amounted to a^{n-2}.
At end of nth year the 3rd £1 will have amounted to a^{n-3}.
At end of nth year the 2nd last £1 will have amounted to a.
At end of nth year the last £1 will have amounted to 1 (just due—no interest).

The sum of these amounts $S_{\overline{n}|} = 1 + a + a^2 + a^3 + \ldots\ldots + a^{n-1}$
This is a Geometrical Progression.

Hence **Formula II.**—$S_{\overline{n}|}$ **(at Compound Interest)**

$$= \frac{1(a^n - 1)}{a - 1} \text{ or } \frac{(1+i)^n - 1}{(1+i) - 1} = \frac{(1+i)^n - 1}{i}, \text{ or } \frac{a^n - 1}{i}.$$

(The amount for £P is obtained by multiplying the answer by £P.)

Examples—

(i.) *Find the amount of an annuity of £30 a year if left unpaid for ten years, allowing Simple Interest at 4 per cent. per annum.*

Using Formula I.—$n = 10$ and $i = \cdot 04$. P = £30

$$S_{\overline{n}|} = \frac{n}{2}\{2 + (n-1)i\}$$
$$= 5(2 + 9 \times \cdot 04)$$
$$= 11 \cdot 8$$

$S_{\overline{n}|}$ for £P $= 11 \cdot 8 \times P = 11 \cdot 8 \times$ £30

= **£354.**

(ii.) *Find the yearly value of an annuity which would amount to £1128, if left unpaid for eight years at 5 per cent. Simple Interest.*

$$S_{\overline{n}|} \text{ for } £P = 1128,\ n = 8,\ i = \cdot 05.$$

$$S_{\overline{n}|} \text{ for } £P = \frac{n}{2}\{2 + (n-1)i\} \times P$$

$$1128 = 4(2 + 7 \times \cdot 05) \times P$$

$$1128 = 9 \cdot 4 \times P$$

$$\textbf{£120} = \textbf{P}.$$

(iii.) *Find at Compound Interest the amount of an annuity of £25 a year for five years at $3\frac{1}{2}$ per cent. per annum.*

Using Formula II.—$n = 5$, $a = 1 \cdot 035$, $i = \cdot 035$

$$\therefore S_{\overline{n}|} = \frac{a^n - 1}{i} = \frac{(1 \cdot 035)^5 - 1}{\cdot 035} = \frac{1 \cdot 18768 - 1}{\cdot 035}$$

$$= 5 \cdot 3623$$

$$S_{\overline{n}|} \text{ for } £P = 5 \cdot 3623 \times 25$$

$$= \textbf{£134, 1}s.\ \textbf{2}d.$$

(iv.) *Find the yearly value of an annuity which at 5 per cent. Compound Interest would, at the end of eight years, amount to £248, 5s. 6d.*

$$S_{\overline{n}|} \text{ for } £P = \frac{a^n - 1}{i} \times P$$

$$248 \cdot 275 = \frac{(1 \cdot 05)^8 - 1}{\cdot 05} \times P$$

$$= \frac{1 \cdot 47745544 - 1}{\cdot 05} \times P$$

$$\therefore \frac{248 \cdot 275 \times \cdot 05}{\cdot 47745544} = P$$

$$\therefore P = \textbf{£26}.$$

(v.) *In how many years will an annuity of £100 a year, if left unpaid, amount to £954, 18s. 2d., allowing Compound Interest at 5 per cent. per annum?*

$$S_{\overline{n}|} \text{ of } P = \frac{a^n - 1}{i} \times P$$

$$954 \cdot 908 = \frac{(1 \cdot 05)^n - 1}{\cdot 05} \times 100$$

$$\therefore \frac{954 \cdot 908}{100} \times \cdot 05 + 1 = (1 \cdot 05)^n$$

$$1 \cdot 477454 = (1 \cdot 05)^n$$

$\therefore$ **$n = 8$. (Calculate by logarithms; or see table, page 254.)**

(*b*) $a_{\overline{n}|}$ = **present value of an annuity of £1 per annum that has to be paid for *n* years.**

The present value of an annuity is the sum of money that would require to be set aside when the annuity is entered upon so as to pay the annuities as each became due.

The present value of £1 due in *n* years is represented by v_n.

Taking the formula, p 239,

$A = a^n \times P$, if $A = 1$ and $n = 1$, then P is the present value of £1 in 1 year, *i e.* v_1, becomes $v = \frac{1}{a}$. Similarly if $n = 2$, then $v_2 = \frac{1}{a^2}$ and so on, and since $\frac{1}{a^2}$ is the square of $\frac{1}{a}$, v_2 becomes v_1^2 and $v_n = v^n$.

Hence—

v_1 present value at beginning of 1st year of 1st £1 (paid at end of 1st year) $= \frac{1}{a} = v$

v_2 present value at beginning of 1st year of 2nd £1 (paid at end of 2nd year) $= \frac{1}{a^2} = v^2$

v_3 present value at beginning of 1st year of 3rd £1 (paid at end of 3rd year) $= \frac{1}{a^3} = v^3$

v_{n-1} present value at beginning of 1st year of 2nd last £1 (paid at end of $(n-1)$th year) $= \frac{1}{a^{n-1}} = v^{n-1}$.

v_n present value at beginning of 1st year of last £1 (paid at end of *n*th year) $= \frac{1}{a^n} = v^n$.

The sum of these $a_{\overline{n}|} = v + v^2 + v^3 + \ldots + v^{n-1} + v^n$.

This is a Geometrical Progression.

$$a_{\overline{n}|} = \frac{v(1 - v^n)}{1 - v} = \frac{\frac{1}{a}\left(1 - \frac{1}{a^n}\right)}{1 - \frac{1}{a}} = \frac{1 - \frac{1}{a^n}}{a - 1} = \frac{1 - \frac{1}{a^n}}{i} = \frac{1 - v^n}{i}$$

Formula III $a_{\overline{n}|} = \frac{1 - v^n}{i}$, or $\frac{1 - \frac{1}{a^n}}{i}$

and $a_{\overline{n}|}$ of £P (usually written $V_{\overline{n}|}$) $= \frac{1 - v^n}{i} \times P$.

(vi.) *Find the present value of an annuity of £150 a year for 50 years at 5 per cent. Compound Interest, taking* $\left(\frac{1}{1{\cdot}05}\right)^{50} = {\cdot}08720372$.

Using Formula III.—$V_{\overline{n}|} = \frac{1 - v^n}{i} \times P$

$$= \frac{1 - \left(\frac{1}{1{\cdot}05}\right)^{50}}{{\cdot}05} \times 150$$

$$= \frac{1 - {\cdot}08720372}{{\cdot}05} \times 150$$

$$= £2738,\ 7s.\ 9d.$$

Where the value of v^n is not given, find it from $\frac{1}{a_n}$ from log. tables (p. 358–361).

(c) $\text{a}_{\overline{n}|}$ **= present value of an annuity due** of £1 for n years

An **annuity due** is an annuity the payments of which are made at the beginning instead of the end of each period. In this case—

Since—

1st £1 is paid immediately annuity is entered upon, its present value = 1.
2nd £1 is paid one year later, its present value = v.
3rd £1 is paid two years later, its present value = v^2.
nth £1 is paid $(n-1)$ years later, its present value = v^{n-1}.

The sum of these values $\text{a}_{\overline{n}|} = 1 + v + v^2 + \ldots + v^{n-1}$,
and since $v + v^2 + v^3 + \ldots + v^{n-1} = a_{\overline{n-1}|}$ (see Formula III.), we get

Formula IV. $\text{a}_{\overline{n}|} = 1 + a_{\overline{n-1}|}$.

In finding the sum of such an annuity due, it has to be noted that although the last payment is made at the ***beginning*** **of the last year the sum is calculated to the** ***end*** **of the last year, and so** ***includes the interest for the last year;*** **and since each payment becomes due at the** ***beginning*** **instead of at the** ***end*** **of each year, each sum will, if unpaid, earn one year's extra interest; hence, if an annuity of £1 were paid at the end of the last year also,** $S_{\overline{n}|}$ **of an annuity due would really be the same as** $S_{\overline{n+1}|}$ **of an ordinary annuity, but since no last payment of £1 is made, we must deduct £1 from** $S_{\overline{n+1}|}$ **of ordinary annuity to obtain** $S_{\overline{n}|}$ **of annuity due.**

Hence Formula V.—$S_{\overline{n}|}$ **of annuity due =** $S_{\overline{n+1}|} - 1$.

This may be shown in another way.

$S_{\overline{n+1}|}$ of an ordinary annuity $= 1 + a + a^2 + a^3 + \ldots + a^n$.

$S_{\overline{n}|}$ of annuity due $= \quad a + a^2 + a^3 + \ldots + a^n$.

Subtracting, we get $S_{\overline{n+1}|}$ of ordinary $- S_{\overline{n}|}$ of annuity due $= 1$,

i.e. $S_{\overline{n}|}$ of annuity due $= S_{\overline{n+1}|} - 1$.

(vii.) *Treat Example* (vi.) *as annuity due.*

$$\text{Calculating } a_{\overline{n-1}|} = \frac{1 - v^{n-1}}{i}$$

$$= \frac{1 - \left(\frac{1}{1 \cdot 05}\right)^{49}}{\cdot 05}$$

$$= 18 \cdot 168718$$

$$\mathbf{a}_{\overline{n}|} = 1 + a_{\overline{n-1}|} = 19 \cdot 168718$$

$$\mathbf{a}_{\overline{n}|} \text{ of } £150 = 19 \cdot 168718 \times 150$$

$$= £2875,\ 6s$$

(viii.) *Find the sum of an annuity due of £25 a year for 4 years at* $3\frac{1}{2}$ *per cent. per annum.*

$$S_{\overline{n+1}|} = S_{\overline{5}|} = 5 \cdot 3623. \quad \text{(See Example iii.)}$$

$$S_{\overline{n}|} \text{ of annuity due} = S_{\overline{n+1}|} - 1 = 4 \cdot 3623$$

$$S_{\overline{n}|} \text{ of annuity due of } £25 = 4 \cdot 3623 \times 25$$

$$= £109,\ 1s.\ 2d.$$

(*d*) $_{t|}a_{\overline{n}|}$ = **present value of a deferred annuity of £1,** *i.e.* an annuity the payments of which would begin after t years and then continue for n years. For example, if an annuity were deferred for six years no payments would be made during those years, the first payment being due at the *end of the seventh* year.

The value of a deferred annuity at the end of the t deferred years, *i.e.* at the beginning of the n years it has to run, is $a_{\overline{n}|}$ (Formula III.), and the present value of this $a_{\overline{n}|}$ at the beginning of the t years will give the real present value of the deferred annuity. Now the present value of £1 due in t years is v^t (see under (5)). Therefore the present value of $a_{\overline{n}|}$ at the beginning of the t years equals $a_{\overline{n}|} \times v^t$.

Formula VI. Hence $_{t|}a_{\overline{n}|} = a_{\overline{n}|} v^t$.

This may be shown in another way.

Since the annuity is deferred t years—

The 1st payment is made at end of $(t+1)$ years, so its present value $= v^{t+1}$.

The 2nd payment is made at end of $(t+2)$ years, so its present value $= v^{t+2}$.

The nth payment is made at end of $(t+n)$ years, so its present value $= v^{t+n}$.

The sum of these $= v^{t+1} + v^{t+2} + \ldots v^{t+n} = v^t(v + v^2 + v^3 + \ldots + v^n)$

$= v^t \times a_{\overline{n}|}$ (Formula III.),

or the sum $= (v + v^2 + v^3 + \ldots v^{t+n}) - (v + v^2 + v^3 + \ldots v^t)$

$= a_{\overline{t+n}|} - a_{\overline{t}|}$ (a form which is sometimes used).

(ix.) *Find the present value of an annuity of £60 a year to run 4 years and deferred 5 years at 5 per cent.*

Assume $a_{\overline{4}|} = 3\cdot54595$ and $(1\cdot05)^5 = 1\cdot276282$.

Hence $t/a_{\overline{n}|} = a_{\overline{n}|}v^t = a_{\overline{4}|} \times \dfrac{1}{(1\cdot05)^5}$

$= 3\cdot54595 \times \dfrac{1}{1\cdot276282}$

$= 3\cdot54595 \times 0\cdot783526$

$t/a_{\overline{n}|}$ of £60 $= 3\cdot54595 \times 0\cdot783526 \times$ £60

$=$ **£166, 14s.**

Note.—In all the above and similar examples, should the interest be compounded otherwise than yearly, let a stand for the amount of £1 for the given *period* and n for the number of such periods (see page 240).

304. In connection with annuities certain tables are drawn up giving the values related to £1 at different rates of interest. We give an extract from one of these tables at 5 per cent. per annum.

The different columns give—

(1) Number of years.

(2) Amount of £1 in the number of years at given rate of interest.

(3) Present value of £1 in the number of years at given rate of interest.

(4) Amount of an annuity of £1 in the number of years at given rate of interest.

(5) Present value of an annuity of £1 in the number of years at given rate of interest.

(6) The annuity which £1 will purchase for the number of years at given rate of interest.

(7) The annuity which would amount to £1 in the number of years at given rate of interest.

TABLE 1.

Table of Amounts, &c., at 5 per cent. Interest.

1 Yrs	2 (A of £1 = a)	3 (P V of £1)	4 (Amount of Annuity of £1)	5 (P V. of Annuity of £1)	6 (Annuity produced by £1)	7 (Annuity to Amount to £1)
1	1·05000000	·95238095	1·00000000	·95238095	1·05000000	1·0000000
2	1·10250000	·90702948	2·05000000	1·85941043	·53780488	·4878049
3	1·15762500	·86383760	3·15250000	2·72324803	·36720856	·3172085
4	1·21550625	·82270247	4·31012500	3·54595050	·28201183	·2320118
5	1·27628156	·78352616	5·52563125	4·32947667	·23097480	·1809748
6	1·34009564	·74621540	6·80191281	5·07569207	·19701747	·1470175
7	1·40710042	·71068133	8·14200845	5·78637340	·17281982	·1228198
8	1·47745544	·67683936	9·54910888	6·46321276	·15472181	·1047218
9	1·55132822	·64460892	11·02656432	7·10782167	·14069008	·0906901
10	1·62889463	·61391325	12·57789254	7·72173493	·12950458	·0795050

305. Tables of this kind are easily calculated by applying the principle of Compound Interest. We give a simple method of computing the remaining columns from 1 and 2. The figures in column 2 are the amounts of £1 for the number of years in column 1. The figures in column 3 are the reciprocals of those in column 2. The figures in column 4 are obtained by the successive additions of those in column 2 to the first figures of column 4, *i e* 1·00000000. The figures in column 5 are the sums of the 1st, 1st + 2nd, 1st + 2nd + 3rd, &c, of those in column 3; those in column 6 are reciprocals of those in 5, and those in column 7 are reciprocals of those in 4.

The following table at $3\frac{1}{2}$ per cent. has been calculated from these rules :—

TABLE 2.

Table of Amounts, &c, at $3\frac{1}{2}$ per cent. Interest.

1	2	3	4	5	6	7
1	1·03500	0·966184	1·00000	0·966184	1·035000	1·000000
2	1·07122	0·933512	2·03500	1·899696	0·526400	[illegible]
3	1·10872	0·901945	3·10622	2·801641	0·340747	[illegible]
4	1·14752	0·871446	4·21494	3·673037	[illegible]	[illegible]
5	1·18768	0·841976	5·36224	4·515063	0·221480	[illegible]

The following is the detailed working of the fifth line of figures :—

Column 2. $(1{\cdot}035)^5$ (§ 294)

$$\log. (1{\cdot}035)^5 = 5 \log. 1{\cdot}035$$
$$= 5 \times 0{\cdot}014940$$
$$= 0{\cdot}074700$$
$$\log. 1{\cdot}18768 = 0{\cdot}074700$$
$$\therefore (1{\cdot}035)^5 = 1{\cdot}18768$$

Column 3. $= \frac{1}{1{\cdot}18768}$

$$\log. \frac{1}{1{\cdot}18768} = \log. 1 - \log. 1{\cdot}18768$$
$$= 0 - 0{\cdot}074700$$
$$= \bar{1}{\cdot}925300$$
$$\log. 0{\cdot}841976 = \bar{1}{\cdot}925300$$
$$\therefore \frac{1}{1{\cdot}18768} = 0{\cdot}841976$$

Column 4. $= (1 + 1{\cdot}03500 + 1{\cdot}07122 + 1{\cdot}10872 + 1{\cdot}14730)$
$= 5{\cdot}36224$

Column 5. $= (0{\cdot}966184 + 0{\cdot}933512 + 0{\cdot}901945 + 0{\cdot}871446 + 0{\cdot}841976)$
$= 4{\cdot}515063$

Column 6. $= \frac{1}{4{\cdot}515063}$

$$\log. \frac{1}{4{\cdot}515063} = \log. 1 - \log. 4{\cdot}515063$$
$$= 0 - 0{\cdot}654664$$
$$= \bar{1}{\cdot}345336$$
$$\log. 0{\cdot}221480 = \bar{1}{\cdot}345336$$
$$\therefore \frac{1}{4{\cdot}515063} = 0{\cdot}221480$$

Column 7. $= \frac{1}{5{\cdot}36224}$

$$\log. \frac{1}{5{\cdot}36224} = \log. 1 - \log. 5{\cdot}36224$$
$$= 0 - 0{\cdot}729346$$
$$= \bar{1}{\cdot}270654$$
$$\log. 0{\cdot}186489 = \bar{1}{\cdot}270654$$
$$\therefore \frac{1}{5{\cdot}36224} = 0{\cdot}186489$$

306. In the case of life annuities, the most probable number of years for which the annuity is likely to be paid is calculated, and the annuity becomes an *annuity certain* for that period. The selection of the number of years is made from statistical papers—

Mortality Tables—in which the probability of length of life of the individual is given, and corrections made according to the personal environment of the individual.

307. The most common form of questions are included in the following examples. The student, as in Compound Interest, should make use of the logarithm tables on pp. 358–361.

Examples—

(i.) *What annuity will £1200 buy if the annuity is to be paid for seven years and the rate of interest is 5 per cent. per annum?*

From Table 1, column 6, we get £1 will buy an annuity of £0·17281982, payable for seven years.

∴ £1200 will buy 0·17281982 × 1200
= £207·38378406
= **£207, 7s. 8d.**

(ii.) *What is the present value of an annuity of £40, payable for 5 years, interest at the rate of 3½ per cent. per annum?*

From Table 2, column 5

The present value of an annuity of £1 for 5 years is £4·515063
∴ „ „ „ £40 „ 5 „ £4·515063 × 40
= 180·602520
= **£180, 12s.**

(iii.) *What sum will buy an annuity of £120 for 8 years, interest being at the rate of 5 per cent. per annum?*

From Table 1, column 5.

The present value of an annuity of £1 for 8 years is £6·46321276
i.e. £6·46321276 will buy an annuity of £1 for 8 years,
and ∴ £6·46321276 × 120 „ „ £120 for 8 years.
= £775·6
= **£775, 12s.**

(iv.) *What will be the amount of an annuity of £60 if left unpaid for 4 years, interest at 3½ per cent. per annum?*

From Table 2, column 4

Amount of an annuity of £1 for 4 years is £4·21494
∴ „ „ £60 „ 4 „ £4·21494 × 60
= 252·89640
= **£252, 18s.**

(v.) *Find the present value of an annuity of £100 to run for 5 years, deferred 2 years, at 3½ per cent. per annum.*

From Table 2, column 5.

The present value of an annuity of £1 for 5 years at 3½ per cent. is £4·515063,
∴ „ „ „ £100 „ „ = £451·5063

From Table 2, column 3.

The present value of £1 due in 2 years at 3½ per cent. is 0·933512
∴ „ „ £451·5063 „ „ 0·933512 × £451·5063
= **£421, 9s. 9d.**

Examples 88.

1. What is the value of a perpetual annuity of £40 a year @ $2\frac{3}{4}$ per cent. per annum?

2. What is the value of a perpetual annuity of £120 a year @ $2\frac{1}{2}$ per cent. per annum?

3. A man wishes to invest money to provide a bursary of £52 a year. How much must he invest in the $3\frac{1}{4}$ per cents. at 90?

4. What perpetual annuity will £2000 purchase, the rate of interest being $3\frac{1}{4}$ per cent. per annum?

5. A man leaves £3000 to a college to provide two equal bursaries. If the rate of interest be $3\frac{1}{2}$ per cent. per annum, what is the value of one of the bursaries?

6. A person leaves money to provide a bursary of £60 a year, interest calculated at 4 per cent. per annum. The principal is invested in a company paying $3\frac{3}{4}$ per cent. per annum. What is the value (1) of the principal; (2) of the bursary?

7. A society offers to provide an annuity beginning at the age of 65, to a man 21 years old, for £80. What is the value of the annuity when the man is 65 years old, interest at the rate of 3 per cent. per annum?

8. A society offers to provide an annuity beginning at the age of 60, to a man 25 years old, for £60. They offer the same annuity to a man of 60 for £423. What rate of interest is the society charging on the latter's payment?

9. A man when 30 years of age pays £100 for an annuity of £70, to begin when he is 65 years old. He dies after one payment has been made. What does the company gain at his death, money being worth 3 per cent. per annum?

10. A society charges a man of 60 the sum of £72 for each £10 of annuity beginning at the age of 65. What will be the same society's charge to a man of 35, money being worth $2\frac{1}{2}$ per cent. per annum?

Examples 89.

(Do not use the Tables. Work by Formulæ.)

1. Find the amount of an annuity of £130 for 16 years at $3\frac{1}{2}$ per cent., simple interest.

2. Find the amount of an annuity of £52 a year for 12 years at $3\frac{3}{4}$ per cent., simple interest.

3. Find the yearly value of an annuity which if left unpaid for 10 years would amount to £694, 10s., allowing simple interest at $3\frac{1}{2}$ per cent.

4. Find at compound interest the amount of an annuity of £300 a year for 8 years at 3 per cent. (take $(1{\cdot}03)^8 = 1{\cdot}26677$).

5. Find the amount of an annuity of £100 a year for 9 years at 4 per cent., compound interest $(1{\cdot}04)^9 = 1{\cdot}423311$.

6. Find the yearly value of an annuity which at $3\frac{1}{2}$ per cent. compound interest would, if left unpaid, amount in 3 years to £155, 6s. 3d. $(1\cdot035)^3=1\cdot10872$.

7. Find the yearly value of an annuity which at 4 per cent. compound interest would, if left unpaid, amount in 6 years to £298, 9s. 8d. $(1\cdot04)^6=1\cdot265318$.

8. What annuity, payable for 7 years, will £240 buy, interest being compounded annually at 5 per cent $(1\cdot05)^7=1\cdot40710042$.

9. Find the present value of an annuity of £100 for 5 years at $3\frac{1}{2}$ per cent. given $(1\cdot035)^5=1\cdot18768$.

10. Find the difference between the amount and the present value of an annuity of £100 for 10 years at 5 per cent $(1\cdot05)^{10}=1\cdot6289$.

11. In how many years will an annuity of £60 a year amount to £397, 19s 8d. at 4 per cent ?

log. $1\cdot04=0\cdot0170333$. log. $1265\ 32=3\cdot1022004$.

12. Find the difference between the amounts of an annuity of £65 a year for 8 years at 4 per cent., according as simple or compound interest is calculated.

13. Find the amount of an annuity of £80 a year for 10 years with interest and instalments payable half yearly, the nominal rate being 3 per cent. Assume $(1\cdot015)^{20}=1\cdot34685$.

14. Find the difference between the amount and the present value of an annuity of £70 a year for 9 years at 4 per cent., according as simple or compound interest is taken $(1\cdot04)^9=1\cdot423311$.

15. Find the present value of an annuity of £60 a year to begin 4 years from now and to continue for 6 years, compound interest being calculated at 4 per cent., $a_{\overline{6}|}=5\cdot24224$, $(1\cdot04)^4=1\cdot16986$.

Examples 89a.

Use Tables 1, 2, and of Example 1.

1. Make out an annuity table for 1 to 6 years at 4 per cent., correct to four significant figures.

2. What annuity can be bought for 10 years with £2000, interest at the rate of 5 per cent. per annum ?

3. What annuity can be bought for 5 years with £500, interest at the rate of $3\frac{1}{2}$ per cent. per annum ?

4. What annuity for 6 years will £70 buy, interest at the rate of 4 per cent. per annum ?

5. A man pays a society £120. What annuity, beginning after 20 years, and payable for 8, should he receive, interest being calculated at the rate of 5 per cent. per annum ?

6. A man pays a society £50 on the birth of his son. What annuity, beginning when the son is 65 years old, and calculated for 5 years, should the son receive, interest at the rate of $3\frac{1}{2}$ per cent. per annum ?

7. Find the present value of an annuity of £52, at 5 per cent. per annum, if left unpaid for 9 years.

8. Find the present value of an annuity of £60, at $3\frac{1}{2}$ per cent. per annum, if left unpaid for 4 years.

9. Find the present value of an annuity of £40, at 4 per cent per annum, if left unpaid for 5 years.

10. What is the present value of an annuity of £80, at 5 per cent per annum, commencing ten years hence and payable for 10 years?

11. What sum will buy an annuity of £80, payable for 9 years, rate of interest being 5 per cent per annum?

12. What sum will buy an annuity of £60, payable for 4 years, rate of interest being $3\frac{1}{2}$ per cent. per annum?

13. What sum will buy an annuity of £52, payable for 4 years, rate of interest being 4 per cent per annum?

14. A person wishes to purchase an annuity of £100, beginning after 30 years and payable for 7. What should he be charged if the rate of interest is 5 per cent. per annum?

15. A parent wishes his son to receive £150 a year for 5 years after he reaches the age of 25. How much must he pay on his son's second birthday, interest at the rate of $3\frac{1}{2}$ per cent per annum?

16. What will be the amount of an annuity of £26, if left unpaid for 8 years, interest at 5 per cent. per annum?

17. What will be the amount of an annuity of £50, if left unpaid for 3 years, interest at $3\frac{1}{2}$ per cent. per annum?

18. What will be the amount of an annuity of £45, if left unpaid for 6 years, interest at 4 per cent. per annum?

19. A house is bought at 6 years' purchase for £837. What should be the annual rent, the rate of interest being 5 per cent per annum?

20. A man 25 years of age wishes to purchase an annuity of £60, beginning at the age of 65. If the mortality tables give 70 as his probable age at death, what should he pay to a Company, the rate of interest being $3\frac{1}{2}$ per cent. per annum?

XXXVII.—EVOLUTION.

308. Extraction of the Square Root:—

In § 272 it was stated that the square root of a number is that number which, when squared, gives the original number. It is therefore evident that if we resolve a number into its prime factors (§ 22), the square root of the number will be obtained by taking one of every pair of like factors and finding their product. When any odd factors occur in the number the root must be found by the methods of §§ 315–317, or by logarithms.

Examples—

(i.) *Find, by factors, the square root of* 53361.

53361 sum of digits = 18 ∴ (§ 21) divisible by 3^2 giving
5929 difference of sums of alternate digits = 11 ∴ divisible by 11 giving
539 " " " " = 11 ∴ " " 11 "
$9 = 7^2$

$$\therefore 53361 = 3 \times 3 \times 7 \times 7 \times 11 \times 11$$
and square root of $53361 = 3 \times 7 \times 11 = 231$.

(ii.) *Find, by factors, the square root of* 680625.

680625 divides by 5 giving
136125 " 5 "
27225 " 5 "
5445 " 5 "
1089 " 3^2 "
$121 = 11^2$

$$\therefore 680625 = 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 11 \times 11$$
and square root of $680625 = 3 \times 5 \times 5 \times 11 = 825$.

Examples 90.

Find, by factoring, the square roots of—

1. 4900.
2. 2025.
3. 3969.
4. 7744.
5. 38416.
6. 65536.
7. 104976.
8. 390625.
9. 531441.
10. 1679616.
11. 28×63.
12. $12 \times 28 \times 21$.
13. $45 \times 30 \times 6$.
14. $168 \times 24 \times 252$.
15. $112 \times 126 \times 18$.
16. $18 \times (24 - 6) \times 10 \times (265 - 15)$.
17. $7 \times (18 + 7) \times 11 \times (84 - 7)$.
18. $6 \times (21 + 9) \times 5 \times (21 + 4)$.
19. $21 \times (40 - 1) \times 3 \times (300 - 27)$.
20. $51 \times (154 + 15) \times 5 \times (304 - 49)$.

309. On squaring the numbers 1 to 9 we find that the results consist of one or two digits, and it is not till we square the number 10 that we have three digits in the result. From this we learn that the square roots of all integral numbers of not more than two digits lie between 1 and 10, and, if the given number is an exact square, the square root is one of the figures 1, 2……9.

Thus :— $\sqrt{36} = 6$
$\sqrt{81} = 9$.

Squaring any number of two digits, *i.e.* a number between 10 and 100, we find that the square is a number of three or four digits, and it is not till we pass to the square of 100 (= 10,000) that we have five digits. Again, if we add a 0 to any of the numbers 1, 2……9, two 0's are added to its square, thus $6^2 = 36$,

and $60^2 = 3600$; $5^2 = 25$, and $50^2 = 2500$. It is therefore evident that the square root of any number between 2500 and 3600 lies between 50 and 60.

310. Continuing this process, we see that an increase of one in the number of digits in a number (the new digit being placed on the right of the others) increases by **two** the number of digits in its square, and therefore to find the number of digits in the square root of an integral number, we have only to divide the number into pairs of digits, beginning from the right hand. When the number is fractional or mixed, and in the decimal form, we shall have two decimals in the square for each decimal in the number, and as there must always be an even number of decimal figures in an exact square, it is better to make the rule regarding the marking off read as follows:—

Beginning from the decimal point, mark off the digits in pairs in both directions.

When the number is not an exact square, it may be necessary to add a 0 to make up the last pair of digits in the decimal portion.

311. The method used in finding the square root of a number is obtained from the process of finding the square of a number.

Every number of more than one digit may be expressed as the sum of two numbers, of which one is a multiple of a power of ten, and the other is the remainder.

Thus:— $67 = 60 + 7$

$673 = 600 + 73$ or $670 + 3$

Representing these parts by x and y respectively, we can express any number by $x + y$. If we desire to square $x + y$, we multiply first by the x and then by the y and add the results.

Thus:–

$$\begin{array}{l} x + y \\ \underline{x + y} \\ x^2 + xy \\ \underline{\quad\quad xy + y^2} \\ x^2 + 2xy + y^2 \end{array}$$

$$\therefore (x + y)^2 = x^2 + 2xy + y^2$$

$$(x + y)^2 - x^2 = 2xy + y^2 \quad (\S 134.)$$

$$= (2x + y)\, y$$

It is from the last of these expressions that we obtain the rule for extracting the square root.

312.

Examples—

(iii.) *Find the square root of* 4489.

As 4489 lies between 3600 and 4900 we know (§ 309) that its square root lies between 60 and 70. The square root, therefore, is a number made up of 60 + a remainder, which we may represent by $y = 60 + y$.

$$(60 + y)^2 = 60^2 + 2.60.y + y^2$$
$$4489 = 3600 + 2.60.y + y^2$$

subtracting 3600 from each side, we have

$$889 = 2.60\,y + y^2$$
$$= (2.60 + y)\,y.$$

As y must be less than 10, and is small compared with 2.60 (120), we take 120 as a trial divisor into 889, giving 7. Putting in the value of 7 for y we find that

$$2.60.y + y^2 = 2.60.7 + 7^2 = 840 + 49 = 889.$$

$\therefore$ the square root of 4489 = 67.

The work as usually performed may be written out as follows:—

```
             60 | 44'89 | 60
                | 36 00 |  7  = 67.
     2.60 = 120 |   889
              7 |   889
 2.60+7 =   127 |
```

313. The process may be extended to numbers consisting of any number of digits.

(iv.) *Find the square root of* 452929.

Proceeding as above we get—

```
               600 | 45'29'29 | 600
                   | 36 00 00 |  70
   2.600 =    1200 |  9 29 29
                70 |  8 89 00
   2.600+70 = 1270 |    40 29
```

The square root may now be considered as $670 + y$, and as we saw in last example we may take $2.670 = 1340$ as a trial divisor into 4029, giving 3. Adding the 3 to 1340 and multiplying by 3 we have $1343 \times 3 = 4029$. The example when completed will be—

```
                     |        | 600
                 600 | 452929 |  70 = 673.
                     | 360000 |   3
                1200 |  92929
                  70 |  88900
                1270 |   4029
   2.670 =      1340 |   4029
                   3 |
   2.670+3 =    1343 |
```

314. As in division, we can contract the actual working out by omitting the 0's and the multiplication results. We then have—

```
              6  7  3
      6 | 45'29'29
   127* |  9 29
  1343* |  40 29
```

315. We are now able to formulate the steps to be taken in finding the square root of a number.

(1) Having divided the number into pairs of digits (§ 310), find the root whose square is nearest, but not greater than, the first figure or pair of figures, and subtract this square. (2) Take twice the number obtained in the result and divide the remainder by it in order to get the next part of the root. (3) Add this part to twice that already obtained and then multiply by the new part.* (4) This product subtracted from the remainder gives the new remainder, and then we proceed as with (2) (3) (4) again.

Example—(v.) *Find the square root of* 893830609.

```
                               | 20000
  20000 | 8'93'83'06'09        |  9000
        | 4 00 00 00 00        |   800 = 29897
  40000 | 4 93 83 06 09        |    90
   9000 |                      |     7
  49000 | 4 41 00 00 00
  58000 |   52 83 06 09
    800 |
  58800 |   47 04 00 00
  59600 |    5 79 06 09
     90 |
  59690 |    5 37 21 00
  59780 |      41 85 09
      7 |
  59787 |      41 85 09
```

This may be contracted by omitting the 0's, &c., to

```
            2  9  8  9  7
      2 | 8'93'83'06'09
    49* | 4 93
   588* |   52 83
   5969 |    5 79 06
  59787 |      41 85 09
```

* Care must be taken always to write the new figure alongside and not below the trial divisor.

Examples—

(vi.) *Find the square root of* 658·076409.

2 5·6 5 3

2	6'58·07'64'09'
45*	2 58
506*	33 07
5125	2 71 64
51303	15 39 09

316. The decimal point may be put in the root immediately before decimal figures are taken down or may be determined after marking off in pairs.

Examples 91.

Find the square roots of :-

1. 1156.
2. 1681.
3. 5329.
4. 86·49.
5. 64009.
6. 82369.
7. 998001.
8. 1679616.
9. 5322249.
10. 14356521.
11. 46362481.
12. 63920025.
13. 6443·2729.
14. 47·706649.
15. 236144689.
16. 733922281.
17. ·00064009.
18. 11664000000.
19. 240398·012416.
20. 123456789·87654321.

317. Sometimes a number is given which is not an exact square and we are required to find an approximate root, correct to a certain number of significant figures or of decimal places. In such cases we continue the operation till the necessary number of figures is obtained, using the given figures, and adding 0's if required.

(vii.) *Find the square root of* 3, *correct to* 6 *significant figures.*

1·732050 = 1·73205.

1	3
27	200
343	1100
3462	7100
346405	1760000
3464100	2797500

A knowledge of the square roots of 2, 3, and 5 is very useful in finding square roots by factoring. See "Mental Arithmetic," Section XI.

* See p. 263.

Examples 92.

Find the square roots, correct to 5 significant figures, of—

1. 0·1.
2. 6.
3. 13.
4. 0·0097.
5. 3473.
6. 63·835.
7. 0·00004.
8. 0·009.
9. 65·03754.
10. 3·83927638947.
11. 131.
12. 87630000.

Find the square roots, to 3 decimal places—

13. 7.
14. ·11.
15. 0·016.
16. 0·002.
17. 65·7.
18. 8900.
19. 395·735.
20. 89·85734.
21. 10·00005.
22. 84·8324675.
23. 300.
24. 768000.

318. When we have to calculate the square root of a vulgar fraction, the fraction must be first reduced to its lowest terms. Then, if the denominator is an exact square, find the root of numerator and denominator separately; if the denominator is not an exact square, convert to a decimal fraction, and proceed as above.

Examples—

(viii.) *Find the square root of* $\frac{2187}{4107}$.

$$\frac{2187}{4107}=\frac{729}{1369}$$

```
      3 7              2 7
  3 | 1369         2 | 729
 67 |  469        47 | 329
```

$$\therefore \sqrt{\frac{2187}{4107}}=\frac{27}{37}.$$

(ix.) *Find the square root of* $2\frac{62}{361}$.

$$2\frac{62}{361}=\frac{784}{361}.$$

```
      1 9              2 8
  1 | 361          2 | 784
 29 | 261         48 | 384
```

$$\therefore \sqrt{2\frac{62}{361}}=\frac{28}{19}=1\frac{9}{19}.$$

(x.) *Find the square root of* $\frac{654}{323}$.

323 is not a perfect square.

Suppose the answer is required, correct to two places of decimals, we divide out to 6 places.

$$\frac{654}{323}=2\cdot02'47'68'.$$

```
       1· 4  2  2 = 1·42
   1 | 2·02'47'68'
  24 |  102
 282 |   641
2842 |    7768
          2084
```

Examples 93.

Find the square roots of—

1. $\frac{144}{225}$. 3. $1\frac{335}{961}$. 5. $\frac{8649}{9409}$.
2. $\frac{289}{529}$. 4. $2\frac{1271}{2809}$. 6. $\frac{546121}{454276}$.

Find the square roots, correct to 3 decimal places, of—

7. $\frac{71}{197}$. 9. $\frac{6934}{18372}$. 11. $\frac{132478}{836925}$.
8. $\frac{318}{5133}$. 10. $\frac{1466896}{1234567}$. 12. $\frac{6380246}{14837904}$.

319. Extraction of the Cube Root :—

In addition to the square root, the only roots which it is possible to find by the ordinary methods of arithmetic are the cube root and those roots obtained by repeating the extraction of the square and cube roots, as the fourth, sixth, eighth, ninth, &c. All other roots must be found by using logarithms (§ 294), except in simple cases (see below). The cube root is usually obtained with the use of logarithms, but attention must be paid to the number of figures in the logarithms used (§ 289). Next to the square root, the cube root is the most commonly required, and the arithmetical method of obtaining it corresponds to that for obtaining the square root.

320. As with the square root, we may obtain the cube root by the method of prime factors. The cube root is obtained by taking one of every set of three like factors and finding their product. This principle may be extended to the finding of any roots. When the factors do not make up complete sets, the roots must be obtained by logarithms or by method of § 317.

Examples—

(i.) *Find, by factors, the cube root of* 592704.

592704 divide by 2^3 (§ 21) giving
74088 " 2^3 "
9261 " 3^3 "
1029 " 3 "
343 = 7^3

$\therefore 592704 = 2^3 \times 2^3 \times 3^3 \times 7^3$

and the cube root of 592704

$= 2 \times 2 \times 3 \times 7 = 84.$

or $\sqrt[3]{592704}$

$= \sqrt[3]{2\times2\times2\times2\times2\times2\times3\times3\times3\times7\times7\times7}$

$= 2 \times 2 \times 3 \times 7 = 84.$

(ii.) *Find the fifth root of* 12762815625.

12762815625 is divisible by 5^2 giving
510512625 " 5^2 "
20420505 " 5 "
4084101 " 3^2 "
453789 " 3^2 "
50421 " 3 "
16807 " 7 "
2401 " 7 "
343 = 7^3

$\therefore 12762815625 = 5^5 \times 3^5 \times 7^5$

and the fifth root of 12762815625

$= 5 \times 3 \times 7 = 105.$

Examples 94.

Find, by factoring, the cube roots of—

1. 3375.	5. 681472.
2. 13824.	6. 884736.
3. 35937.	7. 2000376.
4. 262144.	8. 2299968.

Find, by factoring, the fourth roots of—

9. 4096.	10. 3111696.

Find, by factoring, the fifth roots of—

11. 7776.	12. 52521875.

321. On cubing the numbers 1 to 9, we find that the results consist of one, two, or three digits, and it is not till we cube the number 10 that we have four digits in the result. From this we learn that the cube roots of all integral numbers of not more than three digits lie between 1 and 10, and if the given number is an exact cube, the cube root is one of the numbers 1, 2,......9.

Thus:— $\sqrt[3]{216}=6$

$\sqrt[3]{729}=9.$

Cubing any number of two digits, *i.e.* a number between 10 and 100, we find the cube is a number of from four to six digits, and it is not till we pass to the cube of 100 (= 1,000,000) that we have seven digits.

Again, if we add a 0 to any of the figures 1, 2,......9, three 0's are added to its cube, thus $6^3=216$ and $60^3=216000$; $5^3=125$ and $50^3=125000$. It is therefore evident that the cube root of any number between 125000 and 216000 lies between 50 and 60.

322. Continuing this process, we see that an increase of one in the number of digits in a number (the new digit being placed on the right of the others) increases by three the number of digits in its cube, and therefore, to find the number of digits in the cube root of an integral number, we have to divide the number into sets of three digits, beginning from the right hand. When the number is fractional or mixed, and in the decimal form, we shall have three decimals in the cube for each decimal in the number, and as there must always be a multiple of three decimal figures in each exact cube, it is better to make the rule regarding the marking off read as follows:—

Beginning from the decimal point, mark off the digits in sets of three, in both directions.

When the number is not an exact cube, it may be necessary to add one or two 0's to make up the last set of digits in the decimal portion.

323. We have already seen (§ 311) that a number of more than one digit can be represented by $x+y$. If we desire to cube $x+y$, we find the square and multiply it by $(x+y)$, first by x, then by y, and add the products.

$$\begin{aligned}
(x+y)^2 &= x^2+2xy+y^2 \qquad (\S\ 311)\\
&\ x^2+2xy+y^2\\
&\ x+y\\
&\ x^3+2x^2y+xy^2\\
&\ \qquad\ x^2y+2xy^2+y^3\\
\therefore\ (x+y)^3 &= x^3+3x^2y+3xy^2+y^3\\
(x+y)^3-x^3 &= 3x^2y+3xy^2+y^3\\
&= (3x^2+3xy+y^2)y.
\end{aligned}$$

It is from the last of these expressions that we obtain the rule for extracting the cube root.

324. Example—(iii.) *Find the cube root of 300763.*

As 300763 lies between 216000 and 343000, its cube root lies between 60 and 70. The cube root is a number made up of 60+a remainder, which we may represent by y.

$$\text{then } (60+y)^3 = 60^3+3.60^2y+3.60y^2+y^3$$
$$300763 = 21600+3.60^2y+3.60y^2+y^3$$

subtracting 216000 from each side of the equation, we have

$$\begin{aligned}
84763 &= 3.60^2y+3.60.y^2+y^3\\
&= (3.60^2+3.60y+y^2)y
\end{aligned}$$

As in square root, we take the first of these ($3·60^2=10800$) as a trial divisor into 84763, giving 7, and putting in the value of 7 for y, we find that

$$\begin{aligned}
3.60^2y+3.60.y^2+y^3 &= 10800.7+180.7^2+7^3\\
&= 75600+8820+343\\
&= 84763
\end{aligned}$$

$\therefore$ cube root of 300763 = 67.

The work as usually performed may be written out as follows :—

	300763	60 + 7 = 67.
	216000	
3.60^2 = 10800	84763	
3.60.7 = 1260		
7^2 = 49		
12109	84763	

325. The process may be extended to numbers consisting of any number of digits.

Example—

(iv.) *Find the cube root of* 304821217.

Proceeding as above we get—

	304'821'217	600
	216 000 000	70
$3.600^2 = 1080000$	88 821 217	
$3.600.70 = 126000$		
$70^2 = 4900$		
1210900	84 763 000	
	4 058 217	

The cube root may now be considered as $670+y$, and, as we saw in the last example, we may take $3.670^2=1344700$ as a trial divisor into 4058217, giving 3, then

$$3.670^2+3.670.3+3^2=1346700+6030+9$$
$$=1352739$$

and, multiplying by 3, we have $1352739 \times 3 = 4058217$

The example when completed will be—

	304'821'217	600
	216 000 000	70 = 673.
		3
$3.600^2 = 1080000$	88 821 217	
$3.600.70 = 126000$		
$70^2 = 4900$		
1210900	84 763 000	
$3.670^2 = 1346700$	4058217	
$3.670.3 = 6030$		
$3 = 9$		
1352739	4058217	

326. As in square root, the working may be contracted by omitting several of the 0's, but owing to the different items, composing the expression $3x^2+3xy+y^2$, overlapping, it is not possible to adopt a similar arrangement to that of placing the new figure on the right of the figures in the working-out column, as is done in square root. Omitting all possible 0's and using the Italian method of division, we have— 6 7 3

	304'821'217
$3.60^2 = 10800$	88 821
$3.60.7 = 1260$	
$7^2 = 49$	
12109	
$3.670^2 = 1346700$	4058217
$3.670.3 = 6030$	
$3^2 = 9$	
1352739	

327. We are now able to formulate the steps to be taken in finding the cube root of a number.

(1) Having divided the number into sets of three digits (§ 322), find the root whose cube is nearest, but not greater than, the first figure or figures, and subtract this cube. (2) Take three times the square of the number obtained in the result, and divide the remainder by it, in order to get the next part of the root. (3) Add together three times the square of the number formerly obtained, three times the product of that number and the new number, and the square of the new number, and multiply the sum by the new number. (4) This product subtracted from the remainder gives the new remainder, and then we proceed as with (2), (3), (4) again.

Example—(v.) *Find the cube root of* 10546683057.

	10'546'683'057	2000
	8 000 000 000	100
		90 = 2193.
$3.2000^2 =$ 12000000	2 546 683 057	3
$3.2000.100 =$ 600000		
$100^2 =$ 10000	1 261 000 000	
12610000	1 285 683 057	
$3.2100^2 =$ 13230000		
$3.2100.90 =$ 567000		
$90^2 =$ 8100		
13805100	1 242 459 000	
$3.2190^2 =$ 14388300	43 224 057	
$3.2190.3 =$ 19710		
$3^2 =$ 9		
14408019	43 224 057	

This may be contracted to—

	2 1 9 3
	10'546'683'057
$3.20^2 =$ 1200	2 546
$3.20.1 =$ 60	
$1^2 =$ 1	
1261	1 285 683
$3.210^2 =$ 132300	
$3.210.9 =$ 5670	
$9^2 =$ 81	
138051	43 224 057
$3.2190^2 =$ 14388300	
$3.2190.3 =$ 19710	
$3^2 =$ 9	
14408019	

328. It is useful to know that three times the square of the number obtained can be quickly got from the working. If the square of the last number obtained be placed below the sum obtained by § 327 (3) and the four lines added, we get three times the square of the number now obtained, and have only to add two 0's on the right.

E.g. Take the third part of the last example. In the working column we have—

$$
\begin{array}{rr}
3.210^2 = & 132300 \\
3.210.9 = & 5670 \\
9^2 = & 81 \\
\hline
 & 138051 \\
9^2 & 81 \\
\hline
 & 143883 = 3{\cdot}219^2.
\end{array}
$$

Adding the four lines (*i.e.* omitting the top line), we have 143883, which is the answer to 3.219^2. Then, placing two 0's on the right, we have the answer to 3.2190^2.

Example—

(vi.) *Find the cube root of* 28233 316125.

		3 0· 4 5
		28'233·316'125.
$3.30^2 =$	2700	1 233 316
$3.300^2 =$	270000	
$3.300.4 =$	3600	
$4^2 =$	16	
	273616	138 852 125
4^2	16	
$3.3040^2 =$	27724800	
$3.3040.5 =$	45600	
$5^2 =$	25	
	27770425	

329. It has been already pointed out that the cube root is usually obtained by the use of logarithms (§ 319). Using the four figure logarithms on pp. 302–305, we can get an approximate value of the cube root.

Thus—

Example (iv.) Find the cube root of 304821217.

We have log. 304800000 = 8·4840

log. $(304800000)^{\frac{1}{3}}$ = 2·8280

but log. [illegible] = 2·8280

∴ cube root of [illegible] = [illegible]

Example (v.) Find the cube root of 10546683057.

$$\log. 10550000000 = 10·0233$$
$$\log. (10550000000)^{\frac{1}{3}} = 3·3411$$
$$\text{but } \log. 2194 = 3·3411$$
$$\therefore \text{ cube root of } 10550000000 = \mathbf{2194}.$$

Example (vi.) Find the cube root of 28233·316125.

$$\log. 28230 = 4·4507$$
$$\log. (28230)^{\frac{1}{3}} = 1·4836$$
$$\text{but } \log. 30·45 = 1·4836$$
$$\therefore \text{ cube root of } 28230 = \mathbf{30·45}.$$

Examples 95.

Extract the cube roots of—

1. 2197.
2. 6859.
3. 19683.
4. 29791.
5. 85184.
6. 132651.
7. 438976.
8. 912673.
9. 69426 531.
10. 88·716536.
11. 102413232.
12. 251239591.
13. 1045·678375.
14. 72720580024.
15. 219365·327791.
16. 0·000017576.
17. 0·000658503.
18. 0·002406104.
19. 0·000028372625.
20. 0·027054036008.

330. The method of finding approximate cube roots is similar to that of finding approximate square roots, but the student is advised, where possible, to use logarithms.

Examples 96.

Find the cube roots, correct to 3 decimal places, of—

1. 4.
2. 0·8.
3. 15.
4. 0·067.
5. 3139.
6. 5·872.
7. 0·000645389.
8. 7·4567[illegible]3.

Find, by logarithms, the cube roots, correct to 4 significant figures, of—

9. 83.
10. 161.
11. 0·097.
12. 6·83.
13. 456738.
14. 3·936724.
15. 0·000873.
16. 635000.
17. 765·2194.
18. 0·000007.
19. 835639735.
20. 473800000000.

331. As with square roots a vulgar fraction must be first reduced to its lowest terms. If the denominator is an exact cube, find cube roots of numerator and denominator separately; if not, convert to a decimal fraction and proceed as above

Examples 97.

Find the cube roots of—

1. $\frac{27}{64}$.
2. $\frac{512}{1728}$.
3. $\frac{4913}{9261}$.
4. $6\frac{139}{343}$.
5. $19\frac{7155}{12167}$.
6. $\frac{1715}{24565}$.
7. $\frac{9317}{415233}$.
8. $\frac{300763}{804357}$.
9. $11\frac{131436}{455877}$.
10. $13559\frac{50679}{59319}$.

Find, correct to 3 decimal places, the cube roots of—

11. $\frac{35}{73}$.
12. $6\frac{15}{16}$.
13. $\frac{837}{3125}$.
14. $45\frac{835}{967}$.
15. $\frac{23456}{56788}$.
16. $16\frac{2}{3}$.
17. $\frac{345}{877}$.
18. $\frac{4538}{6034}$.
19. $\frac{62347}{61358}$.
20. $\frac{427384}{814352}$.

XXXVIII.—DUODECIMALS.

332. Duodecimals are largely used by surveyors, engineers, carpenters, &c. All measurements under the foot are expressed in twelfths of a foot, twelfths of twelfths of foot, &c. The table thus descends from feet by powers of twelve. We have—

1 foot	=	12 primes (^{i}).
1 prime	=	12 seconds (^{ii}).
1 second	=	12 thirds (^{iii}).

The above table is used for the three measures, viz. :—lengths, areas, and volumes, and

$\frac{1}{12}$ of 1 lineal foot	=	1 prime.
$\frac{1}{12}$ of 1 square foot	=	1 „
$\frac{1}{12}$ of 1 cubic foot	=	1 „

As the foot is distinguished by the terms lineal, square, and cubic, so the prime, second, &c., in the different measures may be

distinguished, the terms in common use being running, superficial, and solid, in place of lineal, square, and cubic.

The power of twelve, indicating the fraction of a foot that any unit is, is represented by a Roman numeral. It is therefore easily distinguishable from the indices used to represent powers of a number.

333. The conversion from the usual units of measurement is easily understood from the table. Since an inch is the $\frac{1}{12}$ of a foot, an inch will be represented by a prime and fractions of inches by seconds, thirds, &c. In square measure 144 (12^2) square inches make 1 square foot, therefore each square inch is a second and every 12 square inches make a prime. In cubic measure 1728 (12^3) cubic inches make a cubic foot, therefore each inch is a third, every 12 inches make a second, and every 144 inches make a prime.

Conversely, in lineal measure, each prime is an inch,
in square " " " is 12 square inches, and
each second is 1 square inch.
In cubic measure, each prime is 144 cubic inches,
each second is 12 cubic inches, and
each third is 1 cubic inch.

Examples—

(i.) *Convert 16 ft. 3¼ inches into duodecimals.*

1 inch = 1 prime = 12 seconds.
¼ " = 3 seconds.
∴ 16 feet 3¼ inches = 16 ft. $3^{I}\ 3^{II}$.

(ii.) *Convert an area of 5 square feet $4^{I}\ 3^{II}\ 6^{III}$ to square feet and inches.*

4^{I} = 48 inches.
3^{II} = 3 "
6^{III} = $\frac{6}{12}$ "
∴ 5 sq. ft. $4^{I}\ 3^{II}\ 6^{III}$ = 5 sq. ft. 51½ sq. in.

(iii.) *Express 16 cubic feet 854¾ cubic inches in duodecimals.*

Each cubic inch = 1 third.
$\frac{3}{4} = \frac{9}{12}$ " " = 9 fourths.
854 " " = 854 thirds.
= 71 seconds 2 thirds.
= 5 primes 11 seconds 2 thirds.
∴ 16 cub. ft. 854¾ cub. in. = 16 cub. ft. $5^{I}\ 11^{II}\ 2^{III}\ 9^{IV}$.

Examples 98.

Express in duodecimals :—

1. 2 ft. 7 in.
2. 3 ft. $5\frac{1}{4}$ in.
3. 3 ft. $4\frac{7}{12}$ in.
4. 26 sq. ft. 12 sq. in.
5. 39 sq. ft. 67 sq. in.
6. 45 sq. ft. $84\frac{3}{4}$ sq. in.
7. 10 sq. ft. $75\frac{135}{144}$ sq. in.
8. 39 cub ft. 479 cub. in.
9. 46 cub. ft. $298\frac{25}{48}$ cub. in.
10. 291 cub. ft. $1099\frac{5}{8}$ cub. in.

Express in feet and inches :—

11. 4 ft. 5^{i}.
12. 6 ft. $2^{i}\ 11^{ii}$.
13. 1 ft. $8^{i}\ 5^{ii}$.
14. 15 sq. ft $3^{i}\ 11^{ii}$.
15. 9 sq. ft. $4^{i}\ 8^{ii}$.
16. 20 sq ft $10^{i}\ 8^{ii}\ 6^{iii}$.
17. 75 sq. ft. $9^{i}\ 11^{ii}\ 4^{iii}$.
18. 147 cub. ft. $1^{i}\ 3^{ii}\ 3^{iii}$.
19. 28 cub. ft. $3^{i}\ 5^{ii}\ 10^{iii}\ 4^{iv}$.
20. 89 cub. ft. $4^{i}\ 3^{ii}\ 6^{iii}\ 7^{iv}$.

334 In multiplication and division by duodecimals it should be noted that the indices used obey laws similar to those of indices (§§ 275–281). This may be seen from the following—

E.g. $6^{i} \times 7^{i} = 42^{i+i} = 42^{ii}$, for $6^{i} \times 7^{i} = \frac{6}{12} \times \frac{7}{12} = \frac{42}{12^2} = 42^{ii}$

and $6 \times 6^{ii} = 36^{0+ii} = 36^{ii}$, for $6^{0} \times 6^{ii} = \frac{6}{} \times \frac{6}{12^2} = \frac{36}{12^2} = 36^{ii}$

so that when multiplying we must add these indices, and when dividing subtract them.

As the units proceed by 12 we may divide by 12 when the product of two numbers exceeds 12; the quotient so obtained is the number of the next higher unit, *e.g.* $42^{ii} = 3^{i}\ 6^{ii}$, and in the following example when we multiply 6^{ii} by 9^{ii} we get $54^{iv} = 4^{iii}\ 6^{iv}$.

Example—

(iv.) *Multiply 5 feet* $8^{i}\ 6^{ii}$ *by 7 ft.* $6^{i}\ 9^{ii}$.

5 ft.	8^{i}	6^{ii}		
7	6^{i}	9^{ii}		
39	11	6		
2	10	8	0	
	4	3	4	6
43 sq. ft.	2^{i}	0^{ii}	4^{iii}	6^{iv}
	12		12	
	24		54	

$= 43$ sq. ft. 24 sq. in. $\frac{54}{144}$ sq. in.

[illegible] sq. in.

Examples—

(v.) *Simplify 50 ft. 3½ in. × 2 ft. 3 in. × 1 ft. 5 in. and find the value of the result at 5d. per cubic foot.*

50 ft.	3^{I}	6^{II}		
2	3^{I}			
100	7	0		
12	6	10	6	
113	1^{I}	10^{II}	6^{III}	
1	5^{I}			
113	1	10	6	
47	1	9	4	6
160 cub. ft.	3^{I}	7^{II}	10^{III}	6^{IV}
	12			
	43			
	12			
	526½			

= **160 cub. ft 526½ cub. in.**

=160 cub. ft. @ 5*d.* per cub. ft.	=	£3	6	8	
3^{I} " "	=	0	0	1¼	
8^{II} " "	=	0	0	0¾	
		£3	6	10	

(vi.) *Divide 5 square feet 128 square inches by 8 inches.*

5 sq. ft. $10^{\text{I}}\ 8^{\text{II}} \div 8^{\text{I}}$
$= 72^{\text{I}}\ 8^{\text{II}} \div 8^{\text{I}}$
$= 9\ \ 1^{\text{I}}$
= 9 ft. 1 in.

Examples 99.

Multiply—

1. 4 ft. 3 in. by 6 ft. 6 in.
2. 12 ft. 7 in. by 13 ft. 8 in.
3. 8 ft. by 5 ft. by 14 ft. 7 in.
4. 9 ft. 4 in. by 18 ft. 11 in.
5. 6 ft. $3^{\text{I}}\ 4^{\text{II}}$ by 7 ft. 4^{I}.
6. 12 ft. $4^{\text{I}}\ 7^{\text{II}}$ by 9 ft. $0^{\text{I}}\ 6^{\text{II}}$.
7. 9 ft. $6^{\text{I}}\ 5^{\text{II}}$ by 5 ft. $8^{\text{I}}\ 11^{\text{II}}$.
8. 8 ft. $3^{\text{I}}\ 7^{\text{II}}$ by 24 ft. $7^{\text{I}}\ 3^{\text{II}}$.

Find the products of—

9. 8 ft. 3 in. × 6 ft. 4 in. × 9 ft. 7 in.
10. 13 ft. 4 in. × 8 ft. 5 in. × 7 ft. 3 in.

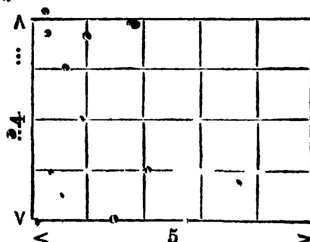
XXXIX.—MENSURATION.

335. Rectangle and Square:—

Mensuration deals with the rules for finding the lengths of lines, the areas of surfaces, and the volumes of solids

The figure most commonly in use is the rectangle; *e.g.*, most dimensions of buildings are usually rectangular.

336.



The area of a rectangle is the product of its length and breadth. This is easily seen from the above figure. If the length is 5 inches, and the breadth 4 inches, we can divide those into 5 and 4 equal parts of one inch respectively. Drawing the lines across the figure, we divide it into 4 rows of 5 divisions each, *i.e.* twenty divisions in all. The sides of each division are 1 inch in length, so that the area of each division is 1 square inch, and the total area is 20 square inches. The area is therefore equal to length × breadth = 5 × 4 = 20 square inches.

337. A square is a particular form of rectangle in which the four sides are equal. We therefore get the **area** by multiplying the length by the breadth, or, since these are equal, by **squaring the length of one side.** Thus, if the above figure were a square of 5 inch side, its area would be $5 \times 5 = 5^2 = 25$ square inches.

338. We have, therefore, for rectangles and squares the following formulæ:—

Let A = area, l = length, and b = breadth.

$A = l \times b$ and where $l = b$ $A = l^2$ or b^2

$\sqrt{A} = l$ or b.

from this we get (§ 135) $\frac{A}{l} = b$

$\frac{A}{b} = l$.

Examples—

(i.) *If the area of a rectangle is 684 square inches, and its length is 36, find its breadth.*

$$A = l \times b$$

$$\therefore \frac{A}{l} = b$$

$$\frac{684 \text{ sq. ins.}}{36 \text{ ins.}} = b$$

19 inches = breadth.

(ii.) *Find the side of a square whose area is 1764 square yards.*

$$A = l^2$$

$$\sqrt{A} = l$$

$$\sqrt{1764 \text{ sq. yds.}} = l$$

$$\sqrt{2^2 \times 3^2 \times 7^2 \text{ sq. yds.}} = l$$

$$2 \times 3 \times 7 = \mathbf{42 \text{ yds.} = \text{side.}}$$

338a. It is not always necessary to express the dimensions in the same unit. Thus, while 5 feet by 2 feet give 10 square feet, so 4 yards by 2 feet might be expressed as 8 yards-feet; but as such double names are not in use, we must by reduction express the product in yards-yards—*i.e.* square yards—or in feet-feet—*i.e.* square feet; still, in the process working of many questions such reduction is unnecessary, and merely adds to the work of solving them. If the dimensions be left in terms of *single* though different units, with their names included, the ordinary cancelling will give the answer required. In this connection, it is important to note that such expressions as square yards must be treated as equivalent to yards × yards and cancelled as such.

(iii.) *Find the length of a rectangle whose area is 25 square feet and its breadth 15 inches.*

$$l = \frac{A}{b} = \frac{25 \text{ sq. feet}}{15 \text{ ins.}} = \mathbf{20 \text{ feet.}}$$

(iv.) *A hall is 16 yards long by 25 feet wide; what length of matting, 30 inches wide, will be required to cover it?*

The length of matting will obviously be the area of the room divided by the width of the matting.

$$l = \frac{A \text{ of room}}{b \text{ of matting}} = \frac{16 \text{ yards} \times 25 \text{ feet}}{30 \text{ ins.}} = \mathbf{160 \text{ yards.}}$$

(v.) *How many tiles, 1 foot 6 inches by 8 inches, will be required to cover a courtyard whose area is 46 square yards?*

$$\text{Number of tiles} = \frac{\text{Total area}}{\text{Area of one tile}} = \frac{46 \text{ sq. yds.}}{1 \cdot 5 \text{ ft.} \times 8 \text{ ins.}} = \mathbf{414.}$$

Examples 100.

Find the areas of the following floors, the dimensions being—

1. Length, 17 ft. 4 in.; breadth, 12 ft. 3 in.
2. Length, 20 ft. 7 in.; breadth, 16 ft.
3. Length, 48·5 cms.; breadth, 30·6 cms.
4. Length, 64·3 cms.; breadth, 42·8 cms.
5. Length, 40 yds. 2 ft. 7 in.; breadth, 18 yds. 1 ft. 3 in.

Find the areas of the following fields—

6. Length, 19 chains, 16 links; breadth, 16 chains, 30 links.
7. Length, 33 chains, 45 links; breadth, 25 chains, 47 links.
8. Length, 85 chains; breadth, 46 chains, 40 links.
9. Length, 73·6 metres; breadth, 68·7 metres.
10. Length, 145·3 metres; breadth, 94·75 metres.
11. The side of a square is 79 ft. Find its area in square yards.
12. Find the number of square yards in a square, whose side is 2465 yds.
13. The side of a square is 6245 links. Find its area in acres.
14. What is the area of a square piece of ground whose side is 34·65 chains? Answer in acres, roods, poles.
15. What is the area of a square piece of ground whose side is 94 chains, 36·5 links? Answer in acres, chains, links.
16. The floor of a room is 18½ ft. by 16½ ft. How many square yards of linoleum will be required to cover it?
17. A railway covers ground to the breadth of 22 yds. How many acres will be covered by a mile of the railway?
18. The area of a square is 5329 sq. yds. Find its side.
19. What is the number of links in the side of a square field measuring 10 acs. 3 rds. 20 pls.?
20. A square field contains 3·6 acs. How many yards of paling will be required to fence it?
21. A square paddock contains 16 sq. pls. and is enclosed by hurdles 2¾ yds. long. How many hurdles are required?
22. A grain of gold, if beaten into gold leaf, will cover 67½ sq. in. Find, in yards, the length of the side of a square that can be covered by an ounce Troy of gold leaf.
23. A square court contains one-third of an acre. Find the length of one side, in yards, correct to 3 decimal places.
24. A square field contains 59 acs. 2 rds. 12 pls. 26 sq. yds. Find the length of one side, in yards, and reduce the result to furlongs, poles, &c.
25. Find the length of the side of a square field containing 11 acs. 1 rd. 10 sq. pls. 3¼ sq. yds., expressing your answer in furlongs, poles, yards.
26. A square piece of land contains 15·21 hectares. Find the length of a side in metres.

27. A plank of wood covers 30 sq. ft. If it is 18 in. broad, find its length.

28. A feu of 2 acs. is to be marked off for building purposes; the frontage is 44 yds. How far back will the ground extend?

29. A rectangular piece of ground is for sale for building purposes. The advertisement states that the ground is 1 ac. 2765 sq. yds. in extent. The frontage of the building at present standing is 195 yds. How far back does it extend?

30. A grass lawn extends to 36 sq. pls., and in rolling it the roller traces out a path 2 furs. 7 chains in length. Find the breadth of the roller.

31. A room 27 ft. long requires 52 sq. yds. of carpet to cover the floor. Find the breadth of the room.

32. What is the length of a hall, the breadth of which is 60 ft. and the floor area is equal to that of another hall, of which the length and breadth are 140 and 35 ft. respectively?

33. The area of a square piece of metal is 8 sq. metres. Find the length of a side, correct to 5 significant figures.

34. Turf is taken from a field 2 furs. 36 pls. 3 yds. 2 ft. long and 2 furs. 14 pls. 2 yds. 2 ft. 3 in. broad, and is found to be exactly sufficient to cover a square garden. Find the length of one side of the garden.

35. How many blocks of wood each $4\frac{1}{2}$ ins. by $2\frac{1}{4}$ ins. would be required to cover a floor 9 yards long by 22 ft. 3 ins. broad?

339. Carpeting floors, papering walls, &c.

Since rolls of carpet and paper are made in standard widths, it is important in dealing with questions connected with carpeting floors, &c., to see that the width of carpet or paper involved divides exactly into the length or breadth of the surface to be covered. If it does, then we may proceed as in Ex. iv., p. 278; but if not, since dealers will not cut a width of carpet or paper, either more or less than is necessary must be taken. In the one case there is so much waste, in the other the part uncovered may be covered with some other materials. In papering walls it is usual to allow extra paper for such contingencies and for other waste occasioned by the proper joining of the pattern. Many carpets again are made of fixed areas and laid on floors with a margin of felt or linoleum. Questions of this type follow the principle of Ex. v., p. 282.

340. It is important to notice that the wall surface of a room is obtained by **multiplying the perimeter by the height.** The perimeter is the measurement round the room, and is made up of twice the length + twice the breadth, *i.e.* A of wall = $2(l + b)h$.

Examples—

(iii.) *How many yards of carpet, 27 ins. wide, will be required to cover the floor of a room 30 ft. 6 ins. by 18 ft. 4 ins, and what will it cost at 4s. 9d. per yard.*

Since neither 30 ft 6 ins. nor 18 ft 4 ins. will divide exactly by 27 ins. we use more carpet than is necessary.

(1) Laying the carpet longways we get $\frac{18\text{ ft. }4\text{ ins.}}{27\text{ ins}} = 9$ widths of carpet, each 30 ft. 6 ins long, giving a total length required of 30 ft. 6 ins $\times 9 = 91\frac{1}{2}$ yards.

$$\text{Cost} = 91\tfrac{1}{2}\text{ yards @ }\frac{4s.\ 9d.}{\text{yard}} = £21,\ 14s\ 7\tfrac{1}{2}d.$$

(2) Laying the carpet broadways we get $\frac{30\text{ ft }6\text{ ins.}}{27\text{ ins}} = 14$ widths of carpet, each 18 ft. 4 ins long, giving a total length required of 18 ft. 4 ins. $\times 14 = 85\frac{5}{9}$ yards.

$$\text{Cost} = 85\tfrac{5}{9}\text{ yards @ }\frac{4s.\ 9d.}{\text{yard}} = £20,\ 6s.\ 5d.$$

(iv.) *If a room be 24 ft. 6 ins in length, 18 ft 3 ins. in breadth, and 11 ft. 9 ins. in height, how much paper $\frac{3}{4}$ yds. wide would be required to cover its walls? Allow one seventh extra for waste.*

A of walls $= 2(l + b)h$

A of paper $= \frac{8}{7}$ of 2(24 ft. 6 ins. + 18 ft. 3 ins.) × 11 ft. 9 ins

$$l = \frac{A}{b} = \tfrac{8}{7}\text{ of }2 \times 42\tfrac{3}{4}\text{ ft.} \times 11\tfrac{3}{4}\text{ ft.} \div \tfrac{3}{4}\text{ yd.}$$

$$= \frac{\overset{2}{\cancel{8}}}{\cancel{7}}\text{ of }2 \times \frac{\overset{19}{\cancel{171}}\text{ ft.}}{\cancel{4}} \times \frac{47\text{ ft.}}{\cancel{4}} \times \frac{\cancel{4}}{\underset{3}{\cancel{9}}\text{ yds.}} = 510\tfrac{2}{3}\text{ ft.}$$

= 170 yards.

(v.) *Find the cost of covering a surface of 645 sq. ft. with paper 21 ins. wide @ 1s. 10d. per piece of 12 yds., allowing 10 per cent. extra for waste, and no fractional pieces of paper to be bought.*

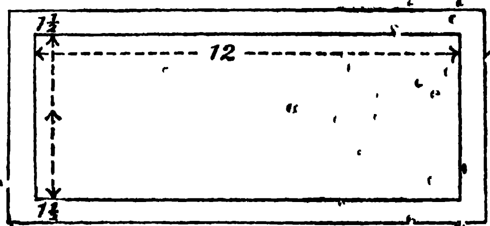
$$\text{No. of Pieces} = \frac{11\cancel{0}}{10\cancel{0}}\text{ of }\frac{\overset{215}{\cancel{645}}\ \overset{12}{\cancel{12}}\text{ sq. ft.}}{21\text{ ins.}}\ @\ \frac{1\text{ piece}}{\underset{3}{\cancel{12}}\text{ yds.}} = \frac{2365\text{ pieces}}{210} = 12\text{ pieces.}$$

$$\text{Cost} = 12\text{ pieces @ }\frac{1s.\ 10d.}{\text{piece}} = £1,\ 2s.$$

341. When the question is involved, the student should clearly understand what is wanted before attempting the solution. In most cases he is recommended to make a figure of surface, &c., as this will make the question more definite.

Example—

(v.) *A path $4\frac{1}{2}$ ft. wide is made round a lawn, 12 yds. long and 5 yds. wide. How many square yards are there in the surface covered by the path?*



Total length, including path = 12 yds. + 2.1½ yds. = 15 yds.
,, breadth ,, ,, = 5 yds. + 2.1½ yds. = 8 yds.
,, area ,, ,, = 15 yds. × 8 yds. = 120 sq. yds.
,, area of lawn = 12 yds. × 5 yds. = 60 sq. yds.
∴ difference, *i.e.* area of path = 60 square yards.

Examples 101.

1. A square courtyard has to be paved at 1*s.* 9*d.* per square foot; the total cost is £91. What are its dimensions?

2. What will it cost to paper the four walls of a room whose length is 16 ft. 9 in., breadth 15 ft. 8 in., and height 10 ft. 6 in., at 9*d.* per square yard?

3. A room is 24 ft. long, $13\frac{1}{2}$ ft. wide, $9\frac{1}{2}$ ft. high. What will it cost to paper the walls at $\frac{3}{4}d.$ per square foot, deducting 16 per cent. of the wall surface for windows, doors, &c.?

4. A wall is 6 yds. 2 ft. long and 5 ft. 3 in. high. What will it cost to paper it with paper $\frac{3}{4}$ of a yard wide, which costs $\frac{3}{4}d.$ a yard, allowing 4 yds of paper for waste?

5. Calculate the cost of paving a courtyard 35 ft. 10 in. long and 18 ft. 6 in. broad, at 6*s.* 3*d.* per square yard.

6. How many yards of carpet 2 ft. 4 in. wide will a square room require, the length of a side being 17 ft. 6 in.?

7. What length of paper 21 in. broad will be required for a room 18 ft. long, 12 ft. broad, and 11 ft. high?

8. What will it cost to paper a room 6 yds. 1 ft. 1 in. long, 6 yds. 0 ft. 4 in. broad, and 12 ft. high, with paper half a yard wide at $4\frac{1}{2}d.$ per yard, allowing $\frac{1}{12}$ extra for waste?

9. 1296 bricks (exposed surface of each = $9\frac{1}{2}$ in. by $4\frac{1}{2}$ in.) are used in paving a certain yard. How many tiles 6 in. square would be required for a yard one-third the size?

10. What will it cost to paper a room 5 yds. 1 ft. long, 5 yds. 2 in. broad, and 4 yds. high with paper 21 in. wide at 1*s.* 2*d.* per yard? Allow one-eight extra for waste.

11. How many yards of matting 2 ft. 6 in. wide will be required to cover a floor 27 ft. long by 20 ft. broad? (The matting may be cut.)

12. What will be the cost of painting the four walls of a room 32 ft. 4 in. long, 15 ft. 8 in. broad, and 10 ft. 6 in. high at 3*s.* per square foot?

13. Find the cost of carpeting a floor 120 ft. by 80 ft. with carpet $1\frac{1}{2}$ yds. wide at 4*s.* 6*d.* per yard.

14. How many planks, each $13\frac{1}{2}$ ft. long and 8 in. wide, will be required for the floor of a hall 150 yds. long and 50 yds. wide?

15. An oblong field is three times as long as it is broad, and its area is 12 acres. Find the cost of fencing it at 2*s.* 4*d.* a yard, neglecting fractions of a yard in final calculation.

16. A room is 19 ft. $10\frac{1}{2}$ in. long, 16 ft. $1\frac{3}{4}$ in. broad, and 10 ft. 3 in. high. What will it cost to paint the walls at 1*s.* 7*d.* per square yard?

17. Find how many yards of paper 2 ft. 6 in. wide are required to cover the walls of a room 20 ft. 6 in. long, 9 ft. 6 in. wide, and 13 ft. 6 in. high; and what will the cost be at 1*s.* 3*d.* per yard?

18. Find how many yards of wall paper 2 ft. 5 in. wide are required to paper the walls of a room 13 ft. 5 in. long, 14 ft. 3 in. wide, and 10 ft. 7 in. high; and what will it cost at 10*d.* per yard? (Allow one-eighth extra for waste.)

19. The interior measurements of a rectangular box are 12 ft. 6 in., 10 ft. 8 in., and 7 ft. 6 in. Find the cost of lining the box (including the lid) with zinc at 6*d.* per square foot.

20. How many meshes are there in a square foot of wire gauze, each mesh being 0·3 of an inch long and 0·1 of an inch wide?

21. A room is 17 ft. long and 13 ft. 6 in. broad. Find (1) how many yards of carpet 2 ft. 3 in. wide will be required to cover the floor; (2) the price of the carpet at 5*s.* 4*d.* per yard.

22. A rectangular tank is 18 ft. long, 14 ft. broad, and 10 ft. deep. Find the cost of painting the sides and bottom of the inside at 1*s.* $1\frac{1}{2}$*d.* the square yard.

23. A square courtyard has to be paved at 5 francs per square metre; the total cost is 1125 francs. What are its dimensions?

24. A room is 8 metres long, 5·5 metres wide, 4·25 metres high. What will it cost to paper the walls at 50 pfennige per square metre?

25. Calculate the cost of paving a courtyard 12·5 metres long, 10·25 metres broad at 6 krones per square metre.

26. What will it cost to paper a room 8·5 metres long, 6·75 metres broad, and 4·5 metres high with paper 50 cms. wide at 50 centimes per metre?

27. A British firm quotes for the painting and renovating of a building 50 metres long, 40 wide, and 45·25 high. If the charge is 7*s.* 6*d.* per square yard, what will be the total cost in francs, the rate of exchange being 25·12 francs = £1?

28. Find the cost of carpeting a floor 120 ft. by 80 ft. with carpets bought in France at 4 francs, 50 centimes per square metre, the rate of exchange being 25·30 francs = £1?

29. How many planks, each 3·4 metres long and 10·6 cms. wide, will be required for the floor of a hall 145 metres long and 45 metres wide?

30. A room is 38 ft. 3 in. long and 25 ft. 6 in. wide. How many yards of carpet 1 yard broad will be required for the floor if a margin of 9 in. in width is left uncarpeted?

31. Find the cost, at 4*s.* per square foot, of paving a yard 21 ft. long by 18 ft. wide, allowing for four unpaved spaces each 3 ft. by 2 ft. 6 in. and for a shed 9 ft. square.

32. If a postage stamp measures $\frac{8}{9}$ of an inch by $\frac{3}{4}$ of an inch, how many will be required to cover the walls of a room whose height is 10 ft. 6 in., length 18 ft., and breadth 12 ft. 2 in., supposing that the door, window, and fireplace occupy 50 square feet?

33. Find the cost of the paper required for a room 19 ft. 9 in. long, 16 ft. 3 in. broad, and 11 ft. high, if the paper be 2 ft. 9 in. broad and cost 3*s.* 6*d.* per piece of 12 yards, assuming that one-sixth of the surface of the walls is not to be papered. Allow 10 per cent. extra. No fractional pieces may be bought.

34. A square enclosure is surrounded by a roadway whose breadth is one-eleventh of the side of the enclosure. The area of the roadway is 1 ac. 2 rds. 28 sq. pls. 5 sq. yds. Find the length of the side of the enclosure in feet.

35. A man endeavoured to paper a wall with postage stamps, of which he had 38,080 measuring $\frac{11}{16}$ of an inch by $\frac{13}{?}$ of an inch. If the wall was 17 ft. 4 in. by 19 ft. 2 in., what fraction of its surface would the stamps cover?

36. A person wishes to cut from a rectangular piece of ground 80 metres long and 43·5 metres wide, a pathway 1·25 metres wide, which will extend right round the ground. What area will be left for cultivation?

37. The length of a room is 20 ft. 6 in., the breadth 15 ft. 9 in., and the height 10 ft. 6 in. What will it cost for plastering, the ceiling costing 8*d.* a square yard and the walls 3*d.* a square yard? Allow for a door 6 ft. 9 in. by 4 ft. 2 in. and a fireplace 5 ft. 6 in. by 5 ft. 3 in.

38. What is the cost of painting (at 2*s.* 6*d.* per square yard) the walls of a room $20\frac{1}{4}$ ft. long, $18\frac{1}{2}$ ft. broad, and 10 ft. high, containing two windows whose dimensions are 7 ft. by 4 ft. each?

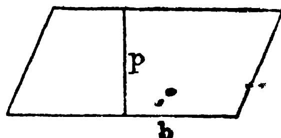
39. What is the cost of painting the walls and ceiling of a room 7·35 metres long, 5·84 metres broad, and 3·25 metres high? The walls cost 2 florins, 2 cents per square metre, and the ceiling 1 florin, 5 cents per square metre. There are two windows 2·16 metres by 1·2 metres each and a fireplace 1·8 metres square.

40. A room is 4·6 metres long, 3·4 metres broad, and 3·85 metres high. It contains two windows 2·8 metres by 1·5 metres each, three doors 2·2 metres by 1·1 metres each, and a fireplace 2·4 metres by 1·2 metres. How many postage stamps will it take to cover the walls (a stamp being 2·4 cms. by 2 cms.), it being supposed that the stamps may be cut if necessary.

342. Other Rectilineal Figures.

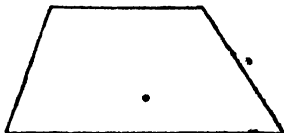
The following are the formulæ from which the areas, &c., of the more commonly occurring rectilineal figures may be obtained. The student is referred to any standard text-book on Mensuration for the proofs of these formulæ. Use should be made of the principles of §§ 133–136 in using them.

Parallelogram.



Area = base multiplied by perpendicular height. $A = bp$.

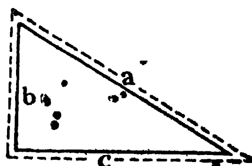
Trapezium.



Area = half the sum of the parallel sides multiplied by the perpendicular distance between them.

$A = up$ (u is uniform, or average breadth).

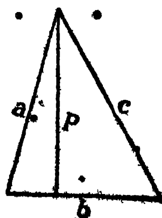
Right-Angled Triangle.



Square of hypotenuse = sum of squares of other sides

$a^2 = b^2 + c^2$

Triangle.



Area = half the product of any side into the perpendicular distance between that side and opposite vertex.

$A = \frac{bp}{2}$, or

Area = square root of product of semi-perimeter into the differences between the semi-perimeter and each of the sides.

$A = \sqrt{s(s-a)(s-b)(s-c)}$

Any regular figure.

(A regular figure is one in which the sides are all equal)

Area = product of semiperimeter into perpendicular distance of centre from any side, divided by 2.

$$A = \frac{sp}{2}$$

Examples—

(vi.) *A four-sided field has two sides parallel to one another. One of these measures 6·7 chains, the other 5·8 chains, and the perpendicular distance between them is 7·4 chains. Find the area of the field.*

The figure is a trapezium.

A up

$= \frac{6·7 \text{ chains} + 5·8 \text{ chains}}{2}$

$= 6·25 \text{ chains} \times 7·4 \text{ chains}$

$= 46·25$ sq chains

$= 4·625$ acres

= 4 acres 2 roods 20 poles.

(vii.) *The diagonal of a square field is 1000 links, find its area in acres.*

The diagonal and two sides form a right angled triangle,

$\therefore a^2 = b^2 + c^2$, but the sides are equal

$\therefore a^2 = 2c^2$

$\frac{a^2}{2} = c^2$

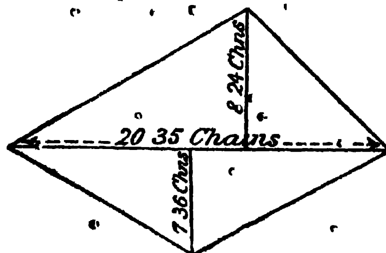
but area of square = square on side

$= c^2 = \frac{a^2}{2}$

$= \frac{1000000 \text{ sq links}}{2} = 500,000$ sq. links.

= 5 acres.

(viii.) *The diagonal of a quadrilateral is 20 chains 35 links, and the perpendicular on it from the opposite angles are 8 chains 24 links and 7 chains 36 links. Find the area in acres, roods, poles.*



Area of a triangle

$= \frac{bp}{2}$

Area of upper triangle

$= \frac{20·35 \text{ ch.} \times 8·24 \text{ ch.}}{2}$ (8·24 / 2 = 4·12) sq. chains.

Area of lower triangle

$= \frac{20·35 \text{ ch.} \times 7·36 \text{ ch.}}{2}$ (7·36 / 2 = 3·68) sq. chains.

$\therefore$ Area of quadrilateral

= 158·73 sq. chains.

= 15 acres 3 rds. 19·68 poles.

Examples 102.

1. A field, in the shape of a parallelogram, is 120 yards long and its breadth is 84 yards. Find its area.

2. A parallelogram is 15 inches wide and contains 5 square yards. Find its length.

3. A field has two parallel sides, the lengths of which are 12 chains 8 links and 7 chains 12 links, and the perpendicular distance between them is 8 chains 25 links. Find its acreage.

4. Find the area of a right-angled triangle field, the length of the sides containing the right angle being 160·4 metres and 63·7 metres respectively.

5. The height of a wall is 50 feet. At what distance from the base of it must the foot of a ladder 120 feet long be placed so that it may just reach the top of it?

6. The foot of a ladder 30 feet long is 14 feet from a house and its top reaches the upper part of a circular window. When the foot is drawn away to a distance of 17 feet from the house the top reaches the lower edge of the window. What is the diameter of the window?

7. Find the acreage of a triangle whose sides are 45, 40, and 13 chains respectively.

8. The sides of a triangle are 39, 42, and 45 feet. Find its area in yards.

9. Find the diagonal of a square that shall be equal in area to a rectangle of which two adjacent sides are 20 and 30 feet.

10. The area of a rectangular field is 87 acs. 1 rd. 26 pls. Find the length of a diagonal if the breadth be 462·5 links.

11. The sides of a quadrilateral ABCD are as follows:—AB 20 metres, BC 65 metres, CD 90·5 metres, AD 57 metres, and the diagonal AC is 80 metres. Find the area of the field and the cost of mowing it at 15 cents per square metre.

12. Find the cost of mowing a triangular field at 15 francs the hectare, the sides being 800, 650, 364 metres respectively.

343. The Circle.

If the circumference of a circle be exactly measured and divided by the length of the diameter, the quotient is 3·14159265...... This value is usually represented by the symbol π, and various approximate values are used. The chief of these, in the order of their correctness, are—

$$\frac{355}{113} = 3{\cdot}1415929......$$

$$3{\cdot}1416$$

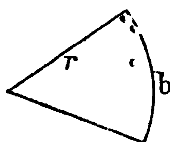
$$\frac{22}{7} = 3{\cdot}142857$$

The circumference of a circle is therefore obtained by multiplying the diameter by π, or if we represent the diameter by $2r$ (twice the radius) we have—

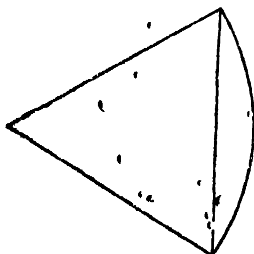
$$\text{circumference of circle} = 2\pi r$$
$$\text{area of circle} = \pi r^2$$

The area of a circle may also be obtained by multiplying the square of the diameter by 0·7854 $\left(=\frac{\pi}{4}.\right)$.

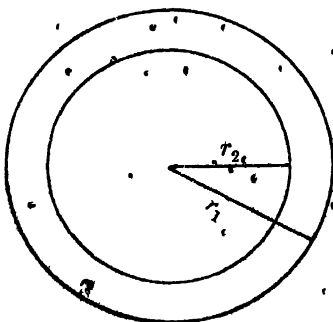
Area of sector $=\frac{br}{2}$



This is similar to a triangle b = length of arc and r is perpendicular height. The areas of sectors of a circle are proportional to the angles at the centre. The sum of the angles at the centre of a circle is 360°.



Area of segment = area of sector − area of triangle formed by radii and chord.



The area of a circular ring is obtained by subtracting the area of the smaller circle from that of the larger.

$$= \pi . r_1^2 - \pi r_2^2$$
$$= \pi (r_1^2 - r_2^2)$$
$$= \pi (r_1 + r^2) (r_1 - r_2).$$

The surface area of a sphere $= 4\pi r^2$

Examples—

(ix.) *Find the circumference of a circle whose diameter is* 13·7 *feet.*

$$\text{circumference} = 2\pi r$$
$$= 13{\cdot}7 \text{ ft.} \times 3{\cdot}1416$$
$$= \mathbf{43{\cdot}04 \text{ feet.}}$$

(x.) *How much will the turfing of a round plot cost at* 4*d. per square yard if the diameter be* 180 *feet and a circular fountain* 18 *feet in diameter be left in the middle?*

Area of whole ground	$= \pi r^2$
	$= 3{\cdot}1416 \times 30^2$ square yards
	$= 2827{\cdot}44$,, ,,
Area of circular fountain	$= 3{\cdot}1416 \times 3^2$,, ,,
	$= 28{\cdot}2744$,, ,,
Area of plot	$= 2827{\cdot}44 - 28{\cdot}2744$ square yards
	$= 2799{\cdot}1656$,, ,,
or area of plot . . .	$= \pi(r_1 + r_2)(r_1 - r_2) = (3{\cdot}1416)(30+3)(30-3)$
	$= 3{\cdot}1416 \times 33 \times 27$ square yards
	$= 2799{\cdot}1656$
cost at 4*d.* per square yard	$= 933{\cdot}0552$ shillings
	$=$ **£46, 13*s*. 0·7*d*.**

Examples 103.

1. Find the circumference of a circle whose diameter is 1·96 yds.
2. ,, ,, ,, ,, ,, 28·342 miles.
3. ,, ,, ,, ,, ,, 16·34 cms.
4. ,, ,, ,, ,, radius is 2·54 ft.
5. ,, ,, ,, ,, ,, 30·6 metres.
6. How many shrubs placed 3 yds. apart can be placed round the edge of a semicircular lawn, the radius of which is 144 ft.?
7. Assuming that 10,000,000 metres is equal to one-fourth of the earth's circumference, find the length of the earth's radius in (1) kilometres, and (2) miles, each answer correct to 4 significant figures.
8. How much linoleum would be required to cover the floor of a semicircular recess, the diameter of which is 4 yds. 1 ft.?
9. The diameter of a shilling is 2·4 cms., and of a florin 3 cms. How much larger is the surface of the one than of the other, neglecting the rims?
10. A square whose side is 500 ft. has a circular garden within it 400 ft. in diameter. What would it cost to pave the part outside the garden at 2*s*. per sq. yd.?
11. A circular bicycle track, 22 ft. wide, and a quarter of a mile round if measured 1 ft. from the inner edge, is to be laid out on a square field. What will be the area, in acres, of the smallest suitable field?
12. Find the diameter of a circle whose area is 1 acre.
13. Find the area of a circular ring whose internal and external diameters are 8 ft. and 15 ft. respectively.

14. The rope by which a calf is tethered is increased from 12 to 16 ft. How much additional ground has it to browse over?

15. A circular plate has a diameter of 12 metres. A sector, the arc of which measures 4·75 metres, is cut out. What area of the plate remains?

16. The diameter of a circle is 13·5 metres. What is the area of a sector of it, the angle at the centre being 60°?

17. A segment is cut from a circular disc of 8·4 cms. diameter. If the arc of the segment measures 2·6 cms., and the chord 2·64 cms., what is the area of the remaining portion?

18. Find the surface area of a sphere, the diameter of which is 12 ft. 6 in.

19. Find the surface area of a sphere, the diameter of which is 16·4 metres.

20. What will it cost to paint a hemispherical dome, the diameter of which is 10 yds. 1 ft., @ 3*s.* per sq. yd.?

21. A painter charges £2, 10*s.* for painting a semicircular panel, the diameter of which is 10 ft. What is the charge per square foot?

22. Find the diameter of a circle, the area of which is 684 sq. ft. Answer correct to 4 significant figures.

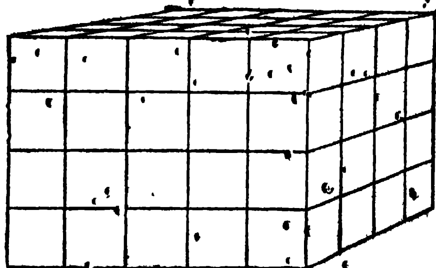
23. Find the diameter of a sphere, the surface area of which is 156 sq. cms.

24. If a grain of gold, when beaten into gold leaf, will cover 67½ sq. in., what will be the weight of gold required to cover a hemispherical dome, the diameter of which is 16 yds.?

XL.—SOLIDS.

344. Rectangular Solid.—

In paragraph 336 it was shown that the area of a rectangle was found by multiplying the length by the breadth. Suppose we



build up a solid, which has the rectangle as one of its faces, by placing cubic inch blocks on each of the squares of the figure. It is evident that we should require 20 of these blocks, and the

volume of the solid should be 20 cubic inches. If we now place another set of these blocks on the first, we shall have 40 cubic inches, and the volume will be 40 cubic inches, or 20×2 cubic inches. If a third set be added the volume will be 60, or 20×3 cubic inches, and with a fourth the volume becomes 80, or 20×4 cubic inches. It will thus be seen that the volume of a rectangular solid is obtained by multiplying the length by the breadth by the height, *i.e.*

$$V = l \times b \times h$$

and given the values of any three of these, we can find the fourth.

345. A **cube** is a special form of rectangular solid in which the length, breadth, and thickness are equal, and therefore the volume is the cube of any one of them, *i.e.*

$$V = a^3.$$

346. Prisms and Cylinder :—

A **prism** is a solid, two of whose faces (the bases) are equal and parallel figures, and the other faces are parallelograms. When the two equal and parallel faces are regular figures, and the other faces are rectangles, the solid is a **regular prism**. The rectangular solid and cube are particular forms of the regular prism. The volume of a prism is obtained by multiplying the area of the base by the height.

$$V = Ah.$$

347. The **Cylinder** is a special form of prism in which the base is a circle, and its volume is obtained by multiplying the area of the circular base by the height, *i.e.*

$$V = Ah \text{ i.e. } \pi r^2 h.$$

Examples—

(i.) *Find the volume of a tank, the length being 60 ft., breadth 40 ft., and depth 45 ft.*

$$V = l \times b \times h = 60 \text{ ft.} \times 40 \text{ ft.} \times 45 \text{ ft.}$$
$$= 108000 \text{ cubic feet.}$$

(ii.) *A block of granite is 17·7 ft. broad and 9·4 ft. thick. What length must be cut off so as to contain 554·6 cubic feet?*

$$V = l \times b \times h.$$
$$\frac{V}{b \times h} = l.$$
$$l = \frac{554{\cdot}6 \text{ cub. ft}}{17{\cdot}7 \text{ ft.} \times 9{\cdot}4 \text{ ft.}} = 3 \text{ ft. } 4 \text{ in.}$$

Examples—

(iii.) *Find the edge of a cubical block containing* 29 *cub. yds.*, 23 *cub. ft.* 219 *cub. in.*

Volume = 29 cub. yds., 23 cub. ft., 219 cub. in.
= 1860867 cub. in.
but Volume = a^3
$\therefore a = \sqrt[3]{1860867 \text{ cub. in.}}$
= 123 in.
= 3 yds. 1 ft. 3 in.

(iv.) *Find the cubical content of a cylinder, the radius being* 8 *cms. and the height* 1 *metre.*

Volume = $\pi r^2 h$
= $3{\cdot}1416 \times 8^2 \times 100$
= **20106·24 c. cms.**

(v.) *How many bricks* 9 *ins. by* $4\frac{1}{2}$ *ins. by* 3 *ins. will be required in building a wall* 15 *yds. long by* 8 *ft. high and* $13\frac{1}{2}$ *ins. thick?*

$$\text{No. of bricks} = \frac{V.\text{ of wall}}{V.\text{ of 1 brick}} = \frac{15 \text{ yds.} \times 8 \text{ ft.} \times 13\frac{1}{2} \text{ ins.}}{9 \text{ ins.} \times 4\frac{1}{2} \text{ ins.} \times 3 \text{ ins.}}$$

= **5760 bricks.**

Examples 104.

Find the volumes of the following solids—

1. Rectangular prism; length, 6 ft.; breadth, $4\frac{1}{2}$ ft.; height, $6\frac{1}{4}$ ft.
2. Rectangular prism; length, 12 ft.; breadth, $6\frac{3}{4}$ ft.; height, $8\frac{1}{4}$ ft.
3. Rectangular prism; length, 4 yds. 2 ft.; breadth, 4 yds. 2 ft.; height, 8 yds.
4. Rectangular prism; length, 4·6 cms.; breadth, 2·3 cms.; height, 5·7 cms.
5. Rectangular prism; length, 7·25 cms.; breadth, 5·34 cms.; height, 10·5 cms.
6. Cube; sides, 2 ft. 3 in.
7. Cube; sides, 6 ft. $4\frac{1}{2}$ in.
8. Cube; sides, 2 yds. 3 in.
9. Cube; sides, 6·4 cms.
10. Cube; sides, 6·35 metres.
11. Triangular prism; sides, $4\frac{1}{2}$ ft., $3\frac{1}{2}$ ft., $6\frac{1}{4}$ ft.; height, 6 ft.
12. Triangular prism; sides, 3 yds. 1 ft., 2 yds. 1 ft. 6 in., 2 yds. 1 ft. 9 in.; height, 3 yds.

13. Triangular prism; sides, 6·4 cms., 7·3 cms, 5·6 cms.; height, 7·25 cms.

14. Right-angled triangular prism; sides, $6\frac{3}{4}$ in., $8\frac{1}{2}$ in.; height, 7 in.

15. Right-angled triangular prism; sides, $4\frac{1}{2}$ ft., 3 ft. 7 in.; height, 6 inches.

16. Right-angled triangular prism; sides, 5·43 cms., 8·26 cms.; height, 12·25 cms.

17. Cylinder; radius, $4\frac{1}{2}$ in.; height, 6 in.

18. Cylinder; radius, 3·75 cms.; height, 8·4 cms.

19. Cylinder; diameter, 4 ft. 3 in.; height, 12 ft 9 in.

20. Cylinder; diameter, 2·45 cms.; height, 1·3 metres.

21. A cistern is 13 ft. 4 in. long, 11 ft. 3 in. wide, and 4 ft. 2 in. deep. How many cubic feet of water will it contain?

22. The length, breadth, and thickness of a rectangular block of stone are proportional to 4, $2\frac{1}{3}$, $1\frac{1}{2}$; the volume is 24,192 cub. in. Find the dimensions.

23. How many bricks, each 9 in. long, $4\frac{1}{2}$ in. broad, and $2\frac{1}{2}$ in. thick, will be required to build a wall $1\frac{1}{2}$ miles long, 8 ft. high, and $2\frac{1}{2}$ ft. thick?

24. A room is 49 ft. 6 in. long, 37 ft. 4 in. broad, and 13 ft. 2 in. high. What is the largest number of persons it can hold, so that each shall have at least 80 cub. ft. of air?

25. A wall is covered with plaster $1\frac{1}{4}$ in. thick. How many square yards will be covered by a cubic yard of plaster?

26. Determine the height in feet of a cubical cistern which contains 3,581,577 cub. in. of water.

27. What length must be cut from a block of wood 10 in. by $4\frac{1}{2}$ in to form a cubic foot?

28. A reservoir is 56 ft. 8 in. long by 17 ft. 6 in. broad. How many cubic feet of water must be drawn off to make the surface sink 2 ft. 6 in.?

29. The length of a rectangular room is 23·7 ft.; its breadth is 18·4 ft. It contains 5000 cub. ft. Calculate its height in feet, correct to 3 significant figures.

30. The dimensions of a cistern are 12, 9, and 6 ft. What must be the height of a second cistern equal in content to the first, supposing the bottom of the second cistern to be a square having its sides 9 ft. long?

31. Find the length, in feet and decimal of a foot, to 3 places, of the edge of a cube, the volume of which is 4397 cub. ft.

32. A block of steel 1 ft. 3 in. long, 10 in. broad, and 9 in. deep is rolled out into a rod which has a uniform section of 1·08 sq. in. Find the length of the rod in yards, feet, and inches.

33. A closed packing case measures externally 32, 25·5, and 15·5 cms., the wood being 0·72 cms. thick and the weight 50 kilogrammes. Find the weight of a cubic decimetre of the wood.

34. A rectangular block of granite is 7 ft. 8 in. long, 3 ft. 4 in. broad, and 2 ft. 10 in. thick. What is the cost at 5*s*. 9*d*. per cubic foot?

35. A block of stone 4 metres long, 2·5 metres broad, and 1·2 metres thick, weighs 40 tonneaux. Find the weight, in grammes, of a cubic decimetre of the stone.

36. A litre contains a cubic decimetre. Taking a metre as 3·28 ft., and a gallon as containing 0·16 cubic ft., find what decimal fraction a litre is of a gallon, correct to 2 decimal places.

Examples 105.

1. Find the inner edge of a cubical vessel which holds $1398665\frac{71}{216}$ oz. of water, given that a cubic foot of water weighs 1000 oz.

2. The base of a regular prism is an equilateral triangle with a side of 7 in. and its height is 24 in. Find its cubic contents.

3. Assuming that "a pint of pure water weighs a pound and a quarter," and that a gallon contains $277\frac{1}{4}$ cub. in., find by how much the weight of a cubic foot of water differs from 1000 oz. Express answers in ounces, to 2 places of decimals.

4. The water in a reservoir 26 ft. 6 in. long and 18 ft. 9 in. wide is 6 ft. deep. Find in tons, cwts., &c., the weight of the water, assuming that a cubic foot of water weighs 1000 oz.

5. A gallon of water weighs 10 lbs. and a cubic foot of water weighs 1000 oz. If a rectangular cistern be 6 ft. long and 4 ft. broad and contain 200 galls., what will be the depth of the water in the cistern?

6. A gallon of water weighs 10 lbs. avoirdupois and a cubic foot of water 997 oz. Find how many gallons are contained in a cistern 12 ft. long, 10 ft. broad, and 6 ft. deep.

7. A cubic foot of water weighs 1000 oz. If a piece of ground, an acre in extent, be flooded by 2000 tons of water, find the average depth of the water in inches, correct to 2 decimal places.

8. If a gallon contain 277·29 cub. in., how many gallons will be contained in a rectangular cistern whose length, breadth, and depth are 13 ft., 7 ft. 6 in., and 6 ft. 7 in.?

9. A rectangular tank is 14 ft. long and 10 ft. wide. What must be the depth, that it may contain $31\frac{1}{4}$ tons of water?

10. A tank, 56 ft. long and 16 ft. wide, has supports capable of bearing safely 77 tons of water. What is the greatest depth of water that can be safely allowed if a cubic foot of water weighs 1000 oz.?

11. Assuming that a cubic foot of water contains $6\frac{1}{4}$ galls., find how many gallons a reservoir will contain, the sides of which are 41 yds. 2 ft., 30 yds., and depth 80 ft.

12. A marble slab 6 ft. 3 in. long, 2 ft. 8 in. broad, and 4 in. thick, and weighing 8 cwt. 1 qr. 20 lbs., cost £4, 0*s*. $6\frac{3}{4}$*d*. How much was the cost per cubic foot, and what is the weight of 1 cub. ft. of marble?

13. Find the cubical contents of a block of stone 17 ft. 9 in. long, 14 ft. 3 in. broad, and 5 ft. 6 in. thick, and its price at 4*d*. per cubic foot.

14. A cube contains 1 953125 cub. ft. Express the area of its complete surface in square inches.

15. A cubical box contains 9261 cub. in. Find the expense of gilding the inside at $1\frac{1}{2}d$. per square inch.

16. The sum of the areas of the six faces of a cube is 94 yds. 6 ft 6 in. Find the edge of the cube in feet and inches.

17. The diagonal of a cube is $21\sqrt{5}$ ft. Find the cost of painting its surface at 4*d*. per square yard.

18. Find the thickness and solid contents of an armour plate 22 ft by 15 ft., weighing 25 tons, having given that $5\frac{1}{2}$ cub. ft. of iron weigh a ton.

19. What is the value of a circular pillar whose diameter is 5 ft. and height 17 ft., at 9*d* per cubic foot?

20. A cubic foot of water weighs 1000 oz. and a gallon weighs 10 lbs. Find in inches, correct to 4 significant figures, the depth of a cylindrical vessel which contains $2\frac{1}{2}$ galls., and whose internal diameter is 10 in.

21. A cistern, 10 ft. long, 7 ft. broad, and 5 ft. deep, is filled with water which runs into it at the rate of 8 ft. per second, through a pipe whose diameter is one inch. How long will it take to fill?

22. What must be the depth of a cylindrical tank to hold 50,000 galls.; its diameter being 12 ft.; given that a gallon contains 277·274 cub. in.?

23. A lump of iron containing 4 cub. ft. is rolled into a bar 16 yds. long. What will be the diameter of the bar?

24. A lump of iron containing 11 cubic decimetres is drawn out into a cylindrical rod 15 metres long. Find the thickness of the rod.

25. A rectangular cistern 4 ft. 2 in. long, 2 ft. 8 in. wide, and 2 ft. deep is one-third full of water. Find the least number of heavy cubical blocks, each measuring 6 in. in every way, that can be introduced into the cistern to make the water *just* overflow.

348. Pyramids and Cone:—

A pyramid is a solid, all, or all but one, of whose faces are triangles. When one of the faces is not a triangle, that face is called **the base**; in other cases, any of the triangles may be taken as base. **A regular pyramid** is one whose base is a regular rectilineal figure, and the line joining the common vertex of the triangles to the middle point of the base is perpendicular to the base.

349. The volume of any pyramid is one-third of that of the corresponding prism, *i.e.*

$$V = \frac{Ah}{3}.$$

A cone is a special form of pyramid, and its volume is one-third of that of the corresponding cylinder, *i.e.*

$$V = \frac{\pi r^2 h}{3}.$$

The volume of a **sphere** is obtained from the following formula—

$$V = \frac{4}{3}\pi r^3.$$

350. The volume of a **solid circular ring** is obtained by multiplying the area of its cross section by the mean of the inner and outer circumferences.

When any portion of a solid is required, the student should adopt the method of § 341, or from a rough diagram find the shape of the new solid.

Examples—

(v.) *Find the volume of a square pyramid, the side of the base of which is 6 ft. and height 8 ft.*

Area of base = 6 ft. × 6 ft. = 36 sq. ft.

$$\text{Volume of pyramid} = \frac{Ah}{3} = \frac{36 \text{ sq. ft.} \times 8 \text{ ft.}}{3}$$

$$= \textbf{96 cubic feet.}$$

(vi.) *Find the volume of a sphere in cubic inches, correct to 4 significant figures, the radius of which is 8 inches.*

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.1416 \times (8 \text{ ins.})^3 \quad (\text{1·0472})$$

$$= 4 \times 1{\cdot}0472 \times 512 \text{ cub. ins.}$$

$$= \textbf{2145 cubic inches.}$$

(vii.) *Find the volume of a regular cone, the slant height being* **13** *inches and the radius* 5 *inches.*

The slant height, radius, and perpendicular height form a right-handed triangle, of which the slant height is the hypotenuse.

$$\text{Then } a^2 = b^2 + c^2$$

$$a^2 - b^2 = c^2$$

$$(a+b)(a-b) = c^2$$

$$(13 \text{ in.} + 5 \text{ in.})(13 \text{ in.} - 5 \text{ in.}) = c^2$$

$$144 \text{ sq. in.} = c^2$$

$$12 \text{ in.} = c$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3{\cdot}1416 \times 25 \text{ sq. in.} \times 12 \text{ in.}$$

$$= \textbf{314·16 cubic inches.}$$

Examples 106.

1. A spire is completed by a stone in the form of a square pyramid, the side of the base being 1 ft. 3 in. and the height 2 ft. 2 in. Find the volume of the stone.

2. The stone at the top of a conical spire has a base 15 in. in diameter and a height of 2 ft. 6 in. Find the weight of the stone if a cubic foot of the material weighs 56 lbs.

3. Find the volume of a regular cone, the slant height being 20 ft and the radius of the base 12 ft.

4. A circular plate of lead 2 in. thick and 8 in. in diameter, is converted without loss of weight into spherical shot, each of 0·05 in. diameter. How many shot does it make?

5. A piece of metal 5·12 in. long, 3·43 in. wide, and 1·25 in. thick, is melted and recast into a perfect sphere. Find the radius of this sphere, and the difference made in the amount of surface. $\sqrt[3]{\frac{4}{3}\pi}=1{\cdot}6$.

6. What are the cubical contents of a cone, the area of the base being 80 sq. in. and the height 10 ft.?

7. The circumference of the base of a conical mound of earth is 25 metres, and the height 650 cms. How much would it cost to cart it away at one florin per cubic metre?

8. In a square pyramid the side of the base is 15 ft. and the height 24 ft. Find the volume.

9. The three sides of the base of a triangular prism are 12, 13, and 14 ft. respectively, and the height is 16 ft. Find the volume.

10. If the diameter of the earth is taken as 7960 miles, what is its volume, correct to 4 significant figures, when considered as a sphere?

11. Find the volume of the metal required for an anchor ring, the inner radius being 15 in. and the outer 18 in.

12. What is the height of a cone whose volume is 16·4 c. cms. if the diameter of the base is 5 cms.?

Examples 107.

1. Find the area of a quadrilateral plot, whose sides taken in order are 24, 32, 35, and 25 ft. respectively, the angle contained by the first two sides being a right angle.

2. The diameter of a circular plot is 78 ft., a walk a yard wide runs round it. Find the area of the walk and the cost of laying it with gravel, at the rate of $4\frac{1}{2}d.$ per sq. ft.

3. In forming a circular tank 21 ft. in diameter, the soil removed was spread over a neighbouring plot of ground 84 ft. long by 60 ft. broad, thereby raising its surface 3 ft. Find the depth of the tank.

4. The inner diameter of a circular fort is 43 ft. 2 in., and the thickness of the wall is 3 ft. 6 in. Find the amount of stone in the wall, its height being 12 ft.

5. The sides of a field taken in order are 36, 48, 60, and 72 yds. respectively, and the first two sides are at right angles to each other. Determine the area of the field.

6. Within a circle 154 ft. in circumference a square plot is placed so that its corners touch the circumference of the circle. Find the cost of turfing it at 3s 6*d.* per sq. yd. Find also the amount of gravel necessary to cover the remainder of the circle, supposing 1 ton of gravel is used to cover 20 sq. yds.

7. One of the equal sides of an isosceles triangle is to the third side in the ratio of 3 : 2. The sum of the 3 sides being 40 ft., find (correct to 2 decimals) the area of the triangle in square feet.

8. The diameter of a circular tank is 7287 ft. How many times will a bicycle wheel turn in going three times round it, the radius of the bicycle wheel being 1 ft. 2 in.?

9. A straight reed is observed, when there is no wind, to project at a point *P*, 5 in above the water. A wind arises and it is driven away and just disappears at a distance of 30 in. from *P*. What is the depth of the water?

10. The diagonals of a rhombus being 88 and 234 ft., find (*a*) the area, (*b*) the length of a side, and (*c*) the height of the rhombus.

11. Find the area (correct to 3 places of decimals) of an equilateral triangle inscribed in a circle whose radius is 8 ft.

12. A bicycle track is circular. The outer circle is $1885\frac{5}{7}$ ft. in circumference, while the inner circumference is $1697\frac{1}{7}$ ft. Find the breadth of the track and the cost of paving it at 1*s.* 2*d.* per sq. yd. (take $\pi = \frac{22}{7}$).

13. Find the cost of enclosing, with a wire fence at 2*s.* 6*d.* per yd., a regular hexagonal plot whose area is 1039·2 sq. yd.

14. A square pedestal stands on a circular plot, whose diameter is 14 ft., in such a way that its corners are 1 ft. from the edge of the circle. Find the area of remainder of plot.

15. The volume of a cylindrical rod of metal is 5 cub. ft., and the area of a section is 1000 sq. in. What is its length?

Examples 108.

The following *pro-forma* invoice may be worked out in sections.

THE METROPOLITAN TUBE WORKS,
BRIXTON ROAD,
LONDON, *25th Sept.* 1901.

MESSRS. FRASER & CO.

To JAMES MUIRHEAD & CO, LIMITED,
Manufacturers of
Iron Tubes for Marine, Locomotive, Stationary, and Portable Boilers.

		£	s	d	
6 Lens. Galv. Tube, 4′ × 15′ = ft	5/6				72½%
6 ,, ,, ,, 3½′ × 15′ = ,,	4/6				,,
48 ,, ,, ,, 2½′ × 15′ = ,,	3/-				,,
24 ,, ,, ,, 2¼′ × 15′ = ,,	2/6				,,
24 ,, ,, ,, 3″ × 15′ = ,,	3/6				,,
48 ,, ,, ,, 1½′ × 15′ = ,,	1/3				70%
48 ,, ,, ,, 2″ × 15′ = ,,	1/9				,,
12 ,, ,, ,, 1¼″ × 15′ = ,,	1/-				,,
12 ,, ,, ,, 1″ × 15′ = ,,	8½*d.*				,,
12 ,, ,, ,, ¾″ × 15′ = ,,	6*d*				,,
11/6 10/- 8/6 5/- 4/-					
12 Flanges, Galv., each 4′ 3½′ 3″ 2½′ 2¼	39/-				72½%
2/9 2/- 1/9 1/4 1/2					
12 ,, ,, ,, 2″ 1½″ 1¼′ 1′ ¾	9/-				70%
6/- 5/- 3/4 2/6 1/9					
12 Sockets, Galv., each 4″ 3½″ 3″ 2½′ 2¼″	18/9				72½%
1/1 8½*d.* 6½*d.* 4½*d.* 3½*d*					
12 ,, ,, ,, 2″ 1½ 1¼ 1′ ¾′	3/-				70%
28/- 22/- 14/- 9/- 6/3					
12 Elbows, Galv., each 4′ 3½″ 3″ 2½″ 2¼″	79/3				72½%
3/10 2/5 1/10 1/4 1/-					
12 ,, ,, ,, 2″ 1½″ 1¼″ 1″ ¾″	10/5				70%
5/6 4/6 3/6 2/3 1/9					
24 Jam Nuts, Galv., each 4′ 3½″ 3″ 2½″ 2¼″	17/6				72½%
1/1 8*d.* 3*d.* 5*d.*					
24 ,, ,, ,, 2′ 1½′ 1¼″ 1″	2/8				70%
3½*d*					
36 ,, ,, ,, ¾″	3½*d.*				,,
32/6 25/- 18/- 12/-					
6 Bends, each 4″ 3½″ 3″ 2½″	87/6				80%
5/-					
12 ,, ,, 2″	5/-				77½%
Forward					

		£	s.	d.	
Forward . .					
6 Lens. Tube, 3½" × 15′ = ft. . .	4/6				80%
12 ,, ,, 3" × 15′ = ,, . .	3/6				,,
12 ,, ,, 2½" × 15′ = ,, . .	3/-				,,
12 ,, ,, 2¼" × 15′ = ,, . .	2/6				,,
24 ,, ,, 2" × 15′ = ,, . .	1/9				77½%
24 ,, ,, 1½" × 15′ = ,, . .	1/3				,,
24 ,, ,, 1¼" × 15′ = ,, . .	1/-				,,
24 ,, ,, 1" × 15′ = ,, . .	8½d.				,,
24 ,, ,, ⅞" × 15′ = ,, . .	8½d				,,
24 ,, ,, ¾" × 15′ = ,, . .	6d.				,,
48 ,, ,, ⅝" × 15′ = ,, . .	6d				,,
12 Lens. Hyd. Tube, 2¼" × 15′ = ft.	1/3				Net
12 ,, ,, ,, 2" × 15′ = ,,	1/1				,,
12 ,, ,, ,, 1" × 15′ = ,,	10d.				,,
12 ,, ,, ,, ⅞" × 15′ = ,,	9d.				,,
24 ,, ,, ,, ⅝" × 15′ = ,,	8d.				,,
10/- 8/6 5/- 4/- 12 Flanges, each 3½" 3" 2½" 2¼" . .	27/6				80%
2/9 2/- 1/9 1/4 1/4 12 ,, ,, 2" 1½" 1¼" 1" ⅞"	9/2				77½%
1/2 1/2 12 ,, ,, ¾" ⅝"	2/4				,,
5/- 3/6 2/6 1/9 24 Sockets, each 3½" 3" 2½" 2¼" . . .	12/9				80%
1/1 8½d. 6½d. 4½d. 4½d. 3½d. 3½d 24 ,, ,, 2" 1½" 1¼" 1" ⅞" ¾" ⅝"	3/8				77½%
22/- 14/- 9/- 6/3 12 Elbows, each 3½" 3" 2½" 2¼" . . .	51/3				80%
3/10 2/5 1/10 1/4 1/4 1/- 1/- 12 ,, ,, 2" 1½" 1¼" 1" ⅞" ¾" ⅝"	12/9				77½%
4/6 3/6 2/3 1/9 36 Jam Nuts, each 3½" 3" 2½" 2¼" . .	12/-				80%
1/1 8d. 6d. 5d. 5d. 3½d. 3½d. 36 ,, ,, 2" 1½" 1¼" 1" ⅞" ¾" ⅝"	3/8				77½%
72½% £364, 4s. 0d. = £					
70 % £145, 1s. 0d. = £					
80 % £211, 13s. 0d = £					
77½% £153, 1s. 0d. = £					
Net £47, 5s. 0d. = £					
2½%					
F.O.B., London.		£			
Lengths are assumed at 15 feet each for making up this invoice, but of course actual lengths would be charged at same rates.					

Miscellaneous Examples.

1. Divide 9803·1 by 0·0797.

2. Resolve 159201 into its prime factors.

3. Calculate the premium of insurance on £3573, 7s. 6d. at the rate of £2, 16s. 8d. per cent.

4. Find the value of £384, 17s. 6d. in francs, supposing the course of exchange is £1 = 26·04 francs.

5. If 16 cwts. 2 qrs. 12 lbs. of sugar cost £30, 3s. 9d., how much should I get for £17, 2s. $1\frac{1}{2}$d.?

6. Calculate the bankers' discount on £387, 10s. 6d. due on July 12th and discounted on April 18th, at $5\frac{1}{2}$ per cent. per annum.

7. Which investment would yield the better return—the three and a half per cents. at $92\frac{1}{8}$ or the three per cents. at 86? Find the difference from investing £2303, 2s. 6d. in each.

8. £350 is to be raised by taxation in a town the rateable value of which is £21,658. If the expenses of collecting the tax amount to £20, find to the nearest sixteenth of a penny the tax that must be levied to secure the desired amount, and also find what surplus will remain if all the taxes save £15 be paid.

9. A merchant sells an article which costs him 16s. 8d. for 17s. $0\frac{3}{4}$d. If his marked prices are 5 per cent. above cost price, what rate of discount does he allow?

10. Calculate to 4 significant figures the weight (in tons) of water in a section of a canal 10 miles long, its breadth being 20 feet and the depth averaging $5\frac{1}{2}$ feet. A cubic foot of water weighs $62\frac{1}{2}$ lbs.

11. Extract the square root of 6789265609.

12. What decimal multiplied by 7 would give the sum of 0·03, 14·36, 8·14, 12·5, 0·003, 8·4?

---

13. Reduce 16 miles 351 yards 2 feet to inches.

14. A chest of tea containing 115 lbs. was sold at 3s. $4\frac{1}{2}$d. per lb. Calculate what it must have cost if there was a clear profit of £1, 7s. $6\frac{1}{2}$d.

15. Divide half a guinea between 4 persons in proportion to the numbers 3, $3\frac{1}{2}$, 4, and $4\frac{1}{2}$.

16. How many sacks of flour will support 500 men during a siege of 21 days when in ordinary time 14 sacks will support 35 men for 10 days, supposing that during the siege the men's rations are reduced by $\frac{1}{5}$?

17. A bankrupt has liabilities amounting to £5373, 10s., while his assets amount to £813, 17s. 6d. Preference claims take up £125, 10s. of his assets and the expenses of the winding up amount to £32, 7s. 6d. What dividend can be declared?

18. Calculate to the nearest penny the simple interest on £345, 10s. 6d. from 3rd September to 18th January, at 4 per cent. per annum.

19. Find a multiplier to convert shillings per stone into marks per kilogramme (1 kilo. = 2·204 lbs., £1 = 20·43 marks).

20. What income is derived from investing £10,400 in three per cent. stock at 78, after deducting 1*s.* 2*d.* in the £ for income tax?

21. Find the amount a person ought to pay just now if he owes £238, 12*s.*, due at the end of 5 months, reckoning money to be worth $3\frac{1}{2}$ per cent. per annum.

22. Find the weight of a block of marble whose length is 4 ft. 6 in., depth 3 ft. 6 in., and width 2 ft. 9 in., when a cubic foot of the stone weighs 180 lbs.

23. Find, by logarithms, the cube root of 517, correct to 4 significant figures.

24. A house built for £664 is sold for £830; what is the gain per cent.? If it had been built for £830 and sold for £664, what would be the loss per cent.?

---

25. Multiply 62·12854 by 3·976875, correct to 5 significant figures.

26. A rectangular field, whose length is 3 times its breadth, contains 45 acres. Calculate its breadth to the nearest foot.

27. Find the value of $\frac{14{\cdot}44 \times 0{\cdot}0133}{0{\cdot}0057 \times 1{\cdot}805}$.

28. Find the cost of 3175 articles at £4, 6*s.* $10\frac{1}{4}$*d.* each.

29. Determine the height in feet of a cubical cistern which contains 3581577 cubic inches of water; and assuming that a cubic foot of water contains $6\frac{1}{4}$ gallons, find how many gallons the cistern will contain correct to three places of decimals.

30. If 18 tons of coal are carried 250 miles by rail for £7, 10*s.*, how much will it cost to send 280 tons a distance of 360 miles by ship whose charges are $\frac{3}{10}$ of the charge by rail?

31. Express a quotation of 3*s.* 6*d.* per lb. troy in marks per kilogramme, given that £1 = 20·4 marks and 2·679 lbs. = 1 kilogramme.

32. How many hours have elapsed between 6 A.M. on 1st April and 25th August at 9.30 P.M.

33. Find the simple interest on £3595, 5*s.*, at $3\frac{1}{4}$ per cent. from 2nd January 1892 to 3rd May 1892.

34. A mass of auriferous sand weighing 12 cwt. (avoir.) is known to contain 3·8 per cent. of gold. From it $\frac{11}{12}$ of the sand is removed by washing and the part removed is found to contain 0·6 per cent. of gold. How many lbs. troy of gold are contained in the remaining cwt.?

35. A tax of £530 is to be raised from three towns, the rentals of which are respectively £25,000, £30,000, and £42,000. How much should each town pay?

36. What is the rate per cent. per annum if a sum of money doubles itself in 17 years at compound interest?

---

37. Find the value of 0·03125 of a mile at 5s. per yard.

38. Find the eighteenth power of 3.

39. Four men rent a field. A puts in 14 oxen and keeps them for 6 weeks, B 10 oxen for five weeks, C 15 oxen for 10 weeks, and D 8 oxen for 7 weeks. If A's share of the rent is 8 guineas find the total rent and the amount each of the other men pay.

40. From $9\cdot\dot{6}$ of £1 take $5\cdot\dot{0}3\dot{7}$ of a guinea and express the result in decimals of a shilling.

41. A house is valued at £282, 6s. 6d., being an increase of 15 per cent. on its purchase price. Calculate the purchase price.

42. Express in English money, correct to the nearest penny, the amount of the following bill in French money:—3·35 articles @ 6·37 francs, 23·63 @ 1·34 francs, 15·37 articles @ 7·15 francs. (Exchange: £1 = 25·42 francs.)

43. If a dishonest dealer uses a measure of 7·8 pints instead of a gallon, what per cent. does he gain by the fraud?

44. Find the thickness and solid contents of an armour plate 22 ft. by 15 ft., weighing 25 tons, given that $5\frac{1}{2}$ cubic feet of iron weigh 1 ton.

45. Find the difference between the true and the banker's discount on £2020 for three months, at 4 per cent. per annum. Show that the difference is equal to the interest on the true discount for 3 months at 4 per cent.

46. A man deposits £5000 in a bank paying $2\frac{1}{2}$ per cent. compound interest, and invests a like sum in the four per cents., at £$133\frac{1}{3}$. At the end of how many years will the accumulated interest at the bank exceed the sum of all the dividends from the stock?

47. What is a perpetual annuity of £150 worth, reckoning interest at 4 per cent. per annum?

48. Find, by duodecimals, the volume of air contained in a room 17 feet 3 inches long, 14 feet 7 inches wide, and 11 feet 2 inches high. What will the answer become when expressed in cubic feet and cubic inches?

49. Evaluate to 3 decimal places $\dfrac{937\cdot518 \times 0\cdot146 \times 0\cdot03}{2\cdot772 \times 143\cdot36}$.

50. If the imperial gallon contains 277·27 cubic inches, and a cubic foot of water weighs 1000 ounces, find the weight of a pint of water.

51. If a metre equals 3·29 feet, how many miles will a man walk in 9 hours 46 minutes 40 seconds, at the rate of 6 kilometres per hour?

52. If the sides of a rectangle, correct to three decimal places, are 6·324 and 14·527 inches, find the area in square inches correct to two decimal places.

53. A cubic centimetre of a certain metal weighs 15·64 grammes, and a cubic centimetre of another weighs 18·35 grammes. Find the weight in kilogrammes of a cubic decimeter of an alloy of 3 parts by weight of the former, and 5 parts by weight of the latter.

54. Express 64·4136 as an improper fraction in its lowest terms.

55. Coffee at £5, 12s. 6d. per cwt. and chicory at £2, 5s. 5d. per cwt. are mixed in the proportion of 2 of chicory to 5 of coffee. If the mixture is retailed at 1s. 3d. per lb., calculate the gain per cent.

56. If I can buy 8 apples and 21 oranges for 9d., or 4 apples and 12 oranges for 5d., find the price of apples and oranges per dozen.

57. Find the duty at 4d. in the pound payable on 320 chests of tea averaging 2 cwts. 23 lbs. in weight.

58. Calculate the brokerage at $\frac{1}{8}$%, on a sale of stock at 97$\frac{3}{4}$ if the total money realised was £2443, 15s.

59. Find the cube root of 1342·17728, correct to 4 significant figures.

60. In six years a merchant had 12 bankrupts among his customers. 2 paid 6s. 8d. in the pound, 4 paid 10s. 4d., 4 paid 16s. 8d., 1 paid 17s. 6d., and the last 5d. Find the average loss per £ that the merchant suffered through these bankruptcies, supposing that each bankrupt owed the same amount.

---

61. Find the value of 56,439,281 ÷ 357, correct to 3 significant figures.

62. Find the cube root of 64532984·7, correct to 3 significant figures.

63. Find the sum of all the prime numbers between 0 and 60.

64. Calculate a traveller's commission on a sale amounting to £35, 12s. 6d., at 7$\frac{1}{2}$ per cent.

65. It took 4 men, working 9 hours a day, 5 days to unload 36 tons of metal. How many tons could 5 men have unloaded in 8 days of 10 hours each?

66. In the second innings of South Australians v. MacLaren's XI. the following scores were made:—One player made 7 runs, one 41, one 80, one 11, one 31, one 19, one 2, one 6, and one 4, the remaining 2 being dismissed for 0. The extras counted 6. Find the average number of runs for each player.

67. A tradesman fails for £6687, but managed to pay 18s. 8d. in the £. What was his estate worth?

68. A man insures a house at a premium of 4 guineas per cent., endeavouring to arrange so that he shall recover cost of building and insurance in case of loss. He insures every £100 of the building as if it were valued at £104$\frac{1}{5}$. What is the amount of his actual loss if the house is valued at £1000?

69. Divide £794, 12s. 10$\frac{1}{2}$d. by 385, and express the answer in £'s, florins, cents, mils.

70. A and B, who trade in opposition, resolve to form a partnership. On the date of entry on the agreement A transfers his stock of 179 cwts. 3 qrs. 25 lbs. of flour, valued at £3, 12s. 3d. per cwt. on the average, to B's premises, where B already has 212 cwts. 3 qrs. 19 lbs. of an inferior quality, valued at £1, 13s. 2d. per cwt. Besides this each puts £100 into the business. When they have gained £400 they divide it proportionally. Find how much each should get.

71. A man, at the age of 37, pays a society the sum of £100, for which he is to receive an annuity of £35, beginning when he is 70. He dies after the second payment is made How much does the society gain, compound interest at the rate of 3 per cent. per annum being calculated on the money ?

72. A merchant having bought 23, 24, and 25 quarters of wheat at 38*s*., 39*s*, and 40*s*. per quarter respectively, mixes them. At what price per quarter must he sell the mixture to gain 20 per cent. on the purchase ?

---

73. Simplify the expression $\frac{45}{308}$ of $\frac{176}{1017}$ of $\frac{63}{220}$.

74. Find the fifth power of 0·008.

75. Reduce £35, 4*s* 6*d* to dollars and cents, taking 4·6 dollars as equal to £1.

76. Find the value of 13 tons 17 cwts 1 qr. $10\frac{1}{2}$ lbs. at £5, 18*s*. 10*d*. per ton.

77. A sum of money is divided between A, B, and C in such a way that A gets 30 per cent. of the whole, B 60 per cent of the remainder, and C the rest If C's share amounts to 7*s*. find the total divided, and A's and B's shares.

78. A merchant allows customers a discount of 5 per cent for cash payment, but requires full payment in 3 months A customer owes him £500 and decides to sell some stock in order to pay his account at once If his money is invested in the 3 per cents. at 95, and a dividend is due in a month, what does he gain or lose by his action ?

79. What rate of interest does a man get for money invested in the 4 per cents. at $66\frac{5}{12}$, brokerage having been charged at the rate of $\frac{1}{4}$ per cent. ?

80. Calculate the difference between the compound and the simple interests on £350, 10*s*. for 6 years, at 3 per cent. per annum

81. At what price per yard must a man sell cloth that costs him 6*s*. 8*d*. per metre so as to gain at least 10 per cent. on his outlay. 1 metre = 39·37 inches.

82. A pond whose area is 3 acres is frozen over with ice $2\frac{1}{2}$ inches thick. Find in tons, correct to 3 significant figures, the weight of all the ice, if a cubic foot of it weighs $57\frac{1}{2}$ lbs.

83. Divide 64·83 by 8·4316 correct to 5 significant figures.

84. Find the cost of paper for a room 11 metres 7 decimetres long, 5 metres 8 decimetres wide, and 4 metres 2 decimetres high, at the rate of 1 franc 50 centimes per piece, being 10 metres long and 7 decimetres 5 centimetres wide.

[illegible] Simplify [illegible]

86. Find the net income of a man whose taxable income is £840, the tax being at the rate of 7*d.* in the £.

87. A rate of £328, 6*s.* 8*d.* was raised from three parishes whose rentals were £4325, 10*s.*, £2689, 17*s.* 6*d.*, and £1192, 19*s.* 2*d.*, respectively. Calculate what each should pay.

88. Express a quotation of 35 marks per quintal in pence per lb., taking 20·5 marks = £1, and 1 kilogramme = 2·2 lbs.

89. By selling tobacco for £126, 10*s.* 1*d.* I lost 3 per cent. What did it cost?

90. A person mixes 3 cwts. 27 lbs. of one quality of tea that costs 3*s.* per lb. with 2 cwts. 50 lbs. which he had been selling at 2*s.* 6*d.* per lb., gaining 25 per cent. How much did he gain or lose by selling the mixture at the rate of 2*s.* 8*d.* per lb.?

91. What rate of interest is obtained by investing in the 3 per cents. at 70?

92. It costs 10 guineas to have 12 tons 6 cwts. 3 qrs. carried 325 miles. What ought to be the charge for carrying 5 tons 5 cwts. 3 qrs. double the distance, the charge per mile for distances over 600 miles being $\frac{3}{4}$ of that for distances under?

93. A person standing on a railway platform 264 yds. long, noticed that a train passed the platform in 20 secs., and himself in 8 secs. Find the length of the train and its rate per hour.

94. A square field contains 10 ac. A man walks at the rate of 3 miles per hour along one side, along a diagonal, along another side, and then returns along the other diagonal to his starting-point. Find, correct to 5 significant figures, the number of minutes he takes.

95. The successful candidate at an election polled $\frac{3}{5}$ of the voters, and had a majority of 756 over his rival; $\frac{1}{6}$ of the constituency remained unpolled. Required the number that voted for each candidate, and the gross number of electors.

96. Find the area of the largest circular plate that can be cut out of a square sheet of iron containing 25·281 sq. in.

---

97. Find the sum of $0·25\dot{4}\dot{5}$ of 4*s.* 6*d.*; 0·0125 of 8*s.* 4*d.*; $4·\dot{0}\dot{9}$ of 1*s.* 10*d.*; and $0·\dot{1}\dot{3}$ of 1*s.* 10½*d.*

98. Change 32470 roubles 40 kopecs into British money, correct to nearest penny, at 3*s.* 4½*d.* per rouble.

99. Find a multiplier to convert rupees per maund (80 lbs.) into pence per ounce, taking a rupee equal to 1*s.* 4*d.*

100. A shilling weighs 3 dwts. 15 grs., and 7½ per cent. of its weight is alloy. If pure silver be worth 5*s.* per oz., find the value of the silver contained in 100 shillings.

101. Find the Greatest Common Measure of 26187 and 33350.

102. A bankrupt pays a dividend of 8*s.* 4*d.* in the £. If his total liabilities amounted to £650, of which £25 was preferential, find his total assets.

103. An article cost 12*s.* 8*d.* per stone How many lbs should be sold for 6*s.* 8*d.* so that 5 per cent of the return may be profit?

104. Find, to the nearest penny, the cost of painting a border in a room 6 metres 50 centimetres long, by 5 metres 25 centimetres broad, in the middle of which is a carpet 5 metres 80 centimetres long by 4 metres 50 centimetres broad, at 1*s* $10\frac{1}{2}$*d* per square metre

105. A person owes £400, due in 3 months. What sum ought his creditor to accept as immediate payment, money being worth $3\frac{1}{2}$ per cent. per annum (to nearest penny)?

106. A section of a pipe is 4 feet in diameter. If the mean flow of water through the pipe is 3 miles per hour, how many gallons can the pipe supply per hour, supposing 25 galls to equal 4 cub ft ?

107. Find the compound interest on £2080 for 8 years at 4 per cent. per annum, neglecting fractions of a penny.

108. What would £1 be worth at Paris if a bill at Amsterdam of 12 florins 15 cents sells for £1, and 9 florins 50 cents are worth 20 francs at Paris?

---

109. Reduce 0·58$\dot{3}$ and 0·4$\dot{3}$46153$\dot{8}$ to vulgar fractions

110. Divide £5, 16*s* 6*d* into 4 parts in the proportion of $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{5}{6}$.

111. Find, to the nearest penny, without unnecessary calculation, the value of 0·318426 of £184, 10*s* $4\frac{1}{2}$*d*.

112. If 5 lbs of tea be worth 9 lbs of coffee, and 4 lbs. of coffee worth 17 lbs of sugar, and 8 lbs of sugar worth 3 lbs of butter, and 2 lbs. of butter worth 7 lbs. of rice, find the value of 40 lbs of tea if the rice cost 2d. per lb.

113. If the cost of producing 250 copies of a work is £12, 10*s*, find the cost of producing 1000 copies, when, owing to scarcity of material the price of every additional edition of 250 increases by 10 per cent. on that of the preceding one.

114. After using $\frac{3}{8}$ of the contents of a cask, a publican sold $\frac{4}{5}$ of the remainder for 27*s.* 6*d.* Find the cost of the whole cask.

115. There is a meadow 81 perches long and 48 perches wide, with a square pond in the centre which occupies just $\frac{1}{3}$ of the ground. How far are the sides of the pond from the sides of the meadow parallel to them?

116. A cistern, 12 ft. long, 8 ft. deep, and 5 ft. 6 in broad, is filled with water. If the waste pipe has a cross section of $\frac{3}{4}$ of a sq. inch, and the water flows out through it at the rate of 14 ft. 6 in. per minute, in what time will it be emptied?

117. A person going abroad banks £200. Twenty-five years later his son returns to claim the money. If the rate of compound interest has been steady at 2 per cent. per annum on the sum, how much ought the son to receive?

118. Find the side of a square field that shall contain as many acres as a rectangular field 301 yds. 2 ft. long, and 153 yds. 1 ft. broad,

119. What sum will amount to £540, 15*s.* in 6 years at $3\frac{1}{2}$ per cent. per annum, compound interest?

120. If the weight of 1 cub. ft. of water is 62·35 lbs. avoirdupois, find the error in calculating the weight of 1000 cub. ft. on the approximate assumption that 1 cub. ft. weighs 1000 oz.

---

121. Multiply 69·857626 by 3·842753, correct to 6 significant figures.

122. Resolve 18304, 20475, 24255 into prime factors, and find the least whole number by which their product must be multiplied to make the resulting number a cube.

123. A sovereign weighs $123\frac{1}{4}$ grains, of which $\frac{11}{12}$ are pure gold. If gold be $17\frac{1}{2}$ times as valuable as silver, what weight of silver will be equal in value to the gold in 153 sovereigns. Give the answer in pounds and decimals of a pound, avoirdupois.

124. Find the cash balance, and by whom due, on the following account on the 1st July 1901, the bank paying 3 per cent. interest, and exacting 4 per cent. on all money overdrawn. Answer to nearest penny. Balance on 30th June 1900, £500; July 30th, deposited £250; August 9th, withdrawn £700; Sept. 14th, deposited £560, Dec. 14th, withdrawn £800, Jan 17th, 1901, deposited £750; April 4th, withdrawn £900; May 16th, deposited £450.

125. A merchant buys 324 cwts of sugar at $5\frac{1}{4}d.$ per lb. He sells $\frac{1}{3}$ of it at $6\frac{1}{2}d.$ per lb, $\frac{1}{4}$ of it at 6*d* per lb., and the rest at $5\frac{1}{2}d.$ per lb. What profit per cent does he make?

126. Calculate to the nearest penny the cost of 12 lbs. 9 oz. 15 dwts. 4 grs, @ £3, 4*s* 4*d* per lb.

127. An agent purchased goods to the amount of £486, 7*s.* 6*d.*, and paid £6, 11*s.* 9*d* for packing, porterage, &c, and £14, 10*s.* 8*d.* for Custom House dues. If he charged a commission of $3\frac{3}{4}$ per cent. on the money laid out, find to the nearest penny the total amount of his bill.

128. The base of a rectangular prism is an equilateral triangle with a side of 7 in, and its height is 24 in. Find its cubic contents.

129. Find the difference between the simple and compound interest on £1296, 10*s.* for 3 years at 4 per cent. per annum.

130. The value of a railway share is £75, and the half-yearly dividend is £4, 4*s.* What rate of interest does a person get for money invested in the Company?

131. A merchant mixes 45 lbs. of tea at one price with 30 lbs. at a dearer price. By selling the mixture at 4*s.* per lb. he gained 20 per cent. on his outlay. Find the value of each kind of tea, the difference in price being 1*s.* 8*d.* per lb.

132. A person bought 160 bags of flour. If the gross weight amounted to 16 tons 11 cwt., and the tare of each bag was $3\frac{1}{2}$ lbs., find the price of the flour @ £4, 4*s.* per cwt.

---

133. Add together 0·2625 of £1, 0·0625 of 13*s.* 4*d.*, and 8·25 of 9*d.*

134. Find the sixth root, correct to 4 significant figures, of 8437.

135. A speculator sold £2250 three per cent. stock at 110½ and invested the proceeds in the 5 per cents. @ 130: Find the alteration in his income (the prices quoted cover brokerage).

136. Find, to the nearest penny, the compound interest on £5938, 10*s.* for 5 years, @ 3 per cent. per annum. Find also what it would have differed by had interest been added half-yearly.

137. A piece of cloth apparently measures 14 yards 2 feet, but the yard measure was ⅜ of an inch too long. Find the true length of the cloth.

138. A man bought equal quantities of oranges at 4 for 2½*d* and at 3 for 2*d.* What per cent. (to 2 decimals) does he gain or lose by selling @ 7 for 4½*d.*?

139. Find the area of a triangle the sides of which measure 6·7 cms., 8·54 cms., 3·6 cms. respectively.

•**140.** In French silver money 1 franc is worth 9½*d.* English money when silver is at 5*s.* the ounce. What sum of French money is worth £262, 12*s.* 8*d.* when silver is at 5*s.* 1½*d.* the ounce?

141. A gallon of water weighs 10 lbs., a cubic foot of water weighs 1000 ounces, a cubic centimetre of water weighs 1 gramme, and a centimetre is 0·3937 of an inch. Find the number of grammes in a pound and the number of cubic centimetres in a gallon, each to 2 places of decimals.

142. What sum put out at simple interest will amount to £1310, 14*s.* in 6 years, at 4⅝ per cent. per annum?

143. The circumference of the driving wheel of an engine is 16 feet 6 inches and the train is going at the rate of a mile in 4 minutes. In what time will the wheel make one revolution?

144. On a railway 1209⅝ miles long the receipts in one week were £96168, 9*s.*, while in the corresponding week of the preceding year the receipts were £87055. Calculate the increase in the average receipts per mile per week.

---

145. Multiply 924·05764 by 8·[illegible]935, correct to 7 significant figures.

146. Divide 63·84715 by 436·27827, correct to 4 decimal places.

147. The population of a town is 932022. What was it a year ago if during that time there has been an increase of 5 per cent.?

148. A man spends equal sums in buying oranges at 4 for 3*d.* and at 3 for 4*d.* By selling them at a penny each he gained 4*d.* Find the number of each kind he bought.

149. A building is insured for £7010, which is [illegible] of its real value. If A owns [illegible], B ⅓, C ¼, and D the rest, find what loss each would sustain if the building were totally destroyed by fire.

150. A square field contains 99856 square yards. Find the length of each side.

151. How many dollars may a person obtain for 545 francs, at 18·4 dollars per 100 francs?

152. Calculate, to the nearest penny, the difference between the simple and compound interests on £1500 for 3 years, at 2 per cent. per annum.

153. Divide £71, 18*s.* 9*d.* among 14 persons, so that each of 5 shall get 3 times and each of 6 twice as much as the remaining three.

154. If the wholesale and retail profits are respectively 4 per cent. and 25 per cent., what part of the price paid by the consumer is profit?

155. A person owning £1000 3 per cent. stock has to sell as much as will pay a debt of £105, 15*s.* If the 3 per cents. are at 94, how much stock has he to sell out?

156. A bankrupt has assets £600 and liabilities £930, of which latter amount £210 are preferential claims for rent, taxes, &c., and must be paid in full. Find how much in the pound the creditors ought to receive.

---

157. Express 6 hours 4 minutes 6 seconds as the decimal of a day.

158. Find the G.C.M. and L.C.M. of 94248, and 105336 by the method of prime factors.

159. A grocer buys 14 dozen eggs at 1*s.* 2*d.* per dozen. He sold [illegible] of them at $1\frac{1}{2}$*d.* each and the rest at $1\frac{3}{4}$*d.* each. Find his gain per cent.

160. What sum will amount to £7852, 19*s.* in $6\frac{1}{2}$ years, at 4 per cent. simple interest?

161. A freehold house, the rent of which is £65 per annum, is sold for 28 years' purchase, the buyer leaving £220 on mortgage, at 5 per cent. per annum. What rate of interest does he receive on the investment?

162. A person borrows £500 at 3 per cent. per annum. If he sets aside £50 each year to pay interest and reduce the debt, how much will he be owing at the beginning of the 4th year?

163. If 2 horses are worth 7 mules, and 28 ponies worth 25 mules, 9 ponies worth 5 cows, and 72 sheep worth 21 cows or 135 lambs, find the cost of 10 horses if the price of 1 lamb is 16*s.*

164. A person sold 120 pairs of stockings, at 3*s.* 2*d.* per pair, to a customer, from whom he bought in return equal quantities of cloth at 5*s.* 3*d.* per yard, of cambric at 9*d.* per yard, and of calico at 6*d.* per yard. How many of each did he get for the stockings and a further payment of £1, 16*s.*?

165. If London remit to Paris at 25·25 francs per £, thence to Denmark at 5 francs for 3 krone, find the arbitrated rate of exchange between London and Denmark.

166. I bought 25 shares of a certain company for £135 each, the paid-up capital for each share being £100. During the year I got a dividend of $5\frac{1}{2}$ per cent. on the paid-up capital and a bonus of £35 on my 25 shares. What rate of interest did my investment bring me?

167. A room is $12\frac{1}{2}$ feet broad and $22\frac{1}{2}$ feet long. What will it cost to cover it with carpet $\frac{3}{4}$ yard wide, at 4*s.* 6*d.* per yard?

168. A nugget weighing 3 lbs. 14 oz. avoirdupois was purchased at £39, 4*s.* per lb. avoirdupois. On separating the quartz from the gold it was found that the gold was to the quartz as 21 to 4. The gold was then sold at £3, 17*s.* 4*d.* per oz. troy. Find the gain or loss.

---

169. Multiply 0·073241 by 0·2382993, correct to 6 decimal places.

170. Express a link as the decimal of a yard.

171. Find the difference between 6·75 times £0, 15*s.* $10\frac{1}{2}$*d.* and 3·518 of £1, 7*s.* 6*d.*

172. What will be the cost of 2 yards 6 inches if 3 yards 1 foot cost £2, 11*s.* 8*d.*?

173. Raise 1·01 to the fourth power, and extract the cube root of 19814511816.

174. A and B buy oranges at 11 for 9*d.* A sells his at a penny each and B at 1*s.* 3*d.* per dozen. If B sells twice as many as A, and A's gain is 6*s.* 8*d.*, what is B's gain?

175. By selling an article for £111, 0*s.* 10*d.* a man would gain $2\frac{1}{2}$ per cent. What would be the gain or loss by selling at £105, 12*s.* 6*d.*?

176. If £1095 lying at interest for 117 days become £1118, 8*s.*, what is the rate of interest per cent. per annum?

177. A merchant can draw on Naples for 2000 ducats at 40*d.* per ducat. What would he gain or lose by an indirect course *viâ* Paris, Leghorn, and Naples, the exchange on Paris being 25·2 per £; of Paris on Leghorn 525 centimes per pezza; of Leghorn on Naples 100 pezzas for 124·5 ducats.

178. Calculate the true discount on £1545, 19*s.* 2*d.* from Sept. 3rd to Jan. 15th, at $4\frac{1}{2}$ per cent.

179. Find what a sum of £100 deposited at the beginning of the year 1900 would amount to at the end of the year 1999, reckoning compound interest at 2 per cent. per annum.

180. Find by duodecimals the cubic contents of a room 19 ft. 9 in. long, 14 ft. 8 in. wide, and 11 ft. 3 in. high; also the area of the four walls.

---

181. Divide 2173·68 by 31·76814, correct to 5 significant figures.

182. Find the ratio between $\frac{3}{8}$ of 5 cwts. 26 lbs. and $\frac{5}{14}$ of 10 cwts. 3 qrs. 8 lbs. 12 oz.

183. Divide £845, 16*s.* 8*d.* between two persons, in the ratio of $\frac{3}{8}$ to $\frac{5}{8}$.

184. A person sold 43 cwts. 3 qrs. 23 lbs. of sugar for £71, 15*s.* $10\frac{1}{2}$*d.*, gaining thereby $\frac{3}{7}$ of the selling price. What did it cost him per lb.?

185. An article cost 3 guineas. What must it be marked at to enable the merchant to allow a discount of 10 per cent. and yet to make a profit of $8\frac{1}{3}$ per cent. on his outlay?

186. In how many years would £314, 12*s*. 11*d*. amount to £456, 4*s*. 8$\frac{3}{4}$*d*., reckoning simple interest at 5 per cent. per annum?

187. Find the compound interest on £850, 10*s*. for 3 years, at 2$\frac{1}{2}$ per cent. per annum.

188. What must a man earn per annum if he spends on the average £56 every 42 days, and saves £120 a year?

189. A person A owes B a sum of money due in 6 months. B offers to accept payment at once and allow true discount at the rate of 4$\frac{1}{2}$ per cent. per annum. A refuses, and B gets the bill discounted at the bank at 4$\frac{1}{2}$ per cent., thereby losing 13*s*. 6*d*. Find (1) the amount of the bill, (2) of both discounts.

190. Given that a cubic foot of water weighs 62$\frac{1}{2}$ lbs., find the weight of water in a 6-gallon cask.

191. Find the value of £32 in francs and centimes, the course of exchange being 25 f. 80 c. per £1.

192. The earth obtained from the excavation of a square pond 63 ft. long by 12 ft. deep, was spread evenly over a garden 252 yds. long by 36 yds. broad. How much did it raise the surface?

193. Reduce $\frac{6480}{117936}$ to its lowest terms.

194. Find, correct to 3 decimals, the value of $0{\cdot}06727 \times 2{\cdot}\dot{8}\dot{1}$.

195. Find the cost of 4860 articles at £3, 6*s*. 9$\frac{3}{4}$*d*. each.

196. A merchant marks his goods so as to gain 5 per cent. profit. What is the greatest discount per cent. he can allow so as still to gain 2 per cent. at least?

197. A dairyman buys 319 pints of milk for £9, 16*s*. 6*d*. He adds to it 44 pints of water. At what price must he now sell it to gain one-third of his outlay?

198. If a man owes me £2000, half of which is payable in 6 months and the rest in 9 months, what sum should I accept as immediate payment, allowing true discount at the rate of 3$\frac{1}{2}$ per cent. per annum?

199. A prize valued at £5220 is taken by a cruiser, and it is proposed to divide the sum amongst the ship's company in proportion to their pay and the time each had been on board. If the captain has served 20 months at £6 a month, 2 chief mates 15 months at £3, 10*s*. a month, 3 second mates 12 months at £3 a month, and 157 sailors 27 months at £3 a month, calculate the share of each.

200. The floor of an office, which is an exact square, requires 66 yds. 2 ft. 1 in. of linoleum to cover it. If the linoleum is 2 ft. 1 in. wide, find the length of the side of the office.

201. A person invests equal sums in the 4 per cents. and in the 4$\frac{1}{2}$ per cents, and obtains equal incomes. If the 4 per cents. are at a discount of 4, find the quoted price of the 4$\frac{1}{2}$ per cents.

202. Find the thickness of a piece of timber 16 ft. long and 14·4 in. broad, which can be cut up into 1719 cubical blocks, each of 10 cub. in. volume.

203. Find, to the nearest penny, the present worth of a bill for £1000, 5*s*., drawn March 14th, at four months, and discounted April 5th, at 4 per cent. Calculate with (1) True discount, (2) Banker's discount.

204. Find the different amounts of £100 at the end of 12 years at 3 per cent. per annum, compound interest, the interest being payable (1) half-yearly, (2) yearly.

---

205. Divide 83·291436 by 3·745619, correct to 4 decimal places.

206. Find the value of $\sqrt[3]{0 \cdot 001331} \times \sqrt[3]{926859375}$.

207. An egg merchant spent equal sums in buying eggs at 5 for 6*d*. and at 11 for 9*d*. By selling at 1*s*. 7*d*. per dozen he gained £3, 7*s*. 9*d*. Find the number of each kind he bought.

208. If 15 per cent. of the return be gained by selling an article at 50*s*. per cwt., how much per cwt. will be gained by selling at £45, 10*s*. per ton?

209. A man spent £1200 in purchasing pictures at an average price of £50 each. While in his possession, some were destroyed by fire, but he disposed of the remainder for £960, thus realising a profit of 20 per cent. on the cost of each. How many were destroyed?

210. A, B, and C commence business together, putting in £1000, £800, and £2000 respectively. After four months A put in £500 more, and at the end of other two months B withdrew £300, but two months later he put in £1000. C took out £600 at the end of three months, but six months afterwards he put in £1200. If the net gain for the year amounted to £2500, find the share of each.

211. How many bricks $8\frac{1}{4}''$ by $4''$ by $2\frac{1}{2}''$ will be required for a wall 164 yds. long, 18 in. thick, and 7 ft high, deducting space for three doors, each 6 ft 4 in. by 4 ft., and calculating the mortar used fills up 20 per cent. of the wall?

212. Find the value in British coinage of 64 tons 5 cwts. of coal at 4 dollars 25 cents per ton, taking the dollar as equal to 4*s*. 2*d*.

213. If a grain of corn bears on the average 60 full ears per annum, find the total number of grain produced in four successive years from the original one.

214. What percentage should I clear by transferring my money from the 4 per cents. at 95 to a Railway Company which pays 5 per cents., and whose shares are at 114?

215. A room is 18 ft. 7 in. long, 14 ft. 3 in. broad, and 12 ft. high. What will it cost to paper its walls with paper 27 in. wide, at $2\frac{1}{4}d$. per yard? Deduct for four windows each 10 ft. high and 3 ft. 6 in. broad, and a doorway 6 ft. 6 in. by 3 ft. 6 in. Charge fraction of a yard as a whole yard.

216. At what rate per cent. per annum will the compound interest on £2400 amount to £649 in 5 years?

---

217. Evaluate to 4 decimals $\frac{63·827 \times 53·18 \times 1·432923}{0·3279}$.

218. Find the continued product of 12683459, 30845, and 628, correct to 6 significant figures.

219. Calculate the fourth root of 1874161.

220. A person owes three separate accounts: £60 due in 3 months, £150 due in 8 months, and £250 due in 1 year. If he desires to pay all at once, when ought he to make a payment of £460?

221. A British merchant owes £318, 7*s.* 9*d.* in Hamburg. Find how many marks and pfennige this represents, taking 20·43 marks as equal to £1.

222. An American firm buys goods in Germany at 103·75 marks per quintal. If it costs 28*s.* per cwt. for carriage, at what price per lb. must it be sold in New York to gain at least 5 cents per lb.? (Use par values of exchange.)

223. The area of a rectangular field is approximately 20 acres. If its length is 15 chains 80 links calculate to 3 significant figures its approximate breadth in chains.

224. A dealer marks his goods so as to gain $12\frac{1}{2}$ per cent., and allows his customers a discount of 5 per cent. Calculate his actual percentage gain.

225. If a man who has an annuity of £120 banks it as he gets it, what sum will it have amounted to at the end of 4 years after the first banking, allowing compound interest at $3\frac{1}{2}$ per cent. per annum?

226. If railway stock bought at 28 per cent. premium pays $7\frac{1}{2}$ per cent. on the investment, how much per cent. would it pay if it were bought at 10 per cent. discount?

227. Find the cost of papering a room 27 ft. 5 ins. long, 19 ft. 3 ins. broad, and 13 ft. 8 ins. high (in which there are 2 windows, 5 ft. by 3 ft. 6 ins. each; 2 doors, 7 ft. by 4 ft. each; and a fire-place, 5 ft. by 6 ft. 6 ins.) with paper 27 ins. wide, at 4*s.* 6*d.* per piece of 12 yards, allowing 1 piece in 10 for waste, and charging for every broken piece.

228. A person borrowed £75, 12*s.* 6*d.* for 18·25 years, at the rate of £0·045 simple interest, for every sovereign per year. Find what sum ought to be paid back on the expiry of the loan.

---

229. Express in the smallest integers the ratio of $4\frac{7}{10} : 5\frac{4}{15}$.

230. Find what factor is wanting to make 266200 a perfect cube.

231. A square courtyard is found to contain 96177249 square inches. Find the length in the side to yards in 3 decimals.

232. Find the cost of repairing a road 64 miles 6 furlongs 110 yards, at an average cost of £3, 16*s.* 8*d.* per mile.

233. How much carpet will be required to cover the floor of a square room 17 ft. 6 ins. long, the width of the carpet being 2 ft. 4 in.? Find also its cost at 3*s.* 9*d.* per yard, allowing no fraction of a yard to be bought.

234. A bankrupt can pay 6s. 8d. in the £ If he had £125 more he could pay 7s 6d. in the £. Calculate his assets and liabilities.

235. A person borrowed £2500, at 4 per cent. per annum, on condition of paying back £200 every year towards clearing off his debt and paying the interest. Calculate to the nearest penny what he owes at the end of six years.

236. Exchange 6824 rupees with francs, the course of exchange between Calcutta and London being $23\frac{1}{4}d$ per rupee, and between London and Paris £1 = 25 francs 60 centimes.

237. If rabbits increase at the rate of 10 from each pair per annum, calculate the number of descendants of a pair born in the tenth year.

238. Find the cost of 385 9 metres of cloth, at 3 fls 7 cents. 5 mils per metre. Express answer in decimal coinage and also in £ s d.

239. Find, by duodecimals, the area of the walls of a room whose length is 42 ft 9 ins, breadth 27 ft. 4 ins, and height 11 ft 5 ins, allowing 50 sq ft. 18 sq ins for windows and door

240. What principal will amount to £720, 6s. 6d. in 4 years, at 3 per cent. per annum compound interest? If invested at simple interest, what principal would be required?

---

241. The sum of two numbers is 342, and one is to the other as 2^6 is to 5^2. Find the numbers

242. Find the number of square kilometres in a square mile, the yard being 0 914383 of a metre.

243. After paying income tax of 6d in the pound, my income from the 3 per cents. at 85 is £780. How much did I invest?

244. Find, to the nearest centime per metre, the difference between 1s. per yard and 1 mark per metre, taking exchange as £1 = 25 2 fr. = 20 4 marks.

245. Find the cost in British money of 562 hectolitres of wine at 6 francs 50 centimes per litre, when £1 = 25·24 francs.

246. Find the cost of papering the walls of a room 28 metres 15 cms. long, 18 metres 9 cms. broad, and 3 metres 75 cms. high, with paper 80 cms broad, at 2 fls. 5 cents. per piece of 10 metres, allowing for 3 doors, each 2 metres 50 cms. by 1 metre 25 cms., and 2 windows, each 2 metres by [illegible] cms., and a fire-place 1 metre 50 cms. high and 2 metres broad. Allow 10 per cent. extra for waste

247. Find, correct to [illegible] significant figures, the square root of 19.

248. Find the present value of a bill for £279, 2s. 6d. due in $4\frac{1}{2}$ months, at 4 per cent. per annum. (Allow true discount)

249. How many square tiles, each 25 cms. long, would be required to pave a courtyard 37 metres 85 cms. by 25 metres 50 cms.? Also find the cost at 6 francs 50 centimetres per 100.

250. Find the cost of 27684 grammes, at £3, 7s. 6d. per kilogramme. Give the answer also in francs, taking 25 francs = £1.

In the following examples use should be made of the Logarithm Tables on pp. 358–361.

251. A water wheel whose diameter is 14 feet makes 33 revolutions per minute. Find, correct to 4 significant figures, the number of miles per hour traversed by a point on the circumference of the wheel.

252. If light travels 186,000 miles per second, find the distance traversed in a year, correct to 4 significant figures.

253. The mean solar year exceeds the civil year by 0·2422 of a day. Find (1), correct to 4 significant figures, in what time this fraction equals one day; (2) the least exact number of years in which the fraction equals an exact number of days; and (3) if an allowance of 97 days in every 400 years is made by means of leap years, in how many years will the civil year be exactly one day behind the mean solar year?

254. If the number of persons born and the number of persons who die are respectively $\frac{1}{37}$ and $\frac{1}{67}$ of the population per year, find (1) the yearly increase per cent of the population, and (2) the increase at the end of a year on a population of 2,563,940.

255. A crystallised salt is heated to drive off the water of crystallisation. If the salt weighed 1·834 grammes before and 1·597 grammes after heating, what is the percentage of water in the crystals? If the theoretical value is 12·5 per cent, find the percentage of error.

256. A man who owes £600 determines to pay at the end of each month 10 per cent of what he then owes till the debt is reduced to less than £100, when he will pay the balance. How many months will he take to pay his debt?

257. The population of a country was 4,314,000 to begin with, and increased by 3·168 per cent every year for 5 years. What was the population at the end of the 5 years?

258. A square pond is surrounded by a strip of lawn whose breadth is the same on all four sides. If the side of the pond is 32 yds. 2 ft. and the breadth of the lawn 25 yds. 1 ft., find the area of the lawn.

259. An estate, which consists of a circular piece of ground $1\frac{1}{4}$ miles in diameter, is left to 5 sons, 4 of whom get each what is equivalent to a circle 400 yards in diameter, while the fifth gets the remainder. If the area of a circle is $\frac{355}{452}$ of the square on the diameter, what percentage of the estate goes to each son?

260. A person walks from Rugby to London in 4 days and back in 5 days. Each day he walks one mile less than on the preceding day. What is the distance between the two places?

261. A path 4 feet wide is to be made within a rectangular lawn measuring 45 yards by 40 yards so as to leave 5 feet between the walk and each side of the lawn. How many square yards will the walk contain?

262. A rectangular box is made of wood $\frac{3}{4}$ of an inch thick, which weighs 50 lbs. per cubic foot. If the external dimensions of the box are 2 feet, 1 foot 4 inches, and 1 foot, and its contents weigh 60 lbs. per cubic foot, find the weight of box and contents.

263. 20,000 tons of water per minute flow out of a reservoir through an opening 14 yards broad and 4 yards high. Find, correct to two decimal places, at how many miles per hour the water flows, assuming that a cubic foot of water weighs $62\frac{1}{2}$ lbs.

264. The par of exchange between England and France is 25·2 francs for £1; a money-changer gives 25 francs for £1, and 15*s.* 9*d.* for a twenty-franc piece. A traveller changes £50 into French money, and on his return 12 twenty-franc pieces into English money. Find in shillings and pence the whole profit made by the money-changer on both transactions.

265. A bankrupt estate is expected to yield 18*s.* in the £. The assets, however, realise £425 less than the estimate, and the costs of administration amount to 10 per cent. of the dividend distributed, which is 12*s.* 6*d.* in the £. Find the amount of the liabilities.

266. A sheet of lead is 10 feet long, 6 feet broad, and $\frac{1}{4}$ of an inch thick. Find how many bullets, weighing $\frac{1}{2}$ an ounce each, can be made from it, if a cubic foot of lead weigh $6\frac{1}{4}$ cwt.

267. Find, correct to the nearest penny, without unnecessary calculation, the value of ·296445 of £1743, 16*s.* 9*d.*

268. A man borrows £1000, interest to be payable at the rate of 4 per cent. per annum. At the end of each year he pays £100, the balance after payment of interest being applied to the reduction of the debt. Find, to the nearest penny, the amount of his debt immediately after he has made his fourth payment.

269. A dealer buys coal at 19*s.* per ton and sells it at 1*s.* 6*d.* per cwt. If he sells, on an average, 25 cwt. daily, and his expenses amount to 8*s.* 6*d.* a day, find what percentage of his receipts is profit.

270. A sum of money is put on deposit at 4 per cent., the interest, omitting fractions of a penny, being added to the principal half-yearly. After two years the amount on deposit was £280, 16*s.* 9*d.* Find the sum originally deposited.

271. A money-changer pays persons going to France 25 francs for one pound, and persons returning from France a guinea for 27 francs. If he make an equal profit from the same number of francs on each occasion, express the real value of a franc in pence.

272. In 1895 the National Debt amounted to £656,998,941, the interest paid on this was £24,977,912. What was the average rate of interest per cent.?

273. A person spends one-tenth of his money, and then 12 per cent. of what remains. He then finds he has £11,880. Find the sum of money he had.

274. A property burdened with a feu-duty of £103, 6*s.* 8*d.* is purchased for £27,863. The gross rental is £1503, and 22 per cent. of this is absorbed in taxes, repairs, factorage, &c. After paying interest on a bond for £18,000 at $3\frac{1}{4}$ per cent. (less tax at 8*d.*), find what return per cent. the purchaser receives on his investment.

275. A bankrupt's debts amount to £5430, 6*s.*, and the dividend is

reduced from 3s to 2s. 4d. by the admission of a claim to a preferable ranking Find the amount of such claim.

276. A family of 14 persons has provisions for 38 days. After 26 days, 4 more persons arrive. How long will the food last?

277. If 30 horses and 206 sheep can be kept for $4\frac{1}{2}$ days for £75, 15s., what sum will keep 5 horses and 66 sheep for 8 days, if 5 horses eat as much as 41 sheep?

278. What is the net amount of a man's income after deducting income tax at 1s 2d. in the £, if the increase of the tax from 1s. to 1s. 2d. makes a difference of £8, 3s. 4d.?

279. Fifty gallons of beer contain 70 per cent. of water and the rest alcohol. How much water should be added to the beer to raise the proportion of water to 90 per cent.?

280 The true discount on a sum due 8 months hence is £30. The interest on the same sum for the same time at the same rate is £30, 18s. Find the sum and the rate per cent.

281. A farmer bought 18 cows and 15 lambs for £265, 2s. 6d. and sold them for £297, 18s. 9d, thereby gaining $12\frac{1}{2}$ per cent. on the former and 10 per cent. on the latter. What was the cost of a cow and of a lamb?

282. A sum of money invested at compound interest amounts to £8820 at the end of the second year and to £9261 at the end of the third year. Find the sum originally invested.

283. Find the sum which in two years will amount to £1000 interest 4 per cent, payable quarterly. Allow for income tax at 1s. per £.

284. What is the value at 5 per cent. interest of an annuity certain of £60 for ten years, deferred seven years?

285. What is the value, at 4 per cent., compound interest of an annuity certain of £70 for four years, deferred six years?

286. Find approximately in how many years £100 will amount to £300 if accumulated at 3 per cent. compound interest.

287. A man owes £6500, and pays off $4\frac{1}{2}$ per cent. of it, then $5\frac{1}{2}$ per cent. of the remainder, then $8\frac{1}{4}$ per cent. of the second remainder, and then $12\frac{3}{4}$ per cent. of what is left. How much does he still owe?

288. Find the amount (to the nearest £) of £650 lent for 10 years at 4 per cent. compound interest, and thereafter for 6 years at 5 per cent. compound interest.

289. An article in passing from the producer to the consumer passed through the hands of 3 dealers, each of whom added 10 per cent. to the price at which he bought it. If the final selling price was £22, 3s. 8d, what was the original cost?

290. A sum of money lent at compound interest amounts in 2 years to £2756, 5s. and in 4 years to £3038. 15s. 4d.. find the sum and the rate per cent.

EXAMINATION PAPERS

SCOTCH LEAVING CERTIFICATE

Lower Grade, 1909.

1 Three new pennies weigh one ounce avoirdupois Find in cwts, qrs, lbs, and ounces the weight of 12,549 new pennies

2. Find the simple interest on a loan of £724, 10s for $4\frac{1}{2}$ years, the yearly rate of interest being $3\frac{1}{4}$ per cent

3. Express 7875 and 9450 as products of prime factors, and hence express their greatest common factor and their least common multiple as products of prime factors

4. A cubic centimetre of water weighs a gram Express in kilograms the weight of the water contained in a cistern 1 metre long, 81 centimetres broad, and 28 centimetres deep

5. A dealer buys 750 sheep at £1, 7s 6*d* each, and sells them at £1, 10*s* 4*d* each Find the amount of his profit on the transaction. Express this profit as a percentage of his outlay

6 A mil is the thousandth part of £1 Explain the following step n pence $=£\frac{n}{240}=\frac{1000n}{240}$ mils $\left(4n+\frac{n}{6}\right)$ mils

Hence convert into mils 9*d*, 2*s* $4\frac{1}{2}d$, 16s 7*d* Without further calculation write down these sums as decimals of £1 to 4 places

1910

1. Find the smallest number which added to 17589 will produce a number divisible without remainder by 276

2. Simplify $\frac{\frac{5}{8}+\frac{7}{16}}{3\frac{7}{8}-2\frac{1}{4}}\times\frac{1\frac{4}{5}}{\frac{2}{3}-\frac{2}{5}}$, and find the product of 3 14159 by 62 4817 to 3 decimals

3. Find correct to the nearest penny the interest on £482 for 60 days at $3\frac{1}{2}$ per cent

4. A solid block 18 metres long, 12 metres broad, and 6 metres high is built of bricks, each of which is 22 5 cm. long, 12 5 cm broad, and 7 5 cm thick; find the number of bricks required.

5. A man pays income tax at 9*d* in £1 and has £688, 3*s* 9*d*. left. Find the amount of the tax deducted.

6. Taking a scale of 6 inches to 1 mile, draw on square paper

(1) A line AB to represent a distance of 1000 yards,

(2) A square PQRS to represent an area of 160 acres.

What fraction of a square mile does a square inch on the paper represent?

1911.

1. A workman earned 40*s.* a week, and found that after paying all necessary expenses he could save 5*s.* a week. He then emigrated to a country where he earns 30 per cent. more, but where his expenses are 35 per cent. higher. Find how much he can now save in a week.

2. A man received a legacy of £2000. He invested £1250 at $4\frac{1}{2}$ per cent. and the rest at $3\frac{1}{2}$ per cent. Find his annual income from the legacy after paying income tax on the whole of the interest at 1*s.* 2*d.* in the pound.

3. Express the following fractions as decimals correct to two places and arrange them in ascending order of magnitude : $\frac{3}{4}$, $\frac{5}{8}$, $\frac{5}{7}$, $\frac{6}{7}$, $\frac{7}{9}$, $\frac{7}{10}$, $\frac{8}{11}$. How many decimal places must be calculated in order to decide which of the two fractions $\frac{10}{13}$ or $\frac{253}{329}$ is the larger ?

4. A contractor undertakes to do a certain piece of work in 175 days and employs 72 men for the purpose. After 130 days one-third of the work has still to be done. How many additional men will he require to employ during the remaining days in order to carry out his contract ?

5. A metre is known to be more than 3·280 feet and less than 3·281 feet. Find the corresponding numbers of square feet between which a square metre must lie. From your results state the number of square feet in a square metre correct to as many decimal places as possible. Taking as your scale one inch to the foot, draw on your squared paper figures representing a square decimetre, a square foot, and a square metre.

1912.

1. Passengers by a certain line of steamers are allowed 20 cubic feet of luggage. A passenger wishes to take two boxes each 3 feet long and $2\frac{1}{2}$ feet broad and of the same height. What is the greatest height he can have them so as not to exceed the limit allowed ?

2. A father leaves £5600 to be divided between his sons aged 30, 34, and 41, so that their shares may be in the same ratio as their ages. Find the share of each.

3. A tank is 2·45 metres long, 1·46 metres broad, and ·56 metres deep. Find its volume in cubic metres, and also how many litres it contains, a litre being equal in volume to a cubic decimetre.

4. From January to July of last year a Scottish Railway Company sold 821,670 first-class tickets and 18,150,737 third-class tickets, receiving £60,966, 0*s.* 8*d.* for the first-class and £814,993, 6*s.* for the third-class. Find the average cost of a first-class and of a third-class ticket correct to the nearest farthing.

5. A building is insured for an annual premium equal to $\frac{9}{80}$ per cent. of the value of the building. If this premium is £5, 2s. 4½d., find the value of the building.

6. The following table gives the times at which a train passed the different stations (A, B, &c.), and the distance in miles of each station from the starting point. Taking a horizontal inch on the square ruled paper to represent 20 minutes, and a vertical inch to represent 10 miles, draw a curve to show the relation between the time spent and the distance travelled, and find from your curve between which stations the train went most quickly and between which it went most slowly, explaining how you come to your conclusion.

	A	B	C	D	E	F	G	H	I
Time . . .	9·0	9·16	9·24	9·45	10·0	10·10	10·24	10·48	10·54
Distance . .	0	5	8	17	25	30	39	58	62

COMMERCIAL ARITHMETIC.

SECOND PAPER, 1909.

Marks are given for neatness and style. Algebraic symbols may be used if properly explained.

1. A St. Petersburg "standard" of timber contains 165 cubic feet. If it is delivered in planks 12 feet long, 11 inches broad, and 3 inches thick, how many planks are contained in the "standard"?

2. A bankrupt pays 5s. 4½d. in the pound. Find the amount received by a creditor to whom he owed £425, 6s. 8d.

3. Gold is beaten out so thin that an ounce avoirdupois covers 20 square yards. Find to the nearest penny the value of the gold required to cover 3½ square feet, assuming an ounce of gold is worth £3, 11s.

4. A merchant buys articles at 5s. 7½d. a dozen, and sells them at 6¾d. each. Find how much profit per cent. he makes on his outlay.

5. The units of length, surface, and volume in the metric system are the metre, the are, and the litre. Express an are in square metres, and a litre in cubic metres.

What is the unit of weight in this system, and what is its relation to the metre?

6. Find to the nearest penny the compound interest on £2500 for 4 years at 4 per cent. interest added yearly.

7. What is meant by saying that the currency of a country is on a gold basis? Name one country in which the currency is on a gold basis, and one in which it is on a silver basis.

Explain what is meant by a depreciated paper currency.

8. An English merchant has to pay 10,000 francs in Paris. If he pays by a cheque on a London bank he will be credited with 25·2 francs per £1. If, however, he pays by a cheque on a Berlin bank, 81 marks will be accepted as worth 100 francs, while the Berlin bank will credit him with 20·5 marks for £1. Find how much he will save if instead of sending a London cheque to his creditor in Paris he pays him by a cheque on Berlin.

SECOND PAPER, 1910.

1. What is the cheaper article, one that costs 2*s.* $8\frac{1}{2}$*d.* per lb. or one that costs 6*s.* per kilogram? (1 kilogram = 2·204 lbs.)

2. British standard gold is 22 carats fine, that is $\frac{22}{24}$ of it is pure gold and the rest alloy. French standard gold is 900 millièmes fine, that is $\frac{900}{1000}$ of it is pure gold. What weight of alloy must be added to 27 lbs. of British standard gold in order to reduce it to the French standard?

3. In 1906 a man invested £3600 in $2\frac{1}{2}$ per cent. Consols at 90. In 1909 he sold them at $82\frac{1}{2}$ and invested the proceeds in Transvaal 3 per cents. at $95\frac{1}{4}$. Find the rise or fall in his income per annum.

4. During a certain year income tax was 1*s.* in £1, and during the following year it was 1*s.* 2*d.* A company pays a dividend of 7 per cent. less income tax on 9500 shares of £2 each for a period including 9 months in the first year and 3 months in the second. What is the total net sum required to pay the dividend?

5. Explain what information is given by each item in the following newspaper paragraph:—

"Exchange Rates—Latest Quotations.

Paris cheques, 25·21; Germany 8 days, 20·44; Germany sight, 20·48; Stockholm sight, 18·26.

A merchant in London owes 1500 francs to a correspondent in Paris and buys a draft for this amount from a bank. What will this cost him, to the nearest penny, the rates of exchange being as given above?

6. A man living in Great Britain holds 175 shares in an Indian tea garden, which declares a dividend of $7\frac{1}{2}$ rupees per share. He receives a cheque for £87, 16*s.* 10*d.* as the equivalent of his dividend, neglecting fractions of a penny. Find the value of the rupee in shillings, pence, and sixteenths of a penny.

7. A dealer sells goods value £440, and agrees to accept a bill at

3 months in payment. For how much should he draw the bill if he deducts 10 per cent. as trade discount, and adds interest at 4 per cent. per annum on the net amount, and the cost of a bill stamp at the rate of 1*s.* for each £100 (or part of £100) on the gross amount of the bill?

SECOND PAPER, 1911.

1. A map is drawn on the scale of 6 inches to the mile. What is the length on the map of the side of a square wood of 810 acres?

2. Buildings are valued in a company's books at £50,000. What will be the valuation of them after 3 years, if depreciation be deducted half-yearly at the rate of 10 per cent. per annum on the diminishing value? Give your answer to the nearest penny.

3. What is meant when it is stated that the rate in London for sight drafts on Paris is 25·25. Is this rate constant? and if not, what causes it to vary?

4. A man sells an article at 10 per cent. above cost price to B, who adds $12\frac{1}{2}$ per cent. and sells it to C. D buys it from C at 15 per cent. more than C paid for it, giving £9, 9*s.* 9*d.* What did the article cost originally?

5. A bankrupt's liabilities are £7455, and his assets realise £4042. After payment of Preferential creditors (amounting to £918) in full and legal expenses £497, one of the unsecured creditors receives £1073. Find the amount originally due to him.

6. A man places on deposit at his bank the following amounts on the dates mentioned bearing interest at $3\frac{1}{2}$ per cent. per annum:—

March 31st	£1000
April 14th	200
May 25th	2500
May 29th	300
June 17th	1000

Calculate the interest receivable on 30th June.

7. In 1907 A receives £201, 17*s.* 6*d.* as one year's interest on $2\frac{1}{2}$ per cent. Consols. He then sold out at $85\frac{1}{2}$. In 1909 he reinvested in the same stock the money thus received, and found that he now got £10, 12*s.* 6*d.* more interest in one year. Find the nominal amount of his present holding in Consols and the price of purchase per cent. (Ignore income tax and brokerage.)

SECOND PAPER, 1912.

1. A man buys 20 gallons of milk at $3\frac{3}{4}d.$ per quart, and after diluting it with water sells it at 4*d.* per quart, and makes a profit of 12 per cent. on his outlay. How much water did he add?

2. A floor measuring 100 feet by 37 feet 6 inches is paved with wooden blocks 12 inches in length, 3 inches in breadth, and 4 inches in depth. How many blocks would be required, and what would be the cost at 3*s*. 7*d*. per cubic foot?

3. A company made a profit of £9700, which was distributed as follows:—

7 per cent. as bonus to the staff.
Dividend on £70,000 Preference Stock at 5 per cent.
Dividend on £50,000 Ordinary Stock at $6\frac{1}{4}$ per cent.
£1500 to reserve.

What was the balance carried forward?

4. 1000 piculs of sugar are bought in Java at 10·50 guilders per picul and sold in London at 15*s*. $3\frac{3}{4}d$. per cwt. Assuming the rate of exchange on Java to be 12 guilders to £1, and that a picul = 136 lbs., find what profit is made on the transaction.

5. Explain what is meant by each of the following terms:—

(*a*) Compound Interest; (*b*) Commission; (*c*) Discounting a Bill; (*d*) Bill of Exchange.

6. A Bill of Exchange for £1095 drawn and accepted on 1st October at 90 days after date was discounted on the 10th October, the Bank charging $3\frac{1}{2}$ per cent. per annum. Taking into account that three days' grace are allowed in addition to the 90, find what amount was received.

7. What is the value of £1 sterling in marks and pfennigs if one quarter's interest (less income tax at 1*s*. 2*d*. on £1) on £10,000 $2\frac{1}{2}$ per cent. Consols realises 1205·33 marks?

HIGHER LEAVING CERTIFICATE.

The following questions in Arithmetic have been set in recent years.

1. If p pounds accumulate for n years at r per cent. compound interest payable annually, find an expression for the amount at the end of that time. Find to the nearest hundred pounds the amount due after £5000 has accumulated for 25 years at $4\frac{1}{2}$ per cent.

2. The dimensions of a block of metal are 1·237, ·659, ·484 metre, and the weight of a cubic centimetre is 8·465 grams, find the weight of the block by means of logarithms, expressing your answer in kilograms.

3. Prove that $\dfrac{A}{\left(1+\dfrac{r}{100}\right)^n}$ represents the present value of A pounds due n years hence at r per cent. compound interest, payable annually.

Find the present value of £15,000 due 8 years hence at $4\frac{1}{2}$ per cent. compound interest, using logarithms.

4. A man pays income tax at the rate of 1*s.* 2*d.* in £1 on one-third of his income and at the rate of 9*d.* in £1 on the rest. If his net income is £1146, 13*s.* 4*d.*, find how much income tax has been deducted.

5. Explain what is meant by (1) Simple Interest, (2) Compound Interest.

Write down an expression for the amount produced, when £x has accumulated for n years at r per cent. compound interest payable annually.

If a certain sum amounted to £5625, after accumulating for 3 years at compound interest payable annually, and to £6084 after 5 years, find the rate of interest.

6. A sphere of wax 6 inches in diameter is melted and part is poured into a cylindrical jar of 3 inches diameter until it is filled to the height of 10 inches. The remainder is poured into a cylindrical jar of 4 inches diameter. Find how high it will be filled.

(The area of a circle is πr^2 and the volume of a sphere is $\frac{4}{3}\pi r^3$ where r = the radius.)

THE INSTITUTE OF CHARTERED ACCOUNTANTS IN ENGLAND AND WALES.

PRELIMINARY EXAMINATION.

December 1910.

1. The table shows the number of coins of various denominations minted during 1909. Find the total number of pieces struck and their approximate value to the nearest pound.

Sovereigns . .	14,448,638	Sixpences . . .	6,688,929
Half-sovereigns .	4,877,[illegible]74	Threepences . .	4,221,109
Half-crowns . .	3,099,503	Pence	19,743,360
Florins	3,523,833	Half-pence . .	8,310,400
Shillings . . .	5,826,824	Farthings . . .	8,861,440

2. Owing to the operation of the Coal Mines Regulation Act, 1908, certain workers had their hours of labour changed. In 1909, of 562,891 people affected, 3212 had their aggregate working time increased by 6399 hours per week, while 559,679 had reductions amounting to 2,398,721 hours per week. What was the net effect of all the changes in the weekly working time of the people affected?

3. The table shows the amount and value of the imports of Canadian wheat since 1905.

Year.	Imports in Cwts.	Value in £.
1905	6,585,785	2,432,055
1906	11,118,555	3,972,354
1907	13,105,606	5,225,447
1908	15,438,180	6,335,329
1909	16,514,007	7,557,036

Illustrate these numbers on squared paper, using a common horizontal axis (of time) and specifying carefully the scales you employ.

Find, in shillings, the average price per cwt. in each year, giving the results to *three* significant figures only.

4. The number 3·1415926... is denoted by π, and represents the ratio of the circumference of a circle to its diameter. Show that $\frac{1}{\pi}$ is nearly equal to $\frac{1}{3}-\frac{1}{150}-\frac{1}{200}$ and more nearly equal to $\frac{1}{3}-\frac{1}{150}-\frac{1}{200}-(\frac{1}{200})^2$. Use each approximation to determine the circumference of a circle whose diameter is 2315·01 cm., giving *five* significant figures in your answer.

5. The time taken by a train to run a series of consecutive miles were 98·3 secs., 95·2 secs., 90·1 secs., 86·2 secs., 92·1 secs., 94·7 secs., 88·9 secs., 85·4 secs., 81·3 secs., and 94·6 secs. What was the average time of the train for one mile? What was the average speed of the train in (i.) miles per hour, (ii.) feet per second? When and where was the train moving most rapidly?

6. Goods sent from a warehouse are marked at 25 per cent. above cost price. A customer buys goods marked £17, 6*s.* 8*d.* and is allowed 5 per cent. on his bill. What profit did the vendor make? What was his percentage profit?

7. The cost of lighting a house by electric light depends on the number of lamps alight, their candle-power, and the time during which they are alight. In two houses the consumption is as follows:—

In house A:—25 sixteen candle-power lamps, each alight on the average for 112 hours.

In house B:—17 sixteen candle-power lamps, each alight on the average for 135 hours.

The bill for light in house A was £3, 7*s.* 5*d.*, what should be the bill in house B?

8. What is the total interest on £436 in 4 years at 2½ per cent. per annum, the interest being paid yearly?

9. What amount of French 5 per cent. Rentes at 99 can be pur-

chased for £3970? What yearly dividend would the holder be entitled to, and what would be the rate of interest received on his money? Brokerage 5s. per cent.

June 1911.

1. The table gives the times occupied on ten outward passages (from pier to pier) by the two Cunard turbine steamers, *Lusitania* and *Mauretania*, during 1908.

Lusitania		*Mauretania*	
Sailing Date	Time of Passage	Sailing Date	Time of Passage
Jan. 25	7 days 5 hrs	Jan. 11	6 days $4\frac{1}{2}$ hrs
Mar. 7	6 " 2 "	Feby. 23	5 " $19\frac{1}{2}$ "
April 4	5 " 16 "	Mar 21	6 " $15\frac{1}{2}$ "
April 25	5 " $22\frac{1}{2}$ "	April 11	5 " 19 "
May 16	5 " $23\frac{1}{2}$ "	May 2	6 " 19 "
June 6	5 " 19 "	May 27	5 " $19\frac{1}{2}$ "
July 4	5 " $13\frac{1}{2}$ "	June 13	5 " 23 "
July 25	5 " $19\frac{1}{2}$ "	July 11	5 " 21 "
Aug. 15	5 " $14\frac{1}{2}$ "	Aug. 1	5 " 17 "
Sept. 5	5 " $19\frac{1}{2}$ "	Aug. 22	5 " 20 "

Find the average time for an outward journey by each steamer.

2. If radium costs £730 per gramme, and 125 milligrammes are required to illustrate a lecture, the charge for admission to which is 5s., how many people would have to be present to pay the cost of the radium used?

3. Some of the following statements are obviously wrong; indicate any errors you can find, explaining your reasons. You need not actually correct the mistakes. The answers are supposed to be correct to *four* significant figures:—

(i.) The area of an equilateral triangle of 10 cm. side is 44·3 sq. cm.

(ii.) $\sqrt{0{\cdot}4816} = 0{\cdot}2194$.

(iii.) $4975 \times 600{\cdot}6 = 2{,}988{,}000$.

(iv.) The area of a circle whose diameter is 32·6 ft. is 8·347 sq. ft.

(v.) A railway has a vertical rise of 1 in 250 horizontal. Therefore the length of rail corresponding to a rise of 4 ft is 1000 ft.

4. The following measures are in use in China:—

1000 *tsun* = 100 *chik* = 10 *chang* = 1 *yin* = 117·5 ft.

Express 150 *yin* in miles, and 1 mile in *tsun*.

5. Sheets of tin-plate are sold in boxes containing varying numbers of plates of different sizes. In one box there are 225 sheets each

15″ × 11″ and of weight 1 cwt. 3 qrs. 19 lbs. What will be the weight of the sheets in a second box of 200 sheets each $13\frac{3}{4}$″ × 10″?

6. The table gives the imports and exports of the United Kingdom for the last eleven years.

Year.	Imports, £.	Exports, £.
1900	523,075,163	354,373,754
1901	521,990,198	347,864,268
1902	528,391,274	349,238,779
1903	542,600,289	360,373,672
1904	551,038,628	371,015,321
1905	565,019,917	407,596,527
1906	607,888,500	460,677,818
1907	645,807,942	517,977,167
1908	592,953,487	456,727,521
1909	624,704,957	469,520,166
1910	487,311,888	397,018,339

Represent these numbers on squared paper, using a common axis of time. What general inferences may be made?

7. Use the figures for 1910 in Question 6, and find in what year the exports would first exceed the imports if the exports increased by 5 per cent. in each year and the imports decreased by 5 per cent. each year.

8. Two stations, A and B, are 63·5 miles apart. A train leaves A and reaches B at 7.50 P.M. Another train starts from B at 6.32 P.M., passes the first train 31 miles from B, and arrives at A at 7.48 P.M. Suppose both trains run at uniform speeds without stops, and determine the time when the first train left A and its speed.

9. A sovereign weighs $123\frac{1}{3}$ grains, and $\frac{11}{12}$ of this weight is gold. A shilling weighs $87\frac{1}{4}$ grains, and $\frac{37}{40}$ of this weight is silver. If gold is worth £4, 4s. $11\frac{1}{2}$d. per. oz. of 480 grains, and silver is worth $24\frac{5}{8}$ pence per oz., find the value of the gold in a sovereign to the nearest penny, and of the silver in a shilling to the nearest farthing.

10. Find the change of income occasioned to an investor who sold £10,000 Chilian 5 per cent. Stock at $100\frac{1}{2}$–$101\frac{1}{2}$, and invested the proceeds in Japanese 6 per cent. at 101–$101\frac{1}{2}$. Brokerage $\frac{1}{4}$ per cent. on each transaction.

11. A merchant blends tea worth 1s. 4d. per lb. with tea worth 2s. If the mixture is worth 1s. 7d. per lb., in what proportion are the teas mixed?

Find, also, the amount of tea at 2s. per lb. which must be mixed with 1 ton 5 cwt. of tea at 1s. 4d. per lb. if the blend is to be sold at £2, 10s. per quarter.

12. Complete the following table :—

Principal.	Simple Interest.	Rate per Cent.	No. of Years.
£ s. d.	£ s. d.		
537 8 9		$3\frac{1}{2}$	10
	34 10 2	3	4
87 16 6	71 2 9	$4\frac{1}{2}$	

December 1911.

1. The figures given below deal with one year's working of British railways. Complete the table. [The increase or decrease per cent. is to be given to two significant figures only.]

	1909.	1910.	Increase or Decrease.	Increase or Decrease per Cent.
	£	£		
Ordinary Stock . .	493,100 000	492,100,000		
Pref. and Guaranteed Stock . . .	469,700,000	472,200,000		
Loans and Debenture Stock . . .	351,600,000	354,200,000		
Total Capital .				
Receipts—	£	£		
1st Class Passengers	3,272,000	3,408,000		
2nd " "	2,408,000	2,127,000		
3rd " "	31,658,000	32,935,000		
Season Tickets .	4,617,000	4,777,000		
Mails, Parcels, &c. .	9,255,000	9,511,000		
Gross Receipts—	£	£		
Passenger Traffic	...	...		
Goods Traffic . .	59·5 millions	61·5 millions		
Total Receipts .				

2. One inch is approximately equal to 2·54 cms. What is the percentage error in assuming that 10 cms. are practically 4 inches?

Express 1 kilometre in miles, correct to *three* significant figures.

Find also the number of square centimetres in 1 square foot.

3. Evaluate, as shortly as possible: (i.) 5 per cent. of 16*s.* 8*d.*; (ii.) $2\frac{1}{2}$ per cent. of 30*s.*; (iii.) the percentage increase in area if the diameter of a circle is doubled; (iv.) $(17{\cdot}5^2 - 12{\cdot}5^2)$.

4. Check the accuracy of the following statements. You are not asked to give the correct answer in each case, but to apply a *rough* check and see if the statement is approximately correct. Specify the nature of the check you apply, and the kind of error which has been made, if any.

(i.) The area of a circle $10\frac{1}{4}$-inch radius is about 3000 square inches.

(ii.) 0·304 tons; that is, nearly 68·1 lbs.

(iii.) At 1*s.* $2\frac{1}{2}$*d.* per square yard, the cost of painting a wall $36\frac{3}{4}$ feet long and $9\frac{1}{2}$ feet high would be, roughly, £7.

(iv.) The square root of 0·02 is approximately 0·014.

5. A bath can be filled from the hot-water tap in 5 minutes, and from the cold-water tap in 3 minutes, while it can be emptied by the waste-pipe in $7\frac{1}{2}$ minutes. How long will it take to fill the bath (i.) if both the hot- and cold-water taps are turned on and the waste-pipe blocked; (ii.) if both taps are on and the waste-pipe is left open?

6. Draw a graph from which one may obtain the amount of £100 at $2\frac{1}{2}$ per cent. compound interest for any number of years up to 5.

What general inference can be made from this graph?

How would it be modified if the interest were added (i.) half-yearly, (ii.) every three months, (iii.) continually?

Can the amount of other sums at the same rate per cent. and for the same time be obtained from your graph? If so, explain how, and read off the amount of £250 for 3 years at $2\frac{1}{2}$ per cent. compound interest.

7. The capital of a trading concern consisted of 4000 shares of £95 each and 3000 shares of £25 each. In sharing profits, 5 per cent. on amount of each share is first allotted, and the remainder, if any, is divided equally among the shareholders. In a certain year the profits were £36,894. How much would be received by the holder of (*a*) 20 shares at £95; (*b*) 100 shares at £25? What percentage does each receive on his money?

8. How much tea at 1*s.* 6*d.* per lb. must be mixed with 3 lbs. of tea at 10*d.* per lb. to make a mixture worth 1*s.* 3*d.* per lb.? If the merchant charge 1*s.* 4*d.* per lb. on the mixture, what is his profit per cent.?

9. A set of circular brass weights are to be 4″ diameter and to have a slot $\frac{1}{2}$″ wide and $2\frac{1}{4}$″ deep cut out. Assume 1 cubic inch of brass weighs 0·29 lb., and determine approximately the thickness of the 1-lb., 2-lb., and 5-lb. weights

June 1912.

1. The table gives the number of inches annual rainfall registered at the Royal Observatory during the last 50 years.

Year.	Inches.	Year.	Inches.	Year.	Inches.	Year.	Inches.	Year.	Inches.
1861	20·45	1871	22·30	1881	25·72	1891	25·04	1901	20·29
1862	26·32	1872	30·02	1882	25·18	1892	22·31	1902	19·34
1863	19·66	1873	23·36	1883	21·91	1893	20·12	1903	35·54
1864	16·38	1874	19·95	1884	18·05	1894	26·89	1904	20·22
1865	28·70	1875	27·97	1885	24·00	1895	19·73	1905	23·02
1866	30·72	1876	24·10	1886	24·21	1896	22·42	1906	24·74
1867	28·46	1877	27·28	1887	19·86	1897	22·13	1907	22·25
1868	25·15	1878	28·98	1888	27·51	1898	18·85	1908	23·78
1869	24·02	1879	31·30	1889	23·28	1899	22·33	1909	25·71
1870	18·55	1880	29·68	1890	21 86	1900	22·31	1910	28 06

Find the average annual rainfall during the last period of 10 years and during the whole period of 50 years. Illustrate your results and the given numbers graphically, carefully specifying the scales you use.

2. The figures given refer to the coal production of 1909. Complete the table.

	United Kingdom.	United States.	Germany.
Total output in tons	263,774,000	441,432,000	146,397,000
Total number of employees	992,330	666,560	613,220
Average price per ton at pit's mouth	8*s*. 0¾*d*.	5*s*. 7½*d*.	10*s*. 2½*d*.
Average output per employee			
Cash value of total output at pit's mouth			

In the same year the United Kingdom exported 86,037,000 tons; what percentage was this of the total output? Locomotives on railways consumed 12,273,800 tons; what percentage was this of the total output?

3. A sovereign is equivalent to 1000 mils. How many mils are equivalent to (i.) a shilling, (ii.) a farthing? Express in mils the value of £2, 7*s*. 2½*d*. Find, preferably by approximate methods, the cost of 6 tons 12 cwt. 1 qr. 7 lb. at 27*s*. 6½*d*. per ton.

4. A grocer buys tea at 2*s*. $2\frac{1}{2}$*d*. per lb. He makes 125 one-pound packets, each containing $15\frac{1}{2}$ oz. tea, and sells them at 2*s*. 6*d*. each. If the paper and string cost 4*s*. 8*d*., what is his total profit? What is his profit per cent.?

If he can and does put 1 lb. of tea into a packet, what must he charge for each in order to make the same profit per cent.?

5. A man pays 8*s*. in the £1 on his debts and I get £8. What did he owe me?

Questions of this standard form *may* be solved by the Unitary method. Discuss and explain the method, making references to the given example where necessary. State the question as a simple proportion and give an alternative solution.

Consider the following question: A rope stretches 1 cm. when loaded with 10 kilograms; how much would it stretch if loaded with 10^6 kilos? Explain why in this case the Unitary method fails. Construct two more examples in standard form to which the Unitary method would not apply, and explain why the method fails in each case.

6. If £4570 Caledonian Stock paying a dividend of $5\frac{1}{4}$ per cent. be sold at 153 and the proceeds invested in Queensland $3\frac{1}{2}$ per cent. Stock at $107\frac{1}{2}$, find approximately the change of income. Brokerage $\frac{1}{8}$.

7. Simplify— (i.) $\frac{.1919}{.2223}$.

(ii.) $\frac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{1}{8}}{\frac{1}{6} \text{ of } 3\frac{1}{3} + \frac{1}{3}} \div \frac{2\frac{1}{2}}{3\frac{1}{3}}$.

(iii.) $\frac{280 \text{ yards} \times 32 \text{ square yards} \times 84 \text{ lb.}}{0\cdot5 \text{ ton} \times 10 \text{ minutes} \times 12 \text{ inches} \times \frac{1}{2} \text{ mile}}$.

In (iii.) use the properties of ratios and interpret the final result.

8. Find the cost of discounting on April 4th, 1912, a bill of £580 maturing June 21st, 1912. (Mercantile discount at the rate of $3\frac{1}{2}$ per cent. per annum.)

SCOTTISH CHARTERED ACCOUNTANTS.

INTERMEDIATE ARITHMETIC.

December 1910.

1. An Indian officer whose annual pay was estimated in rupees lost £145, 5*s*. 10*d*. in one year by a fall in the value of the rupee from 1*s*. $10\frac{1}{4}$*d*. to 1*s*. $7\frac{3}{4}$*d*. What was his salary in rupees?

2. Multiply by contracted method 856·234 by 17·93 to one decimal.

3. Divide by contracted method 1 by 3·14159265 to five decimals.

4. If I lend a sum of money to a friend at $4\frac{3}{4}$ per cent. simple interest and he returns me £877, 12*s.* 6*d.* at the end of 4 years, what is the amount of the loan?

5. If a property is divided into 4 shares in the ratio of $\frac{3}{4} : 3 : 4\frac{1}{2} : 5\frac{1}{4}$, and the third share is £789, 5*s.* 3*d*, what is the total value?

6. What will be received for a bill of £500 drawn on 15th May at 3 months and discounted on 3rd June at $3\frac{3}{4}$ per cent.? Allow days of grace.

7. A man exchanges 45 horses for 700 sheep. If 125 horses are worth 300 cows, and 750 sheep worth 120 cows, find his gain per cent.

8. A person bought £187, 10*s.* of Bank Stock at $243\frac{3}{4}$ and received half-yearly dividends of £7, 17*s.* 6*d.* and £6, 15*s.* What annual rate per cent. of interest had he for his money invested?

9. Find the interest gained in the fourth year if £1000 is invested at 5 per cent. compound interest.

10. A man walking beside a railway at the rate of $3\frac{1}{2}$ kilometres per hour meets a train 154 metres long which completely passes him in 30 seconds, what is the speed of the train in kilometres per hour?

June 1911.

1. If $\frac{3}{8}$ of the agricultural land of a district is arable, $\frac{7}{16}$ pasture, and $\frac{2}{5}$ of the remainder woodland, and the remaining 1800 acres are common land, find the number of acres in the whole and the size of each kind of land.

2. Multiply 845·6 by 23·1783 by the contracted method correct to one decimal.

3. Divide ·0001 by 3141·592654 by the contracted method correct to 10 decimals.

4. Find by practice or otherwise the value of 38·75 acres of turnips at £21, 18*s.* per acre, and the weight of the turnips at 4 tons $12\frac{1}{2}$ cwts. per acre.

5. A tradesman bought a quantity of tea and sold it at 2*s.* per lb., gaining $33\frac{1}{3}$ per cent. If his total gain was £14, what weight of tea did he buy?

6. Find the ordinary discount on a bill for £500 drawn 10th March for 3 months and discounted on 5th April at $3\frac{1}{4}$ per cent. Allow days of grace.

7. Find the compound interest on £1350 for $1\frac{1}{2}$ years at $2\frac{1}{2}$ per cent., interest being added half-yearly.

8. A man sells out £1225 of $2\frac{3}{4}$ per cent. Stock at 109 and invests in the 4 per cents. at 140. Find the change in his income.

9. £1000 was divided between A, B, and C so that B had $\frac{7}{8}$ of A's share and C $\frac{6}{7}$ of B's share. Find the share of each.

10. Having given that a metre is equal to 39·3708 inches, find in miles per hour the speed of a train which takes $31\frac{1}{2}$ seconds to go 1 kilometre.

December 1911.

1. A person has $\frac{7}{8}$ of a property worth £4500. He sells $\frac{1}{3}$ of his share to A and $\frac{2}{3}$ of $\frac{3}{4}$ of the remainder to B. What fraction of the whole property has he left, and what is it worth?

2. Multiply by contracted method 567·23 by 9·845 correct to one decimal.

3. Divide by contracted method 7·325 by 459·6 correct to four decimals.

4. Calculate, by practice or otherwise, correct to a penny the cost of 92 tons 7 cwts. 3 qrs. 14 lbs. @ £1, 12*s*. 7*d*. per ton.

5. One pound of tea and four pounds of sugar cost 2*s*. 6*d*., but if sugar were to rise 50 per cent. and tea 10 per cent. they would cost 3*s*. 1*d*.; find the cost of tea and of sugar per lb.

6. To what does £1890, 16*s*. amount in 2 years at 5 per cent., compound interest added half-yearly?

7. A bill was drawn on 13th March 1906 for £325, 10*s*. at 9 months and discounted 23rd July same year. Find the discount at $3\frac{3}{4}$ per cent. per annum. Allow days of grace.

8. Divide £682, 10*s*. into 4 parts such that the third is half as much again as the fourth, the second half as much again as the third, and the first half as much again as the second. Find the value of the first part.

9. A person finds that if he invests his money in 3 per cents. at 95 his income will be greater by £2 than if he invests it in $3\frac{1}{2}$ per cents. at 114; find the sum invested.

10. A gallon of water weighs 10 lbs and a litre of water 1 kilogram. Having given that 1 litre = 1·76 pints, express 1 lb. as the decimal of 1 kilogram.

June 1912.

1. What fraction is that from which if $\frac{2}{3}$ of $\dfrac{3-1\frac{1}{4}}{2\frac{2}{5}}$ be taken and the remainder divided by $\dfrac{5\frac{1}{3}}{16\frac{2}{3}}$ the result is $\frac{1}{3}$?

2. Multiply 56·78125 by 3·0125 correct to 1 decimal.

3. Divide 9·613425 by 768·26123 correct to 4 decimals.

4. By practice or otherwise find correct to the nearest penny the dividend on £1437, 15*s*. at 8*s*. $5\frac{1}{2}$*d*. in £1.

5. The wages of a man working 8 hours a day are 31*s*. 6*d*. per week. If a woman works 10 hours a day and 3 women do in an hour as much as 2 men, what should a woman's weekly wage be?

6. At the pit mouth coal costs 12*s*. per ton, freight adds $37\frac{1}{2}$ per cent. to the value, and the retailer takes a profit of $8\frac{1}{3}$ per cent. on his

total outlay. What is the retail price of the coal, and how much per cent above its price at the pit mouth?

7. Find correct to the nearest penny the difference between the ordinary and true discounts on £125, 8s 6d. for 6 months at $3\frac{1}{4}$ per cent per annum

8. Divide £954, 9s. between A, B, and C so that A's share may be $\frac{3}{5}$ of B's share, and B's $\frac{10}{11}$ of C's share. Find B's share

9. What ought to be the price of £100 Bank Stock, which pays a dividend of $10\frac{1}{4}$ per cent, in order that it may yield 4 per cent on the money invested?

10. A tavern keeper mixes 200 litres of wine at 60 centimes per litre, 150 litres at 80 centimes, and 80 litres at 90 centimes. What is a litre of the mixture worth? Express this in British money. (25 francs = £1)

December 1912.

1. Simplify

$$\left(\frac{\frac{4\frac{3}{8}}{\frac{7}{10}} - \frac{1\frac{4}{7}}{\frac{8}{21}}}{\frac{1\frac{1}{7}}{4} + \frac{\frac{3}{4}}{2\frac{1}{3}}}\right) + \frac{1762 \times 26 - 0017}{032 \times 015}.$$

2. Multiply, by the contracted method, 537 84 by 9 25 correct to the unit's place.

3. Divide, by the contracted method, 2 516 by 8 479 correct to 3 decimal places.

4. Find, by practice or otherwise, the value of 18 tons 12 cwt. 2 qr. 14 lb. at £7, 12s. per ton

5. If 10 eggs are bought for a shilling, how many must be sold for £1, 2s 6d. to yield a profit of $12\frac{1}{2}$ per cent?

6. A sum of money put out at compound interest amounts in two years to £1760, 8s. 4d., and in three years to £1830, 16s 8d. Find the rate of interest and the original sum invested

7. Find the banker's discount, on a bill of £573, 0s. 8d, dated 4th March at 4 months, and discounted 3rd May at 5 per cent. Assuming the banker pays 5 per cent. for his money, what profit does he make?

8. Eight tons of coal are divided between four families. The first consists of 6 persons, the second of 8 persons, the third of 5 persons, and the fourth of 9 persons. Each person got the same amount. What did each family receive? (Answer in tons and cwts)

9. A man invests £4275 in a $3\frac{1}{2}$ per cent. stock at 95, and sells out at $103\frac{1}{2}$. He invests the proceeds in a 4 per cent. stock at 108. What is the new income?

10. Express 5 francs per metre in terms of pence per yard, assuming that £1 is equivalent to 25 francs, and 1 metre to 39·37 inches.

INTERMEDIATE :—ANNUITIES.

The following are some of the questions set in recent years.

1. Deduce a formula for the amount at compound interest of an annuity certain of £1 payable yearly for n years. Define carefully the notation you employ.

2. Find the value at 5 per cent. interest of a perpetuity of £40 payable in advance.

3. Deduce a formula for finding the present value of an annuity to continue for a certain number of years, allowing compound interest. Define carefully the notation employed.

4. What at simple interest is the amount of an annuity of £40 for 8 years at 5 per cent.? What would be the amount of this same annuity at compound interest?

5. Find the amount of an annuity of £50 for 6 years with interest and instalments payable half-yearly, the nominal rate of interest being 5 per cent. Assume $(1\cdot025)^{12} = 1\cdot34489$.

6. What at compound interest is the present value of an annuity of £40 for 8 years at 5 per cent.? Assume $(1\cdot05)^{8} = 1\cdot4774$.

7. For the amount of an annuity of £1 payable yearly the general formula is $S = \frac{(1+i)^n - 1}{i}$. Explain the notation. Transform the above into a formula which will give the number of years the annuity has to run, when the amount and the rate of interest are given.

FINAL :—ACTUARIAL SCIENCE.

The following are some of the questions set in recent years.

1. What sum at 4 per cent. interest payable half-yearly will in 2 years amount to £1000 after allowing for an income tax of 1*s.* in £1.

2. Find approximately in how many years 2*s.* 6*d.* will become £1 if interest is compounded yearly at 4 per cent.

3. Find the amount of an annuity of £25 for 8 years at 5 per cent. simple interest.

4. Find the effective rate which corresponds to a nominal rate of 5 per cent. convertible quarterly.

5. Explain the difference between *commercial* discount and true discount, and give a formula for true discount, (1) at simple interest, (2) at compound interest.

6. Show that the true discount on 1 due n years hence, interest at i is equal in value to an annuity of i to run for n years, and show by reasoning the correctness of the result.

7. A borrower pays £633, 12*s.* in settlement of principal and interest on a loan for 8 years at 4 per cent. simple interest.

(*a*) What was the sum advanced?

(*b*) How much more would the debtor have to repay if the interest had been compounded yearly?

8. How would you construct and verify tables of amounts and present values of annuities certain?

9. Show that the value of an annuity certain payable *m* times a year is equal to the value of an annuity payable yearly multiplied by the effective rate and divided by the corresponding nominal rate.

10. Find the amount and from it the present value of an annuity certain of £100 for 5 years payable half-yearly, the nominal rate of interest being 6 per cent.

$$(1\cdot03)^5 = 1\cdot15927. \qquad \frac{1}{(1\cdot03)^{10}} = 0\cdot744.$$

11. Find the effective rate corresponding to a nominal rate of 4 per cent. payable quarterly.

12. To what sum will £50 accumulate in 4 years at $4\frac{1}{2}$ per cent. payable half-yearly? Give the answer correct to 3 places.

13. Give two formulæ for the value of a deferred annuity in terms of an immediate annuity, and show how these are arrived at.

14. How would you construct and verify (*a*) a table showing the amount to which £1 will accumulate in any number of years, and (*b*) a table showing the present value of £1 due at the end of any number of years?

15. What is the value at 5 per cent. compound interest of a perpetuity of £10 payable in advance?

16. Find in how many years a sum of money will double itself at compound interest. Given $\log_e 2 = \cdot69$.

17. A 5 per cent. Stock redeemable in 10 years at par is purchased at 105. How much requires to be set aside each year out of the dividend to provide for the loss on redemption? Given $a_{\overline{10}|} = 7\cdot722$.

18. Given $v^{30} = \cdot267000$ and $(1+i)^{20} = 2\cdot411714$. Find v^{50} and $(1+i)^{10}$ correct to 3 decimals.

19. What is the value, correct to 3 decimals, at 4 per cent. compound interest of an annuity certain of £100 for 4 years, deferred 4 years; when would the first instalment be payable? Given $v = \cdot9615$.

20. What sum would require to have been invested 4 years ago to produce £1000 now, interest being accumulated at 4 per cent. half-yearly, allowing for deduction of income tax @ 1*s.* 2*d.* per £1.

21. What is the value at 5 per cent. interest of an annuity certain of £60 for 10 years deferred 7 years?

22. Find the annuity which will repay a loan of £1000 in 8 years at 5 per cent. interest.

INSTITUTE OF BANKERS.

COMMERCIAL ARITHMETIC.

1909.

1. Find correct to 3 decimal places the values of:—

(i.) $246 \cdot 9251 \times \cdot 008729$.
(ii.) $467 \cdot 92 \div \cdot 387$.
(iii.) $346 \cdot 25 \times 3 \cdot 2164 \div 73 \cdot 296$.

2. Decimalise £5, 13*s.* 8½*d.*, and multiply the result by 6938 correct to 3 decimal places, and finally express this product as pounds, shillings, and pence.

3. Find the difference in francs and centimes between 8000 dollars and 70,000 marks, if 4·85 dollars = £1, 25 francs 21 centimes = £1, and 20 marks 51 pfennige = £1. 1 franc = 100 centimes, 1 mark = 100 pfennige.

4. If the rate of exchange between Germany and Britain be 20·52 marks for £1, make a table showing in three columns the values in marks and pfennige of 1, 2, 3 up to 9 £'s; 1, 2, 3 up to 9 shillings; and 1, 2, 3 up to 9 pence; then use this table to write down the values of £69, 15*s.* 6*d.* and £84, 18*s.* 9*d.* in marks and pfennige.

5. Find the equivalent of an interest of 1½*d.* per £1 per month in terms of centimes per 20 franc piece per week. What rate per cent. per annum is this? (Take £1 = 25 22 francs, 1 month = 4 weeks, and 1 year = 12 months.)

6. A money-lender regularly advances to an old age pensioner 2*s.* 6*d.* two days before his pension of 5*s.* per week is due. When the pensioner, who is an old soldier, receives his soldier's pension, at the end of every 28 days, he pays the money-lender 10*s.* 3*d.*, thus paying 3*d.* for the accommodation he has received in that time. How much per cent. per annum does the money-lender receive for his advances? (1 year = 365 days.)

7. Two merchants, A and B, have run accounts with one another. A owes B £150, and B owes A £60. If both parties agree to allow the same rate of discount off their respective accounts, to whose advantage alone is this arrangement? Illustrate your answer by taking a 5% discount. What would the discount be if in settling A has to pay B £76, 10*s.* only?

8. A loan of £200 to a merchant is to be repaid by half-yearly instalments of £40, the first to begin six months after the advance is made. If interest is calculated at 5% per annum on the respective outstanding balances of the loan, what is his indebtedness to the nearest penny after having paid the fifth instalment?

9. If the rate of exchange in London on Paris be 25 23 francs for £1, and the rate of bankers' discount in London be $2\frac{1}{8}\%$ per annum for a three months' bill, what debt can a London merchant discharge in Paris who holds a three months' bill for £600?

10. By investing in the $2\frac{1}{2}\%$ Consols at the present price you can obtain a return of 3% per annum on your investment find (1) the present price of the $2\frac{1}{2}\%$ Consols, and (2) how much *stock* can be bought for £1000. (Include brokerage at $\frac{1}{8}\%$ as part of money invested)

11. A tradesman sells an article for £25 so as to obtain a gain of 15% on the *selling* price What is the *cost price* of this article, and what gain per cent does he make on this *cost price*?

12. A shareholder in an industrial company receives a half-yearly final dividend at the rate of 4% per annum, which, with income tax at 1*s* in the £1 deducted, amounts to £46, 11*s* If the shares are £1 shares, how many of them does he hold?

13. The duty on an article was reduced from 6*d* to 4*d*, but the consumption of that commodity increased by 20% What is the decrease per cent in the revenue derived from this tax What increase per cent in the consumption would have left the revenue unchanged?

14. Half-crown shares in a mining company can be bought for 6*s* 6*d*, if the dividends paid during the year amount to 7*d* a share, what rate per cent is this? What rate per cent per annum does an investor receive who bought these shares at that price, viz 6*s* 6*d*? (Neglect brokerage)

1910.

1. Find the value of any *three* of the following:—

(i) 756 × 89 24 correct to 3 decimal places

(ii) 3 5618 − 81 79 correct to 3 decimal places.

(iii) $(1.08)^{10}$ correct to 6 decimal places.

(iv) $\sqrt[3]{(32\ 157432)}$

2. Taking 4 dollars 86 cents and 25 francs 18 centimes as each equivalent to £1, construct a table showing the value of 1 up to 9 dollars in terms of francs and centimes. Use this table in finding the equivalents of 986 dollars 75 cents and 785 dollars 25 cents in terms of francs and centimes

3. The following sums of money have to be added together — £4639, £3874, £2956, £3348, and £5396. One clerk obtains a total £360 less and another £1800 more than the true total. What makes you suspect that these errors are due to transposition of two consecutive digits in writing down the separate items before adding? And in what items has the transposition occurred in the above two cases in order to produce those errors?

4. Find the amount at compound interest of £50, 7*s*. 6*d*. for 10 years at $2\frac{1}{2}$ per cent. per annum, the interest being added on yearly.

5. Compare the rates of interest yielded by investments in the following Stocks at the quoted prices (neglecting brokerage throughout): Consols $2\frac{1}{2}$ per cent. at $81\frac{15}{16}$; German 3 per cents. at $84\frac{1}{4}$; and French 3 per cents. at $97\frac{1}{2}$. Give your percentages in each case as a whole number and three places of decimals in order to facilitate comparison, and then place them in order.

6. If 155 Napoleons (French 20-franc gold pieces) weigh 1 kilogramme, and 1869 sovereigns weigh 40 lbs. Troy, how many sovereigns contain the same amount of gold as 155 Napoleons? (1 kilogramme = 15,432 grains, 1 lb. Troy = 5760 grains; the Napoleon is $\frac{9}{10}$ fine, and the sovereign is $\frac{11}{12}$ fine.)

7. A person arranges to receive an advance of £50 on the 1st day of each month from January to June inclusive. On June 30th he pays off the loans with interest on them at 6 per cent. per annum. What does the settling payment amount to?

8. An investor puts $\frac{1}{3}$ of his capital in an undertaking yielding 10 per cent. per annum, $\frac{1}{2}$ of his capital in debentures yielding 4 per cent. per annum, and the remainder in preference shares yielding 6 per cent. per annum. His entire income from these investments is £760; what is the average rate of interest that he is receiving on his capital, and what is the amount of this capital?

9. When the railway fare between two places was reduced by 20 per cent. the number of passengers increased by 30 per cent. in the year and amounted to 2,600,000. What was the number of passengers before the reduction was made? And if the original fare was 5*d.*, what difference does the change make in the earnings of the company for the year?

10. The rate of exchange between London and Paris was 25·18 francs per £1, between Berlin and Paris 82·50 marks per 100 francs, and between Berlin and London 20·47 marks per £1. What would it cost a London merchant to discharge a debt of 3000 marks in Berlin (1) by remitting direct to Berlin, and (2) by remitting through Paris?

11. An industrial company has an issued capital of 360,000 5 per cent cumulative preference shares of £1 each, fully paid, and 360,000 ordinary shares of £1 each, also fully paid. The dividends on the preference shares are paid quarterly, and on the ordinary shares an interim dividend of 6*d.* per share was paid at the end of the half-year, and a final dividend of 9*d.* per share at the end of the year. After paying these dividends and placing to General Reserve £15,000, to Capital Reserve £2000, and to Income Tax Reserve £750, a balance of £11,380, 7*s.* 11*d.* is carried forward to the next year's account. What must the net profits for the year's trading have been if a balance of £5357, 0*s.* 2*d.* was brought forward from the previous year? Give also the rate per cent. per annum received by the ordinary shareholder.

12. Find the interest on £735 for 81 days at $3\frac{1}{4}$ per cent. per annum.

13. A merchant marks his goods at 25 per cent. above cost price,

but allows his customers 10 per cent. discount off their bills for cash. What percentage of profit does this give him on the *cost price?*

14. A person holds 400 £1 shares in a refreshment catering company which pays two *half-yearly dividends*—an interim dividend at the rate of 25 per cent. *per annum*, and a final dividend at the rate of 40 per cent. *per annum*—from both of which income tax at the rate of 1*s*. 2*d*. in the £1 has to be deducted. He also holds 600 £1 shares in a mining company which pays quarterly dividends of 1*s*. 3*d*. per share, *free* of income tax. What is his entire income *free* of income tax? And if the catering company's shares are quoted at £6 and the mining shares at £1$\frac{3}{4}$, what rate per cent. per annum does his net income (*i.e.* after paying income tax on the dividends of the former) form of the amount which these shares would realise at the above prices?

1911.

1. Find the value of any three of the following:—

(i.) $3{\cdot}58067 \times 289{\cdot}3785$ correct to 3 decimal figures.

(ii.) $1{\cdot}765489 \div 26{\cdot}493876$ correct to 3 significant figures.

(iii.) $\dfrac{\sqrt{8}}{\sqrt{5}-\sqrt{2}}$ correct to 3 decimal figures.

(iv.) $(1{\cdot}06)^{12}$ correct to 6 decimal figures.

2. Find the difference in £ *s*. *d*. between 125,000 francs and 75,000 marks, when $£1 = 25{\cdot}27$ francs and $£1 = 20{\cdot}47$ marks.

3. The highest price of the 2$\frac{3}{4}$ per cent. Consols in 1896 was 113$\frac{7}{8}$, and in 1911 the price of the 2$\frac{1}{2}$ per cent. Consols is 80$\frac{1}{4}$; what rate per cent. per annum of interest is yielded by investing at these prices respectively?

4. Find a multiplier for converting £s per acre into marks per hectare, and use it in finding the equivalent in marks per hectare of £3, 10*s*. per acre, and the equivalent of 125 marks per hectare in terms of £s per acre. (1 hectare $= 2{\cdot}471$ acres and $£1 = 20{\cdot}46$ marks.)

5. A person on the 27th July applied for 1500 6 per cent. Cumulative Preference shares of a public company and was allotted them on August 2nd. By the terms of the issue, 2*s*. 6*d*. per share is paid on application, 3*s*. 6*d*. per share on allotment, 7*s*. per share on 31st August, and the balance of 7*s*. per share on the 30th September. Find the first half-yearly dividend which he receives on 31st December, the interest being calculated from the dates of payment of the instalments.

6. By selling an article for £12 a manufacturer gains *one and a-half times* as much as he would have *lost* had he sold it for £10. Find (1) the cost price of that article; (2) the gain per cent. made by selling it at the former price; and (3) the loss per cent. sustained by selling it at the latter price.

7. What sum of money would earn as interest 1*s*. 6*d*. per day at 3$\frac{1}{2}$ per cent. per annum?

8. A German manufacturer sells an article for 65 marks, which yields him a profit of 8⅓ per cent. on his outlay. If he could at least double his sales, the cost of manufacture would be reduced by 5 per cent.; at what price now could he afford to sell the article so as to make a profit of 10 per cent. on his outlay?

9. A money-lender advances to a client £5 on the 1st of each month, agreeing to accept interest on his loans at the rate of 8 per cent. per annum. What ought he to receive at the end of the year from his client in complete discharge of the loans?

10. Taking 25 francs 26 centimes and 20 marks 47 pfennige as each equivalent to £1, construct a table showing the value of 1 up to 9 marks in terms of francs and centimes. Use this table in finding the equivalents of 5786 marks 75 pfennige and 4029 marks 25 pfennige, in terms of francs and centimes.

11. An interest of 3 farthings per week on 30*s.* is equivalent to an interest of how many centimes on 20 francs per month? From this answer obtain the rate per cent. per annum.

12. Two merchants run *contra* accounts with one another, agreeing to allow each other the same rate of discount off their accounts at the end of the year. At the end of the year A owes B for goods priced at £200, and B owes A for goods priced at £60. Which of the two merchants benefits by this arrangement about discount? and what rate of discount must have been agreed upon if the year's accounts are settled by A paying B £119?

13. A New York merchant owes 30,000 marks in Berlin, the exchange at New York on Berlin being 95 dollars for 400 marks, and on London 4·86 dollars for £1 sterling. If the exchange at London on Berlin is 20·47 marks per £1, will it be better for the merchant to remit direct from New York or through London, and what difference will it make?

14. In a South African gold mining company the yearly dividends on a £1 share amount to 11*s.*, and in a Westralian gold mining company they amount to 6*s.* on the £1 share. When the price of the former is quoted at £2⅞ and the price of the latter at £1$\frac{1}{16}$, find in each case the rate per cent. per annum obtained by an investor who buys at these prices and obtains these dividends.

1912.

1. Find the values of:—

(i.) $2{\cdot}\dot{1}\dot{5} - {\cdot}005\dot{7}6\dot{8}$ correct to 3 decimal places.

(ii.) $32{\cdot}6784 \times {\cdot}08769$ correct to 5 decimal places.

(iii.) $(1{\cdot}07)^{16}$ correct to 4 decimal places.

2. Find a multiplier for converting grammes per tonne into dwts. per ton, and use it in finding the equivalent in dwts. per ton of 25

grammes per tonne, and the equivalent of 18 dwts. per ton in terms of grammes per tonne.

(1 tonne or metric ton = 1000 kilogrammes = 1,000,000 grammes; 1 ton = 2000 lbs., 1 lb. = 7000 grains, and 1 gramme = 15·43235 grains, 1 dwt. = 24 grains.)

3. Calculate the Mint par of exchange between London and Paris, and between Paris and Berlin; the weight of a sovereign being 123·27447 grains and $\frac{11}{12}$ths fine, of a 20-franc gold piece 99·561 grains and $\frac{9}{10}$ths fine, and of a 20-mark gold piece 122·91795 grains and $\frac{9}{10}$ths fine.

4. What sum of money will earn in 3 years at 5 per cent. per annum £37, 7*s.* 3*d.* more at *Compound* interest (calculated yearly) than it will at *Simple* interest for that period at the same rate?

5. If the present price of the $2\frac{1}{2}$ per cent. Consols is $78\frac{3}{16}$ and the dividends on them were paid free from the deduction of income tax at 1*s.* 2*d.* in the £1, what difference would this concession make in the net income of an investor who spent £7048, 2*s.* 6*d.* in the purchase of Consols at that price and paid a brokerage of $\frac{1}{8}$ per cent.? Also calculate the rise in Consols which would just neutralise the effect of this remission of income tax.

6. In dealing with the addition of sums of money an error occasionally arises from making the last two digits of a number of £'s in a sum of money become shillings and pence respectively. What test would you apply to ascertain if the error in the total is due to that cause, and if an error from that cause has reduced the total by £5332, 11*s.* 6*d.*, what is the sum of money which has been so treated?

7. A merchant who allows 15 per cent. discount for cash on his listed prices and wishes to make 25 per cent. profit on his own outlay, buys an article which costs him £680. At what price should he put this article in his price list?

8. A dealer in house property buys a leasehold house in a London suburb for £150: he spends £100 in converting it into 3 flats, for which he gets 7*s.*, 15*s.*, and 10*s.* a week respectively. Allowing per year £8 for repairs, &c., £16 for local rates, £15 for ground rent, and £5 for redemption fund *to* provide for capital outlay, what percentage does he make on his investment if all 3 flats are constantly let?

9. The American Eagle or 10-dollar gold piece weighs 258 grains and is $\frac{9}{10}$ths fine, and $934\frac{1}{2}$ sovereigns weigh 20 lbs. Troy, the sovereign being $\frac{11}{12}$ths fine. Calculate how many sovereigns contain the same amount of gold as 1500 Eagles. (1 lb. Troy = 5760 grains.)

10. One dealer fixes the credit price of a motor car he has on sale at £420 and another dealer fixes *his* credit price of the same car at £405. The first dealer allows a discount off his credit price for cash half as large again as that allowed by the second dealer off his, and yet their cash prices are the same. Find the respective rates of discount and the cash price of car for both.

11. If 132 half-crowns weigh 60 oz. Troy, calculate the intrinsic value of the silver in one half-crown at the present price of silver, $27\frac{1}{8}d.$ per ounce. (Standard silver is $\frac{37}{40}$ths fine.)

12. Thirteen parts by weight of gold are mixed with seven parts by weight of silver, and the total weight is 49 ounces. What is the value of the mixture, taking the gold and silver to be worth respectively £4, 4*s.* $11\frac{1}{2}d.$ and 2*s.* $3\frac{1}{2}d.$ per ounce.

BANKERS' INSTITUTE OF SCOTLAND

FOR ADMISSION AS ASSOCIATES.

1909.

1. Add $\frac{3}{26}$ of $\frac{3}{48}$ of 8*s.* 2*d.* to ·0625 of £6, 13*s.* 4*d.* and express the sum as the decimal of £2, 0*s.* 9*d.*

2. Simplify $\frac{48\cdot358 + 24\cdot1446}{9\cdot82 - 1\cdot97}$ and find the square of the result.

3. In a bank the interest on overdrafts is applied annually to the principal. At the end of 3 years a certain overdraft amounted to £1041, 17*s.* 3*d.* What sum was originally advanced supposing the interest to have been added yearly at 5 per cent.?

4. How much must be invested in a company paying $3\frac{3}{4}$ per cent. whose £5 shares (£4 only being paid up) are quoted at £5, 10*s.* so as to obtain a dividend of £6. What must be paid by the owner of the shares if the remaining £1 per share is called up?

5. The Deposit Receipt rate on 1st January 1906 was 4%. On 16th January it fell to $3\frac{1}{2}$%; on 23rd January to $2\frac{1}{2}$%; on 5th March to 2%; on 19th March to $1\frac{1}{2}$%; and on 28th May to 1%, at which it continued till the end of the year. What was the average rate during the year? Calculate to the nearest penny the interest due on a Deposit Receipt for £500 for the whole year.

6. A London merchant wishes to pay a debt of 6400 francs to a merchant in Paris when the course of exchange is quoted at 25·40 in London and 25·45 in Paris. Calculate to the nearest penny the gain to the former merchant if instead of remitting he be drawn upon by the latter.

7. An investor holding £4205 of $2\frac{1}{2}$ per cent. Consols calculates that by selling at 84 and investing the proceeds in 5 per cent. railway debentures at the current price he will improve his income by £63, 1*s.* 6*d.* What is the price of the debenture stock?

1910.

1. Find by practice or otherwise the value of 1735 ozs. 13 dwts. 12 grs. of gold at £3, 17*s.* 9*d.* per oz.

2. What is the difference between true and ordinary discount? Find the banker's discount on a bill for £250 drawn on 5th April at 4 months and discounted on 10th April at $3\frac{1}{2}$ per cent. per annum.

3. If 5763 bushels of wheat are imported for £1800, 18*s.* 9*d.* and an import duty of $10\frac{1}{3}$ per cent. is paid on the purchase money, what is the amount of duty paid on each bushel?

4. A property was destroyed by fire and the owner received from the insurance company £2760, which was $\frac{2}{3}$ of the amount of the policy. The property had been insured at $\frac{3}{4}$ of its value, and the premium was £66·24. Find the value of the property and the rate per cent. of the premium.

5. On starting business A had a capital of £6000. B was admitted as a partner 4 months afterwards with a capital of £4000. At the end of the year the profits amounted to £2900. What share of this should each receive in proportion to their capitals supposing the rate of gain before the admission of B was a half more than afterwards?

6. One company pays a dividend of $6\frac{1}{2}$ per cent. on its stock. Another pays at the rate of 14*s.* per share on shares of £20 each. An investor buys the stock at a premium of $37\frac{1}{2}$ per cent., and the shares at a discount of £1, 15*s.* per share. Compare the rates of interest which these investments yield.

7. The exchange between London and Paris is 25·5 francs for £1; between Paris and Amsterdam 117 francs for 55 florins; between Amsterdam and Hamburg 8·25 florins for 13 marks. What is the equivalent in London of 2905 marks in Hamburg?

1911.

1. Simplify (i.) $(12\frac{5}{7} - 10\frac{6}{7}) \div (1\frac{1}{1}\frac{2}{3} + \frac{3}{7}$ of $3\frac{3}{4}$ of $3\frac{4}{5})$.
(ii.) Express £5, 10*s.* 5*d.* as the decimal of £100.

2. If the manufacturer of an article makes a profit of 20 per cent., the wholesale dealer a profit of 25 per cent., and the retail dealer a profit of 40 per cent., what is the cost of manufacturing an article retailed at 17*s.* 6*d.*?

3. The Bank discount on a bill due 8 months hence is £186, and the true discount is £6 less. Find the amount of the bill and the rate per cent. at which it was discounted.

4. The capital and reserves of a bank amount to £3,000,000, the reserves being $\frac{1}{12}$ of this sum. If the bank obtains 4 per cent. for its funds, what dividend can be paid to the shareholders from the revenue derived from the reserve alone?

5. A life policy for £2000 effected in 1888 by payment of a single premium of £52, 10s. per cent. becomes a claim in 1910. What profit on the transaction is made by the insurance company which obtains $3\frac{1}{2}$ per cent. for its investments, it being given that at this rate of compound interest money doubles itself in 20 years?

6. An investor bought £400 bank stock at 452 per cent., and receives half-yearly dividends of £40, less income tax at 1s. 2d. in £1. What rate per cent. per annum had he for his money invested?

7. Bills on Paris are bought in London for £250, 10s. at the rate of 25·30 francs per £1. They are remitted to Vienna and are negotiated there at the rate of 178 francs for 100 florins. What sum is obtained for them?

Had bills on Vienna been bought in London at the rate of 14·15 florins per £1, how much more or less would the holder in Vienna have received?

1912.

1. Find the value of—

$2\frac{3}{11}$ of 4s. 7d. + $\cdot 2\dot{3}$ of 7s. 6d + $\cdot 0\dot{1}$ of $1\frac{1}{8}$ of 1s. 8d.;

and reduce the result to a decimal of £60, 18s. 9d.

2. What is meant by the Highest Common Factor and Least Common Multiple of two numbers? Show that the product of two numbers is equal to the product of their H.C.F. and L.C.M. What is the L.C.M. of 5698 and 6105?

3. A bankrupt's debts amount to £5430, 6s., and the dividend is reduced from 3s. to 2s. 4d. in the £1 by the admission of a claim to a preferable ranking. Find the amount of such claim.

4. A cargo was bought for £10,282 payable 12 months hence, and was immediately sold again for £10,712 payable 8 months hence. What was the gain per cent. on the value of the purchase money, the rate of discount being 6 per cent.?

5. The gross rental of an estate is £2437, 10s., and deductions arising from rates and taxes at 3s. in £1 and from interest on a bond of £2600 amount to 19 per cent. of this rental. What is the rate of interest paid on the bond?

6. The capital of a company consists of £1,100,000 in $4\frac{1}{2}$ per cent. debenture stock, the same amount in 6 per cent. preference shares, and the same amount in ordinary shares. If the net profits of a year's trading are £209,000, calculate the amount per cent. available for dividend on the ordinary shares, after providing for the debenture and preference charges, placing £46,000 to reserve and carrying forward £20,500.

7. When bills on New Orleans are at a discount of 5 per cent., for what sum in dollars must a Manchester cotton broker draw a bill of £520 which he wishes to remit to New Orleans? (Take 1 dollar = 49·29 pence.)

THE LONDON CHAMBER OF COMMERCE
(INCORPORATED).

EXAMINATION FOR JUNIOR COMMERCIAL CERTIFICATES, 1909.

ARITHMETIC.

1. Without finding the square root explain why 1472328 cannot be a perfect square. Having given that $\sqrt{2} = 1.414$, find, by factoring, the square root of 1472328 correct to four significant figures.

2. The number obtained on dividing 1 by a certain integer is 0·0149. Find the integer and the next figure of the quotient.

3. Of integers less than 40 how many are prime and how many are prime to 40? How many sets of numbers are there each composed of two integers prime to each other and having their product equal to 40?

4. A chemist in making a mixture which should contain 9 grains of a drug to the ounce of water, puts by mistake 36 grains only into an 8 oz. bottle; the mistake is discovered when 2 ounces of the solution have been used. How much of the drug should now be added to the remainder of the solution to make it of correct strength?

5. Of 4675 volumes in a section of a library 44 per cent. are in English; in the other section there are 1008 books in English, which is 38·4 per cent. of the number of volumes in that section. Find the percentage of books in English in the library.

6. Workmen go on strike to get an increase of $4\frac{1}{6}$ per cent. in their wages, which will just make good a reduction made three years ago. How much per cent. were their wages reduced on that occasion?

7. A certain map is on a scale of six inches to a mile. What area to the nearest acre is represented by a rectangle 2·4 in. × 3 in., and what area on the map expressed in square inches, correct to three figures, will represent a farm of 550 acres?

8. If the distance from London to Chester is 180 miles, from Chester to Holyhead 84 miles, and from Holyhead to Dublin 70 miles, how long will the boat wait at Holyhead for the trains if a fast train leaves London one hour before the boat leaves Dublin and two hours before a slow train leaves Chester. Average rates.—Fast train, 40 miles per hour; slow train, 24 miles per hour; boat, 16 miles per hour.

9. A year contains $365\frac{1}{4}$ mean solar days and $366\frac{1}{4}$ sidereal days. If two clocks, one keeping mean solar time, and the other sidereal time, are together at 12 midnight, December 31st—Jan. 1st, what does the sidereal clock show at 12 noon on 5th January (mean solar time), and what is the mean solar time when the sidereal clock shows 6 on the afternoon of 5th January? Answers to nearest second.

1910.

1. What do you understand by the following terms: prime number, prime factor, composite number? Find, by the method of prime factors, the Highest Common Factor and Least Common Multiple of 3456, 4212, and 5427.

2. From the following data find the percentage of pensioners (correct to two significant figures) for each country and for Great Britain and Ireland:—

	Population.	Number of Pensioners.
England and Wales . . .	35,756,615	421,432
Scotland	4,877,618	75,134
Ireland	4,374,158	186,202

3. The driving wheel of a locomotive is 14 feet in circumference. If there is no slipping, how many times does it revolve in a journey of 64 miles? If it revolves 11,000 times in 28 miles, what distance, in miles, was lost in consequence of slipping?

4. The area of a cistern is 100 square feet, its depth is 10 feet. How many gallons of water will it hold, supposing 100 gallons occupy 16 cubic feet? If the area of the base were 100 square metres and the depth 10 decimetres, how many litres of water would it hold?

5. A map is drawn to a scale of 1 inch = 5 miles. If the map is 7 feet long and 3 feet 5 inches wide, what area, in square miles, is represented? What is the distance between two towns situated at the extremities of one of the diagonals?

6. A gas meter, after being used for 60 days, registered 14,360 cubic feet as the consumption. The meter is then tested and found to register 12·5 per cent. in excess of the gas actually consumed. Find the amount that ought to be charged at 2*s.* 10*d.* per 1000 cubic feet and the average daily consumption.

7. A bookseller orders 50 dozen copies of a book published at two shillings. If the publisher gives him 13 for the dozen and allows a discount of $3\frac{1}{2}d.$ on the shilling, while the bookseller allows a discount of 3*d.* on the shilling to the public, what is the difference between the money paid by the bookseller and that received for the sale of the 50 dozen?

8. Either of the following:—

(*a*) The following are the times of departure and arrival of week day trains from London (King's Cross) to York. Find, from the table,

(*a*) the arithmetic average of the times taken for the journey, (*b*) the "median" time, and (*c*) the most common time or "mode."

King's Cross. *Departs.*	York. *Arrives.*
3.15	6.55
5.5	9.18
7.15	12.35
10.0	1.42
10.10	2.10
10.35	3.0
11.45	4.40
1.40	5.40
2.20	6.3
3.25	7.35
5.30	9.5
5.45	9.54
6.5	10.20
8.0	11.38
8.45	1.5
11.30	3.12

(*b*) There are two clocks in a building, one of which gains three minutes in four days, and the other loses four minutes in five days. They are set right at noon on Sunday. Determine the time indicated on each clock—to nearest second—when they differ by five minutes.

1911.

1. Find by the method of Prime Factors, the Highest Common Factor and Lowest Common Multiple of 1885, 7917, and 9048. State what is meant by the following terms:—prime number, prime factor, composite number.

2. Calculate the values necessary to complete the following table :—

Year ending Dec. 31st, 1907.	No. of Depositors.	Deposits. £	Average (to nearest £) for each Depositor.
Railway Savings Banks . .	64,126	5,865,072	
Trustee " " . .	1,780,214	61,729,588	
Post Office " " . .	10,692,555	178,033,974	
Total	12,536,895	245,628,634	

3. A yard stick measures 0·14 inches too short. If it is used to obtain the area of a floor, how much per cent. will the area thus obtained exceed the true area?

4. For a certain journey by railway the fares are charged at the rate of 2*d.*, $1\frac{1}{4}$*d.*, 1*d.* per mile for first, second, and third class respectively. If the third-class fare is 1*s.* 2*d.* less than the second, what is (1) the distance and (2) the first-class fare?

5. A joiner takes 47 hours to make a cupboard. If his rate of pay is $11\frac{1}{2}$*d.* per hour, and the material used costs 9*d.* per cubic foot, what charge will be made if the amount of material used is $6\frac{2}{3}$ cubic feet, the cost of sundries is 5*s.* 9*d.*, and the charge is 10 per cent. above the actual cost of labour and material?

6. In a certain chemical compound, potassium, chlorine, and oxygen are present. The proportions (by weight) are 78, 71, and 96 respectively. What percentage of each is present, and what weight of oxygen does a kilogramme of the substance contain?

7. Water is flowing at the rate of 16 kilometres per hour through a pipe of cross section 500 square centimetres. How long will it take to raise the level of a reservoir, 170 metres long, and 79 metres wide, by 5 centimetres, and what volume in litres will have passed through the pipe during that time.

8. There are two liquids which do not contract when mixed, and their specific weights are 0·6 and 0·75. If equal volumes are mixed, what is the specific weight of the mixture? If equal weights are mixed, what is the specific weight of the mixture? [The specific weight is the weight per unit volume.]

1912.

1. Find the least number which, when divided by 63, 119, and 153, leaves a remainder of 7 in each case.

2. The average number of persons admitted to an exhibition during the first five days was 623. If 845 persons were admitted on the sixth day, what is the average for the six days?

3. If a gallon contains 277·3 inches, what is the capacity (to the nearest gallon) of a cistern which is 4 ft. 6 in. long, 3 ft. 8 in. broad, and 2 ft. 10 in. high?

4. A wall is covered with plaster $1\frac{1}{4}$ inches thick; (i.) how many square yards can be covered with a cubic yard of plaster; (ii.) how many cubic yards will be required for the walls of a room 14 ft. long, 10 ft. broad, and 9 ft. high?

5. The floor of a room 6·8 metres by 4·2 metres is covered with rectangular blocks of wood 4 cms. by 6 cms. How many blocks will be required?

6. A tea-dealer blends 100 lbs. of tea at 1*s.* $5\frac{1}{2}$*d.* per lb. with 30 lbs. of tea at 1*s.* $9\frac{1}{2}$*d.* per lb. What is the lowest price at which he can sell the mixture in order to gain at least 20 per cent.?

7. A person puts £15 in the Post Office Savings Bank on 20th January, £10 on 25th April, £15 on 20th September. What interest will be due on 31st December, if interest, at the rate of $2\frac{1}{2}$ per cent. per annum, is allowed on every complete pound deposited, and commences on the first day of the month next following the deposit?

8. By selling an article at 6*s*. $1\frac{1}{2}$*d*. a tradesman gains 5 per cent. What would he gain if he sold it at 6*s*. 5*d*.

9. A man can travel to his office by tube or railway. The railway charges 11*d*. return, but issues a season ticket at £1, 15*s*. per quarter. The tube charge is 3*d*. a single journey. If he travels 6 days per week, how much was saved in the first three months of this year by taking a railway season ticket instead of paying daily (i.) by railway, (ii.) by tube?

COMMERCIAL ARITHMETIC.

1909.

1. A grocer advertises eggs at $1\frac{1}{2}$*d*. each, or nine for a shilling, or 1*s*. 5*d*. the dozen. Which is the lowest price, and what is the difference in the cost between five dozen at the lowest price and five dozen at the dearest?

2. Fifteen hundred copies of an engraving were printed. It cost £310 to prepare the plate and £3, 12*s*. per hundred to print the copies. The first thousand were sold at 10*s*. 6*d*. per copy and the remainder at 2*s*. 6*d*. per copy. A discount of 30 per cent. was allowed to the retailer. What was the profit per cent.?

3. A bicycle was sold at 55 per cent. less than the cost price and a second bicycle was bought with the proceeds and £4, 10*s*. The second bicycle was sold at 30 per cent. less than the cost price and a third bicycle, costing £3, 10*s*. more than the proceeds, was bought. The third bicycle cost £11, 7*s*. 6*d*. What was the price of the first?

4. A grocer buys one kind of tea at £7, 14*s*. a cwt. and another kind at 2*s*. a lb. In what proportion must he mix them to gain 50 per cent. by selling the mixture at 2*s*. 6*d*. per lb.? What is his profit, on each lb. sold, if his working expenses are 30 per cent. of his receipts?

5. A man's income is made up as follows:—£300 from employment and £60 from investments. Calculate his income tax if he pays £25, 10*s*. as insurance premium and the income tax is 9*d*. in the £ on earned incomes and 1*s*. on unearned. [There is an abatement of £160 on all incomes under £400, and money paid as insurance premium is deducted before the charge for income tax is calculated.]

6. A tradesman commences business on 14th March with a capital of £500; on 1st June he takes a partner with £400; on 30th

November another partner is admitted with £450, and at the end of February following a profit of £620 falls to be divided. Find the share of each.

7. Give a short account of the money systems in use in France, America, India, and Japan. Give in the money of each of these countries the value of £1.

8. A holds a bill of B's for £377, 3*s.* 6*d.* due in four months. B holds a bill of A's for £500 due in 18 months. They agree to settle by exchanging the bills and by the payment of a sum of money. If the rate is $4\frac{1}{2}$ per cent. per annum, what is the sum, and who should pay it?

9. The following is taken from a notice of issue of Stock :—

"Payment will be required as follows, viz. :—

£5 per cent. on application.	
£17, 10*s.* . .	26th February.
£25 . . .	15th March
£25 . . .	7th April.
£25 . . .	7th May.

"Payment may be made in full on 26th February, or on any subsequent day under discount at the rate of $2\frac{1}{2}$ per cent. per annum."

If a person, who applied on 15th February, decides to pay in full on 26th February, how much per £100 Stock must he pay on that date? If a dividend at the rate of $3\frac{1}{2}$ per cent. per annum is to be paid on the 1st July for the half year, what rate per cent. per annum does he receive for his money?

1910.

1. Find :—

(*a*) The area of a sheet of iron 8 ft. 5 ins. long, 6 ft. 7 ins. broad.

(*b*) The cubical contents of a tank 5 ft. 4 ins. long, 3 ft. 7 ins. broad, and 4 ft. 5 ins. deep.

2. How many yards of wall-paper (allowing $\frac{1}{10}$ for waste), which is 2 ft. $4\frac{1}{2}$ ins. wide, will be required to paper the walls of a room 16 ft. 5 ins. long, 12 ft. 7 ins. wide, and 9 ft. 4 ins. height if 63 sq. ft. are taken up by door, window, and fireplace? What will be the cost of the paper if pieces 20 ft. long cost 4*s.* 4*d.*?

3. What do you understand by "Absolute error," "Relative error," and "Percentage error" in approximations? The length of a rectangle is 13·8 ins., and its width is 24·4 ins. correct in each case to three significant figures. Find the area as correctly as the measurement allows, and state the approximate values of the maximum relative and absolute errors of the area.

4. A bankrupt's liabilities are £7435, 15*s.* 9*d.* His assets are made up as follows :—goods which are disposed of for £2000, outstanding

debts which bring in £450, and cash £336, 15s. If his liabilities include £130, 5s. 9d. for rent, taxes, and wages (which must be paid in full), and the expenses of the trustees are £65, 6s. 8d., what dividend can be paid to his creditors?

5. A six months' bill of £5394, 10s. is drawn on 1st March. For how much would it be discounted on May 10th at 3½ per cent.?

6. A person buys 100 3s. shares (1s. 6d. paid up) at a cost of 2s. per share. The company then makes a further call of 3d. per share. If a dividend of 4 per cent. on paid-up capital is declared, what amount will be received and what per cent. is obtained on the money invested?

7. From the following table calculate the amounts of the following sums of money for the times stated :—

(a) £400, 0s. for 4 years at $2\frac{1}{2}$ per cent.
(b) £354, 10s. „ 7 „ „
(c) £235, 0s. „ 12 „ „

Amount of £1 at Compound Interest.

Years.	$2\frac{1}{2}$ per Cent.	Years.	$2\frac{1}{2}$ per Cent.
1 . . .	1·025000	6 . . .	1·159693
2 . . .	1·050625	7 . . .	1·188686
3 . . .	1·076891	8 . . .	1·218403
4 . . .	1·103813	9 . . .	1·248863
5 . . .	1·131408	10 . . .	1·280085

8. A man visits the capitals of Europe. What is the standard of currency in each (select six), and what are their values in British money?

9. Find by graphs or otherwise the largest and smallest relative annual increase or decrease in the world's production of sugar as given in the following table :—

Year.	Beetroot.	Cane.	Total.
	Million Tons.	Million Tons.	Million Tons.
1903 . . .	5·56	4·19	9·75
1904 . . .	5·88	4·30	10·18
1905 . . .	4·93	4·37	9·30
1906 . . .	7·22	4·68	11·90
1907 . . .	7·15	4·81	11·96
1908 . . .	7·03	4·80	11·83
1909 . . .	6·52	5·18	11·60

1911.

1. Find, correct to five significant figures :—

(*a*) 64·357 × 395·68.
(*b*) 4·8763 ÷ 395·68.

2. In a district the sum of £6725 is required for the maintenance and repair of highways. If the rateable value is £96,194, what amount, to the nearest hundredth of a penny, will require to be levied in the £, and what will be the charge, to the nearest penny, on a rental of £38?

3. A publisher calculates that the inclusive cost of producing a book will be 4*s*. 3*d*. per copy. At what must the retail price be fixed so that he may sell the book to booksellers at a reduction of 15 per cent. on the retail price and make a profit of 15 per cent. on the cost of production?

4. In the manufacture of an article £40 is spent on plant and each article costs 6*d*. in addition. How many articles must be sold at 1*s*. each (1) to cover cost; (2) to gain 20 per cent. on the cost? Prove graphically.

5. A dealer, who quoted some of his goods in his lists at an increase of 12 per cent. on the cost price, found that, when he allowed a discount of 2½ per cent. on his list prices his business in these goods increased by one-fifth. Find whether his gains on these goods were increased or diminished when he allowed discount, and in what proportion.

6. I have some money invested in 2¾ per cent. Corporation Stock, and my net half-yearly dividend, after deduction of income tax at 1*s*. 2*d*. in the £, is £20, 14*s*. 4*d*. How much Stock do I hold, and what is the cash value if the Stock is quoted at 91½?

7. Compare the rates, per £ on total income, of income tax at 9*d*. per £ paid in the three following cases, if, in each case, the person expends one-twelfth of his income on insurance premium, on which no tax is paid :—

(1) Income £400, abatement £160.
(2) Income £700, abatement £70.
(3) Income £1000, no abatement.

8. If £1 = 20 marks 36 pfennige, what is the present value in English money of a bill for 5386 marks due in 3 months at 3½ per cent. per annum?

9. Complete the following table and find from it the compound interest on £450 for 6 years at 4 per cent. per annum :—

Amount of £1 at 4 per Cent. per Annum.

Years.	Amount.	Years.	Amount.
1 . .	1·040000	6 . .	
2 . .	1·081600	7 . .	1·315930
3 . .		8 . .	1·368568
4 . .	1·169858	9 . .	1·423311
5 . .	1·216652	10 . .	1·480243

1912.

1. Find, correct to five significant figures :—

(*a*) 469·34 × 63·874.

(*b*) 39·735 ÷ 597·03.

2. A sum of £2000 is divisible among A, B, and C in the proportions 8, 7, and 9. If A's share lapses, what will be the increase in B's?

3. Find, in lbs., the difference between a metric ton and a British ton. (1 metric ton = 1000 kg.; 1 lb. = ·4536 kg.)

4. Find the cost, to the nearest franc, of 3 cwts. 40 lbs. of tea at 4 frs. 55 c. per kilogram, given 1 lb. = 0·4536 kg.

5. From the following table, find by how much per cent. (correct to three significant figures) the cost of the articles mentioned have increased :—

Comparative Produce Prices.

		1912, March 1st.	1911, March 3rd.
Wheat, Eng. Gazette Aver.	qr.	34*s.* 6*d.*	30*s.* 2*d.*
Sugar, Tate's Cubes . .	cwt.	25*s.*	17*s.* 10½*d.*
Coffee, Good Av. Santos .	cwt.	59*s.* 10½*d.*	49*s.* 3*d.*

6. Use the table to find :—

(*a*) The amount of £360 for 6 years at 4 per cent. per annum, compound interest (answer to nearest penny).

(*b*) The smallest exact number of pounds which will amount to £360 in 4 years at 4 per cent. compound interest.

Amount of £1 at 4 per Cent. per Annum.

Years.	Amount.	Years.	Amount.
1 . . .	1·040000	4 . . .	1·169858
2 . . .	1·081600	5 . . .	1·216652
3 . . .	1·124864	6 . . .	1·265318

7. A man invests in a $2\frac{3}{4}$ per cent. Government Stock, and finds that, after paying 1*s.* 2*d.* in the £ income tax, his net income is exactly $2\frac{1}{2}$ per cent. on the sum invested. At what price did he purchase the Stock? (Answer to nearest sixteenth of £1.)

8. A man has £5000, which he proposes to invest in a $2\frac{3}{4}$ per cent. Government Stock. While he delays investing, the Stock rises 78 to 83. What is the loss in annual income through the delay? (Answer to nearest penny.)

TABLES OF FOUR FIGURE LOGARITHMS AND ANTILOGARITHMS

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

Antilogarithms.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

ANSWERS TO THE EXAMPLES

ANSWERS TO THE EXAMPLES

Examples 1 (page 2).

	A	B	C	D	E	Totals.
1-5	1,708,773	2,289,886	1,865,883	1,683,105	4,248,996	11,696,943
6-10	2,208,759	1,568,378	2,292,542	998,939	329,207	7,397,825
11-15	1,857,737	1,828,339	458,979	1,538,864	694,343	6,378,262
16-20	1,473,411	649,313	1,557,291	692,446	2,209,869	6,612,330
1-20	7,248,680	6,365,916	6,174,695	4,813,654	7,482,415	32,085,360

Examples 2 (page 3).

	A			B			C			D			E			F			Totals.		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
1-5	1982	12	0	1493	9	5	1224	3	4	2175	16	11	1596	19	2	2178	18	8	10,951	19	6
6-10	1780	19	2	1066	12	8	633	5	10	1569	3	6	1779	18	7	1400	7	10	8,230	7	7
11-15	1730	19	4	1642	4	2	1873	2	11	2301	0	3	1701	15	10	1875	7	1	11,127	9	7
16-20	1342	9	1	1449	10	2	1948	1	5	2255	5	11	706	15	2	1454	5	11	9,156	7	8
1-20	6836	19	7	5651	16	5	5678	13	0	8304	6	7	5785	8	9	7208	19	6	39,466	4	4

Examples 3 (page 5).

1. 530145
2. 2889315
3. 1580192
4. 304055082
5. 636885811
6. 3919688334
7. 5585452992
8. 20908412985
9. 8894355878
10. 14689637600
11. 56376768592
12. 57760583881
13. 22661922
14. 80109120
15. 13950846
16. 57116664
17. 424576663
18. 161379855
19. 2332094013
20. 3397792977
21. 34934273244
22. 19398078546
23. 48395793654
24. 434617131780

Examples 4 (page 7).

1. 63643 || 13
2. 125020 || 11
3. 170651 || 299
4. 2597478 || 59
5. 455034 || 462
6. 938159 || 229
7. 2039136 || 108
8. 1009765 || 510
9. 6899674 || 595
10. 160832889 || 71
11. 6793898 || 347
12. 521391 || 164
13. 16975 || 214
14. 19216 || 3704
15. 15410 || 364
16. 42684 || 2152
17. 50280 || 2448
18. 153535 || 2147
19. 50459 || 2076
20. 116056 || 3453
21. 026616 || 6499
22. 1062658 || 32749
23. 903070 || 5347
24. 984369 || 40869

Examples 5 (page 9).

1. 244258 || 23
2. 9896 || 8
3. 9705 || 11
4. 94327 || 20
5. 111410 || 17
6. 7880 || 23
7. 159068 || 32
8. 114411 || 53
9. 20408 || 19
10. 79184 || 69
11. 19014
12. 6080 || 59
13. 700 || 237
14. 854 || 265
15. 1480 || 214
16. 906 || 289
17. 1082 || 628
18. 2271 || 720

Examples 6 (page 10).

1. 72
2. 4
3. 187
4. 99
5. 1008
6. 2065
7. 629
8. 1
9. 17
10. 3135
11. 193
12. 7

Examples 7 (page 12).

1. $2^5 \times 3^2 \times 7$
2. $5 \times 3^3 \times 11$
3. $2^3 \times 3 \times 5 \times 31$
4. $3^4 \times 7 \times 11$
5. $2^4 \times 3 \times 11^2$
6. $2^3 \times 3 \times 5 \times 11 \times 17$
7. $2^5 \times 11 \times 47$
8. $3^4 \times 13 \times 23$
9. $2^{11} \times 13$
10. $3^5 \times 7 \times 23$
11. $5^4 \times 7 \times 11$
12. $2^8 \times 3^5$

	L.C.M.	G.C.M.		L.C.M.	G.C.M.
13.	9504	16	19.	820512	37
14.	4536	72	20.	303160	53
15.	9240	231	21.	676368	61
16.	471240	85	22.	698544	36
17.	255717	41	23.	9030528	27
18.	97020	105	24.	1596672	24

Examples 8 (page 14).

1. $1\frac{13}{57}$
2. 98
3. $\frac{6}{77}$
4. $416\frac{2}{3}$
5. $1\frac{13}{15}$
6. 41
7. 135
8. 31104
9. $156\frac{13}{32}$
10. $89\frac{3}{8}$
11. 4
12. $126\frac{9}{25}$
13. $1\frac{1}{4}$
14. 36
15. 19
16. $\frac{3}{5}$
17. $\frac{14}{65}$
18. 39
19. $3\frac{277}{360}$
20. 1
21. $1\frac{1}{194}$
22. $4\frac{8}{73}$
23. $\frac{39}{500}$
24. $\frac{175}{4653}$
25. $15\frac{15}{64}$
26. $\frac{1}{3}$
27. $117\frac{1}{2}$
28. $\frac{1474}{11405}$
29. $\frac{121}{224}$
30. 1

(page 16).

31. £2, 8*s.*
32. £1, 10*s.*
33. 378 pages
34. £54
35. £3, 4*s.*
36. £29, 3*s.* 4*d.*
37. 28 miles per hour
38. £2, 10*s.* 5*d.*
39. £12, 15*s.*
40. $1\frac{1}{4}$ lbs. per gallon.

Examples 9 (page 19).

1. 164·82386
2. 856·94157
3. 1921·28331
4. 1015·6169
5. 964·08216
6. 165·27123
7. 0·0016871
8. 132·802005
9. 6·4984
10. 140·47433
11. 27·797496 ; 1·87
12. 7818·684 ; 0·192052
13. 223·77161 ; 31911·84
14. 0·7448 ; 13097·02796
15. 6241664·84012 ; 12·25
16. 1028·85 ; 0·0102885
17. 0·01584 ; 1·584
18. 0·0006241 ; 62410
19. 0·034398 ; 3·4398
20. 36 ; 0·0036
21. 0·2192806
22. 33·9326
23. 126·084
24. 271·74212
25. 0·003096
26. 0·002016
27. 0·10134
28. 0·0059319
29. 212·42
30. 13363·36

Examples 10 (page 21).

1. 0·0501
2. 140000
3. 22600
4. 307·4
5. 0·0234
6. 0·000125
7. 2921·984
8. 17·2056
9. 59·049
10. 4·083396
11. 742
12. 342·03
13. 20·9
14. 7000
15. 0·48
16. 2·04
17. 1094000
18. 2020000
19. 390000 ; 390
20. 2100000 ; 210
21. 4170000 ; 0·000417
22. 0·0025 ; 25000
23. 0·0754 ; 0·0000000754
24. 0·324 ; 3240000000
25. 12672
26. 0·31104
27. 0·0126
28. 0·555
29. 34425000
30. 34500000

Examples 11 (page 23).

1. $\frac{8}{9}$, $\frac{16}{99}$, $\frac{34}{99}$, $\frac{683}{999}$.
2. $\frac{782}{999}$, $\frac{3128}{3333}$, $\frac{698}{1111}$.
3. $6\frac{1}{3}$, $18\frac{2}{3}$, $435\frac{37}{90}$.
4. $\frac{328}{9}$, $\frac{8515}{999}$, $\frac{36280}{999}$.
5. $\frac{283}{740}$, $\frac{5417}{9900}$, $\frac{2069}{5550}$.
6. $16\frac{86}{225}$, $453\frac{967}{990}$, $835\frac{119}{303}$.
7. $\frac{2501}{75}$, $\frac{21321}{1100}$, $\frac{224387}{33300}$.

Examples 12 (page 23).

1. $13\frac{7}{25}$
2. $6\frac{3}{8}$
3. $\frac{7}{100}$
4. $8\frac{104}{125}$
5. $17\frac{4}{5}$
6. $\frac{1}{200}$
7. $4\frac{1253}{5000}$
8. $8\frac{3}{10}$
9. $36\frac{1}{250}$
10. $\frac{37}{250}$
11. $4\frac{101}{125}$
12. $\frac{3}{5000}$
13. 0·3125
14. 4·12
15. 16·21875
16. 0·140625
17. 0·52
18. 18·125
19. 4·0928
20. 1·714843 +
21. 0·0011875
22. 0·033203125
23. 0·8191489 +
24. 0·1038848 +

Examples 12a (page 24).

1. $8\frac{1}{3}$
2. $2\frac{1}{16}$
3. $55\frac{59}{70}$
4. $\frac{128}{15}$ by $\frac{2}{17}$
5. 4
6. $\frac{7}{32}$
7. $\frac{37}{80}$
8. $\frac{9}{64}$
9. $\frac{11}{18}$
10. $\frac{5}{18}$
11. $\frac{1}{42}$
12. $\frac{61}{315}$
13. $21\frac{3}{4}$
14. $\frac{2}{3}$
15. 27 inches
16. $1\frac{1}{3}$ by $\frac{1}{12}$
17. $\frac{1}{4}$
18. $\frac{3}{16}$
19. 2520 miles
20. 378
21. £4500
22. $\frac{2}{3}$
23. $1\frac{53}{585}$
24. 16
25. $3\frac{19}{24}$
26. $\frac{127}{144}$
27. $\frac{20}{110}$; $\frac{26}{78}$; $\frac{29}{82}$
28. $\frac{24}{91}$
29. £2
30. (i) $4\frac{18}{20}$; (ii) $3\frac{78}{76}$
31. $1\frac{18}{47}$
32. 8 pieces ; $\frac{7}{75}$
33. $67\frac{5}{8}$
34. $\frac{95}{1152}$
35. 12
36. $8\frac{1}{2}$
37. $\frac{9}{14}$
38. £4500
39. $\frac{49}{85}$
40. $\frac{227}{315}$
41. $10\frac{18}{25}$
42. $24\frac{5}{8}$
43. £1, 19s. 7d.
44. $\frac{5}{4}$
45. 1
46. £15, 15s.
47. 16s. $10\frac{3}{4}d$.
48. £15, 5s. 11d.
49. $\frac{5}{16}$ and $\frac{11}{16}$
50. £504

Examples 13 (page 27).

	G.C.M.	L.C.M.		G.C.M.	L.C.M.		G.C.M.	L.C.M.
1.	$\frac{1}{45}$	$\frac{4}{3}$	5.	$\frac{5}{9009}$	$\frac{150}{13}$	9.	$\frac{1}{126}$	10
2.	$\frac{1}{16}$	$\frac{9}{8}$	6.	$\frac{2}{765}$	$\frac{8}{17}$	10.	$\frac{1}{168}$	30
3.	$\frac{4}{175}$	$\frac{24}{5}$	7.	$\frac{4}{165}$	$\frac{48}{11}$	11.	$\frac{1}{10}$	140
4.	$\frac{2}{405}$	$\frac{56}{9}$	8.	$\frac{13}{1197}$	$\frac{117}{19}$	12.	$\frac{19}{42}$	114

Examples 14 (page 29).

1. 295·9
2. 254·8
3. 291·860
4. 2508·79
5. 8,020,000
6. 3634
7. 69·10
8. 7·02
9. 99·46
10. 568·786
11. 9236·20
12. 262·1938
13. 361·67528
14. 230·5414
15. 9·08012

Examples 15 (page 30).

1. 74·5
2. 3602
3. 56·1
4. 80300
5. 59·66
6. 38197·26
7. 40·93
8. 60·131
9. 3450
10. 15·019
11. 5·907
12. 8·59911

Examples 16 (page 32).

1. 254,750,000,000
2. 3,418,300,000
3. 5,038,300,000
4. 6,921,300,000
5. 39,500,000,000
6. 67,570,000,000
7. 10,920,000,000
8. 46,400,000,000
9. 32,230,000,000
10. 33,465,000,000
11. 61,840,000,000
12. 129,010
13. 37,606
14. 1735·0
15. 5315·5
16. 463·6
17. 4·496
18. 407·5
19. 603·3
20. 22061
21. 0·8832
22. 3·117
23. 0·000297
24. 0·0004294
25. 31·181
26. 324·069
27. 2925·650
28. 359·857
29. 1787·428
30. 0·1696
31. 68·7993
32. 0·2635
33. 0·00003
34. 0 07436
35. 7·11776
36. 0·27404
37. 5885·169
38. 9·93
39. 6044·2
40. 21784
41. 1
42. 1
43. 20391 × 1
44. 138·514
45. 18441·09
46. 636·1045

Examples 17 (page 35).

1. 14500
2. 1660
3. 17890
4. 12210
5. 66840
6. 16970
7. 41820
8. 22900
9. 248200
10. 85100
11. 18·7
12. 29·7
13. 9·948
14. 111·8
15. 8049
16. 55·572
17. 0·047441
18. 0·086937
19. 1·2162
20. 6476·3
21. 6·1995
22. 285·17
23. 6·935
24. 107·070
25. 1·325
26. 248·4035
27. 2·8893
28. 81·6879
29. 10·83959
30. 4·24184
31. 0·3183
32. 0 014
33. 21·96
34. 0·0000000032
35. 0·7835262
36. 0 61391133
37. 0·90194
38. 0·22148
39. 0·17143
40. 0·7544

Examples 18 (page 38).

1. 148·161
2. 31·132
3. 132·377
4. 31·607
5. 26239·840
6. 59·424
7. 254·702
8. 173·52835
9. 530·61342
10. 439·48275
11. 677·56236
12. 176·95187
13. 8·74912
14. 263·43665

Examples 19 (page 43).

1. 1804 pence
2. 27319 halfpence
3. 2783 farthings
4. 56[illegible]94 farthings
5. 9048 pence
6. £247, 8*s.*
7. 3182*s.* 0½*d.*
8. 768 half-sovs., 2*s.* 3*d.*
9. 14995 lbs.
10. 45[illegible]453 drs.
11. 1[illegible]978 oz.
12. 23 tons 13 cwts. 3 qrs. 15 lbs. 15 oz.
13. 25 cwts. 30 lbs. 2 oz. 7 drs.
14. 10083⅛ lbs.
15. 9542½ yds.
16. 146400 in.
17. 35655 ft.
18. 9 mls. 7 furs. 3 chs. 10 yds. 0 ft. 1 in.
19. 70 mls. 2 furs. 3 chs. [illegible]0 links
20. 1 ml. 6 furs. 37 pls. 1½ yds.
21. 67335 sq. pls.
22. 64[illegible]03424 sq. in.
23. 714 sq. yds. 3 sq. ft. 4[illegible] sq. in.
24. 85 acs. 3 rds. [illegible]9 sq. pls. 5¾ sq. yds.
25. 2 sq. mls. 444 acs. 3522 sq. yds.
26. 2449921 cub. in.
27. 15 cub. yds. 14 cub. ft. 164 cub. in.
28. 3369[illegible] gills
29. 1303 pts.
30. 1470 barrels 26 galls. 1 pt.
31. 49728 grs.
32. 97 lbs. 9 oz. 17 dwts. 11 grs.
33. 12 lbs. (A.) 276 grs.
34. 3700 grs.
35. 15 ℥ 2 ℈ 1 gr.
36. 33600 m.
37. 5 imp. galls. 2 pts. 15 oz. 2 [illegible]
38. 28[illegible]3560 mins.
39. 3075360 secs.
40. [illegible] yr. 218 days 20 hrs. 7 mins.
41. 2272 wks. 1 day 6 hrs.
42. 43200, 40320, 44640 mins.
43. 1.46 A.M., 2[illegible]th October
44. 2647824 secs.
45. 0 right angles 14° 56′ 56″
46. 12240 sheets
47. 8 reams 0 quires 6 sheets
48. 11[illegible]694 yds.
49. 21 hands 1 inch
50. 72 hhds. [illegible] galls.

Examples 20 (page 47).

1. 103840 yds.
2. 188160 lbs.
3. 2·78 mls.
4. 1·75 acs.
5. 946516·48 oz.
6. 53·15 days
7. 20400 pence
8. £78·060
9. 4[illegible]27·20 half-crowns
10. 3[illegible]9·143 oz. (Avoir.)
11. 3[illegible]4 pls.
12. 43 04 54 links
13. 73 ·78 mls. per hr
14. 24 ·4 yds. per sec.
15. 3[illegible]*d.* per in.
16. 1·875*s.* per lb.
17. 12*s.* per gal.
18. £1815 per ac.
19. £485·3 per ton
20. 13·71 pence per hr.
21. £2, 0*s.* 6*d.*
22. 1250 gooseberries
23. He gains 1*d.*
24. [illegible]; 22 yds. per sec.

Examples 21 (page 50).

1. £0·834375; £1·364583; £3·336458
2. £0·690625; £5·428125; £2·4625
3. £0·335417; £2·821875; £8·376042
4. £14810·025
5. £58488·115
6. £34654·222
7. £5·352
8. £1·06185
9. 7s. 10½d.; 12s. 9d.; £12, 16s. 7¾d.
10. £13, 16s. 9d.; £64, 11s. 8d.; £35, 14s. 6¼d.
11. £1, 17s. 7½d.; £19, 15s. 7½d.; £32, 5s. 8¾d.
12. £16, 7s. 2¾d.; £3, 9s. 9d.; £1, 19s. 5¾d.
13. £1058, 5s.
14. £31997, 14s. 10¾d.
15. £13787, 3s. 4¼d.
16. £6926, 7s. 11¾d.
17. £789, 18s. 6¼d.
18. £1743, 13s. 11¾d.
19. £2464, 15s. 7d.
20. £1, 17s. 1¾d.
21. £1, 1s. 1½d.
22. 16s. 6·48d.
23. 2d.
24. 1s. 3·48d.

Examples 22 (page 51).

1. 0·2796875 of a ton
2. 0·16875 of a ton
3. 0·891518 of a ton
4. 0·353795 of a cwt.
5. 3·895926 of a cwt.
6. 0·4765625 lbs.
7. 43 tons 2 cwts. 1 qr. 13 lbs.
8. 24 tons 12 cwts. 100 lbs.
9. 6 tons 16 cwts. 2 qrs. 22 lbs.
10. 17 cwts. 1 qr. 3 lbs.
11. 6 cwts. 1 qr. 11 lbs.
12. 4 cwts. 3 qrs. 10 lbs.
13. 5 lbs. 10 oz. 2 drs.
14. 13 cwts. 0 qr. 18 lbs.

Examples 23 (page 51).

1. 0·946023 of a mile
2. 2·291477 of a mile
3. 0·682955 of a mile
4. 0·861 of a yard
5. 0·432197 of a mile
6. 8 mls. 3 furs. 2 yds.
7. 13 mls. 5 furs. 1 ch. 1 yd.
8. 2 furs. 8 chs. 2 yds. 1 ft. 11 in.
9. 4 mls. 5 chs. 14 yds.
10. 5 mls. 7 furs. 8 chs. 6 yds.

Examples 24 (page 52).

1. 0·024804 of a sq. mile
2. 0·119647 of a sq. mile
3. 0·9 of an acre
4. 1·577594 of an acre
5. 0·490741 of a cubic yd.
6. 3 sq. mls. 437 ac. 1 rd. 30 pls.
7. 13 acs. 2 rds. 28 pls.
8. 8 acs. 9 sq. chs. 53 sq. yds. 2 sq. ft. 23 sq. in.
9. 2 cub. yds. 17 cub. ft. 950 cub. in.
10. 7 sq. chs. 88 sq. yds. 7 sq. ft. 9 sq. in.

Examples 25 (page 52).

1. 15·84375 of a gallon
2. 0·96875 of a gallon
3. 0·578125 of a quarter
4. 33·730469 of a quarter
5. 2 qts. 1 pt.
6. 3 qts. 1 pt. 1 gill
7. 3 pks. 2 qts. 1 pt. 2 gills
8. 81 qrs. 5 bush. 2 qts. 3 gills
9. 8·25 galls.
10. 7 galls. 1 qt. 3 gills

Examples 26 (page 52).

1. 0·614583 of a lb. (Troy)
2. 0·822857 of a lb. (Avoir.)
3. 0·73 of an ounce (Troy)
4. 0·728125 of a gallon
5. 0·2828125 of a pint
6. 10 oz. 8 grs.
7. 3 lbs. 8 ℥ 1 ʒ 2 ℈ 17 grs.
8. 2 galls. 7 pts. 9 ℥ 3 ʒ 31 min.

Examples 27 (page 53).

1. 0·858219 of a year
2. 0·605694 of a day
3. 0·259275 of a year
4. 0·352678 of a week
5. 250 days 22 hrs. 51 min. 1 sec.
6. 14 yrs. 340 days 8 hrs. 42 mins.
7. 6 wks. 1 day 22 hrs. 42 mins. 14 secs.
8. 53 mins.
9. 12 hrs. 20 mins. 49 secs.
10. 296 days ; 0·808743.

Examples 28 (page 53).

1. 0·38024[illegible] of a right angle
2. 0·189576 of a right angle
3. 61° 32′ 15″
4. 17° 50′ 20″·4
5. 0·46 of a bale
6. 0·33375 of a bale
7. 1 bale 8 reams 6 quires 22 sheets
8. 15° C. ; 41°·6 C. ; 93°·3 C.
9. 39°·2 F. ; 122° F. ; 176° F.

Examples 29 (page 55).

1. 417·9 lbs.
2. 12*s.* 3·218*d.*
3. 0·284375 of a ton
4. 37·8125 of 2 shillings
5. 3 pecks 1 gall.
6. 7 tons 14 cwts. 15·4 lbs.
7. 0·1875 of a furlong
8. 56 mins. 15 secs.
9. 0·278125 of a ton
10. 0·822857 of a lb. (Avoir.)
11. 1 ml. 5 fur. 2 chains
12. 10·425 inches
13. 0·187671 of a year
14. 93 cub. yds. 13 cub. ft. 864 cub. in.
15. 0·439375 of a mile
16. 16 yds. 1 ft. 5 in.
17. 1 gall. 0 qts. 0·14 pts.
18. 0·969 of a bush. ; 7 galls. 3 qts.
19. 0·892857 of a cwt.
20. 1·15275 of a mile
21. 19 cwts. 2 qrs. 20 lbs. 10 oz. 1 dr.
22. 39 galls. 3·97 pts.

Examples 30 (page 56).

1.

Ounces.	Lb.	Drachms.	Lb.
1	·0625	1	·00391
2	·125	2	·00781
3	·1875	3	·01172
4	·25	4	·01563
5	·3125	5	·01953
6	·375	6	·02344
7	·4375	7	·02734
8	·50	8	·03125
9	·5625	9	·03515

2.

Inches.	Yard.
1	·02778
2	·05556
3	·08333
4	·11111
5	·13889
6	·16667
7	·19444
8	·22222
9	·25
1 foot	·33333
2 feet	·66667

3. 1 gill = ·03125 galls.
1 pt. = ·125 „
1 qt. = ·25 „

4. 1 pk. = ·031 qr.
1 bush. = ·125 „

5. 1 dwt. = ·00417 lb.
1 oz. = ·08333 „

6. 1 sec. = ·00028 hour
1 min. = ·01667 „

7.

Centigrade degrees		Fahrenheit degrees
1	=	1·8
5	=	9

Fahrenheit degrees		Centigrade degrees
1	=	0·56
5	=	2·78

8. 1 stone = ·00625 ton
1 qr. = ·0125 „
1 cwt. = ·05 „

9. 1 chain = ·0125 mile
1 furlong = ·125 „

10. 1 qt. = ·0069 barrel
1 gall. = ·0278 „

11. 1 double sheet = ·00417 ream
1 quire = ·05 „

12.

Oz (Troy).	Lb (Avoir.)	Oz. (Troy).	Lb (Avoir.).
1	·0686	7	·48
2	·1371	8	·5486
3	·2057	9	·6171
4	·2743	10	·6857
5	·3429	11	·7543
6	·4114	12	·8229

Examples 31 (page 59).

1. £62, 10s. 6d.
2. £176, 6s. 8¼d.
3. £6, 10s. 6¼d.
4. £345, 18s. 1¾d.
5. £33, 4s. 7d.
6. £96, 4s. 10½d.
7. £25,842, 19s. 4½d.
8. £221,448, 1s. 6d.
9. £19,446, 11s. 9¾d.
10. £238,635, 1s. 1½d.
11. £424, 8s. 6d.
12. £492, 16s. 5d.
13. £5791, 7s. 2d.
14. £950, 10s. 7d.
15. £60,311, 0s. 5d.
16. £4,611,168, 12s. 0d.
17. £22,479, 10s. 11d.
18. £4,196,937, 11s. 6¾d.
19. £251,938, 7s. 6¾d.
20. £11,151,755, 4s. 0¾d.

Examples 32 (page 59).

1. 1 ton 5 cwts. 3 qrs. 13 lbs. 2 oz.
2. 291 tons 11 cwts. 1 qr. 6 lbs.
3. 2171 yds. 1 ft. 7 in.
4. 27991 acs. 2 rds. 10 pls.
5. 1828 mls. 1 fur. 2 chains 6 yds.
6. 23425 lbs. 11 oz. 12 drs.
7. 616 lbs. 3 oz. 6 dwts. 12 grs.
8. 41598 days 10 hrs. 3 mins. 45 secs.
9. 8349 yrs. 5 days 13 hrs. 40 mins.
10. 662864 qrs. 3 bush. 2 pts. 1 gill
11. 1288 tons 11 cwts. 3 qrs. 14 lbs.
12. 423 cwts. 2 qrs. 14 lbs. 14 oz.
13. 7389 yds. 1 ft. 9 ins.
14. 96425 acres
15. 142978 mls. 4 fur.
16. 14565 lbs. 4 oz.
17. 9626 lbs. 7 oz. 14 dwts. 16 grs.
18. 96250 days 5 hrs. 58 mins.
19. 4560 yrs. 246 days
20. 9890 qrs. 3 bush. 2 pts. 1 gall. 1 qt.
21. 8806 tons 4 cwts. 2 qr. 2 lbs.
22. 887 mls. 5 fur.
23. 40608 sq. yds.
24. 1725 lbs. 11 oz.

Examples 33 (page 62).

1. £81, 2*s.* 9¾*d.*
2. £165, 2*s.* 0½*d.*
3. £21, 15*s.* 1¼*d.*
4. £12, 9*s.* 4¼*d.*
5. £72, 7*s.* 5½*d.*
6. £132, 8*s.* 11½*d.* $\frac{19}{64}$
7. £26, 12*s.* 1¾*d.*
8. £26, 3*s.* 8¼*d.*
9. 1 ton 17 cwts. 3 qrs. 22 lbs. 4 oz.
10. 14 cwts. 60 lbs. 14 oz. 15 drs.
11. 1 cwt. 107 lbs. 7 oz.
12. 14 mls. 5 furs. 8 chains 5 yds.
13. 773 yds. 1 ft. 11 ins.
14. 6 acres 1 rood 29 poles
15. 2 acs. 2532 sq. yds. 6 sq. ft. 80 in.
16. 3 rds. 2 sq. poles 17 sq. yds.
17. 7 lbs. 6 oz. 2 dwts. 12 grs.
18. 10 qrs. 7 bush. 2 pks. 3 qts. 1 pt.
19. 8 days 12 hrs. 31 mins. 20 secs.
20. 167 days 19 hrs. 35 mins.
21. £2, 13*s.* 2*d.*
22. 12*s.* 5*d.*
23. £1, 7*s.* 4*d.*
24. 5 cwts. 2 qrs. 17 lbs.
25. 1 cwt. 2 qrs. 18 lbs. 15 oz.
26. 3 miles 4 furlongs 18 poles
27. 3 yds. 2 ft. 9 ins.
28. 8 acres 2 roods 9 poles
29. 13 sq. yards 125 sq. inches
30. 6 sq. chains 8872 sq. links
31. 1 lb. 11 oz. 17 dwts.
32. 1 lb. 5 oz. 4 dwts. 4 grs.
33. 1 day 9 hrs. 27 secs.
34. 1 day 12 hrs. 15 mins.
35. 6 bush. 2 pks. 1 qt. 2 gills
36. 5° 39′ 49″

Examples 34 (page 63).

1. £1, 0*s.* 5¼*d.*
2. £1, 19*s.* 4*d.*
3. £49, 17*s.* 3*d.*
4. £5, 9*s.* 7*d.*
5. £47, 18*s.* 2*d.*
6. £1, 8*s.* 1*d.*
7. 15 cwts. 2 qrs. 18 lbs.
8. 5 cwts. 2 qrs. 7 lbs. 3 oz. 8 drs.
9. 26 lbs. 4 oz. 6 drs.
10. 24 miles 7 furlongs 13 poles
11. 3 roods 27 poles
12. 29 acres
13. 541 yards 2 feet
14. £1235, 16*s.* 8*d.*
15. 20 tons 9 cwts. 42 lbs.
16. 10 days 22 hrs. 5 mins.
17. 20 lbs. 1 oz. 15 grs.
18. 3 lbs. 1 oz. 5 drs.
19. £79, 6*s.* 7½*d.*
20. 4 quarters 4 bushels 1 peck

Examples 35 (page 68).

1. 84376·25 decimetres
   12·947628 kilogrammes
   821709·5 decilitres
2. 1294·7385 square metres
   3285·29 square decimetres
   6·1284738 cubic metres
   5837000 cubic millimetres
3. 793·5826 deciares
   0·04298175 hectares
   432·59 decasteres
4. 52·8407 hectares
   4283·205 litres
   3259283·7 cub. decimetres
5. 429385·294 kilolitres
   3842760 cubic centimetres
6. 90,000 metric tons
7. 4000 rails
8. 25,000 pieces
9. 20,000 allotments
10. 500 jugs
11. 6·6 steres
12. 250 cubic centimetres
13. 375 grammes
14. 20,000 pieces
15. 6800 grammes
16. 2666·6 centimetres per second

Examples 36 (page 69).

1. 643·732 kilometres
2. 29,765,507 square metres
3. 77304 cubic yards.
4. 2012 metres ; 354506 metres ;
   19568 „ 2244350 „
   678013 „
5. 39500 kilos. ; 5600 kilos. ;
   3100 „ 830500 „
   11500 „ 10500 „
6. 23226 Hl. ; 2347 Hl. ;
   250 „ 925 „
   764 „
7. 33476409 tonneaux
   35921645 „
   35621032 „
   41841176 „
   44796721 „
8. 8839 metres
9 435 miles 5 furlongs
10. 208430 square miles
11. 87 kilometres per hour
12. 15·43 grains
13. 20 44 hectares
14. 6 hours 13 minutes
15. 76·054 centimetres
16. 16·90 kilometres
17. 3·45 inches
18. 374·51 yards
19. 1·14 litres
20. 2825·44 lbs. ; 9·121 yards
21. 8003380 hectares
22. 48190·2 tons
23. 19368800 gallons
24. 687700 tonneaux
    994000 „
    782100 „
    881200 „
    1001000 „

Examples 37 (page 72).

1. 148 million kilometres
2. 5 kilos. at 5s. 6d. per kilo.
3. 25 quintals ; 1·2 hectares
4. 906·25 miles
5. 461 kilometres
6. 32 kilometres
7. 29·6 inches
8. 1 litre
9. 191 feet
10. 7 grammes
11. 32700 cms. per second
12. 188000 miles
13. 89 kilogrammes
14. 12 lbs.
15. 73 lbs.
16. 14 metres

1. **Examples 38 (page 74).**

Yds.	Kilometres.	Chs.	Kilometres.	Furs.	Kilometres.	Mls.	Kilometres.
1	000914	1	02012	1	·20116	1	1·60933
2	001829	2	04023	2	40233	2	3·21866
3	002743	3	06035	3	60349	3	4·82799
4	003658	4	08047	4	80466	4	6·43732
5	004572	5	10058	5	1·00582	5	8·04665
6	005486	6	12070	6	1·20698	6	9·65598
7	006401	7	14082	7	1·40815	7	11·26531
8	007315	8	16093			8	12·87464
9	008229	9	18105			9	14·48397

2.

Sq. Yds.	Hectares.	Roods.	Hectares.	Acres.	Hectares.
1	000084	1	101168	1	·404679
2	000167	2	202340	2	809359
3	000251	3	303510	3	1·214038
4	000335			4	1·618717
5	000418			5	2·023396
6	000502			6	2·428065
7	000586			7	2·832754
8	000669			8	3·237434
9	000753			9	3·642113

3.

Grs.	Grammes.	Dwts.	Grammes.	Ozs.	Grammes.	Lbs.	Grammes.
1	064799	1	1·55518	1	31·1036	1	373·243
2	129598	2	3·11036	2	62·2072	2	746·486
3	194397	3	4·66554	3	93·3108	3	1119·730
4	259196	4	6·22072	4	124·4144	4	1492·973
5	323995	5	7·77590	5	155·5180	5	1866·216
6	388793	6	9·33108	6	186·6216	6	2239·459
7	453593	7	10·88626	7	217·7252	7	2612·702
8	518392	8	12·44144	8	248·8288	8	2985·946
9	583191	9	13·99662	9	279·9324	9	3359·189

4.

Pints.	Litres.	Quarts.	Litres.	Gallons.	Litres.
1	56755	1	1·13510	1	4·54041
		2	2·27020	2	9·08082
		3	3·40530	3	13·62123
				4	18·16164
				5	22·70205
				6	27·24246
				7	31·78287
				8	36·32328
				9	40·86369

5.

Quintals.	Cwts.	Tonneaux.	Tons.
1	1·9684	1	9842
2	3·9368	2	1·9684
3	3·9052	3	2·9526
4	7·8736	4	3·9368
5	9·8421	5	4·9210
6	11·8105	6	5·9052
7	13·7789	7	6·8894
8	15·7473	8	7·8736
9	17·7157	9	8·8579

6.

Centimetres.	Inches.	Metres.	Chains.
1	393708	1	04971
2	787416	2	09942
3	1·181124	3	14913
4	1·574832	4	19884
5	1·968540	5	24855
6	2·362247	6	29826
7	2·755955	7	34797
8	3·149663	8	39768
9	3·543371	9	44740

7.

Sq. Metres.	Square Yards.	Hectares.	Square Miles.
1	1·196033	1	003861
2	2·392067	2	007722
3	3·588100	3	011583
4	4·784133	4	015444
5	5·980167	5	019306
6	7·176200	6	023167
7	8·372233	7	027028
8	9·568267	8	030889
9	10·764300	9	034750

8.

Litres.	Pints	Hectolitres.	Bushels.
1	1·7608	1	2·7512
2	3·5216	2	5·5024
3	5·2824	3	8·2536
4	7·0432	4	11·0048
5	8·8040	5	13·7560
6	10·5648	6	16·5072
7	12·3256	7	19·2585
8	14·0864	8	22·6097
9	15·8472	9	24·7609

Examples 39 (page 75).

1. 165·097 mm.; 1142·98 mm.; 761·99 mm.; 2870·1
2. 4·166 metres; 2·921 metres; 6·274 metres
3. 27·888 ft.; 20·136 ft.; 27·781 ft.
4. 2065 c. cms.; 566400 c. cms.; 2293600 c. cms.
5. 1589 cub. ft.; 2·4137 cub. ft.; 315·52 cub. ft.
6. 35·756 litres; 82 litres; 291 litres
7. 0·859 galls.; 10·214 galls.; 2·216 galls.
8. 4·0823 grammes; 453·593 grammes; 373·24 grammes
9. 120·57 lbs.; 105·21 lbs.; 2·2425 lbs.; 1 2153 lbs. (Troy)
10. 3307·598 kilogrammes
11. 7731·033 „
12. 18698·45 „
13. (1) 8032·7 yds.; (2) 4·5641 miles
14. (1) 9773 yds.; (2) 5·5527 miles; (3) 5 miles 973 yds.
15. (1) 41·872 miles; 73694 yds.

Examples 40 (page 75).

1. 0·82143 of a cwt. @ 0·83334 of a £ per cwt.
2. 27·669 kilogrammes @ £0·033 per kilogramme
3. £1, 13s. 1d.
4. £59, 18s. 3d.
5. £6, 12s.
6. £3, 5s. 1d.
7. £2, 0s. 3d.
8. £22, 17s. 6d.
9. 56 miles
10. 487·7 centimetres
11. 28320 grammes
12. 1s. 4d.
13. 937·4 metres; 3075·6 ft.

Examples 41 (page 78).

1. £2239, 2s. 6d.
2. £5003, 2s.
3. £317, 3s. 9d.
4. £985, 12s. 7d.
5. £2903, 10s. 5¼d.
6. £52, 14s. 3d.
7. £87, 4s. 2d.
8. £134, 8s.
9. £26, 16s. 5d.
10. £19, 1s.
11. £563, 6s. 6d.
12. £1114, 6s. 6d.
13. £998,998, 19s. 2¼d.
14. £2187
15. £148, 18s. 3d.
16. £268, 18s. 3d.
17. £1395
18. £3, 16s. 7d.
19. £203, 14s. 3d.
20. £2081, 13s.
21. £44,917, 10s. 9d.
22. £13,107, 13s. 3d.
23. £109,888, 0s. 3d.
24. £2661, 5s 2d.

Examples 42 (page 81).

1. £11, 13s. 5¼d.
2. £248, 8s. 11½d.
3. £49, 2s. 6½d.
4. £10, 5s. 3¼d.
5. £418, 13s. 6¾d.
6. £255, 9s. 2¼d.
7. £60, 13s. 2d.
8. £38, 3s. 7d.
9. £367, 18s. 3¾d.
10. £123, 15s. 9d.
11. £127, 9s. 6d.
12. £158, 8s. 8d.
13. £8095, 4s. 2¼d.
14. £1532, 4s. 4d.
15. £27, 2s. 7¾d.
16. £59,626, 17s. 6d.
17. £11, 11s. 8d.
18. 10d. per lb.
19. 20 tons 4 cwts. 1 qr. 26 lbs.
20. £25, 12s.

Examples 43 (page 82).

1. £10, 14s. 11d.
2. £27, 5s. 7d.
3. £54, 2s. 1d.
4. £55, 0s. 2d.
5. £52, 5s. 4d.
6. £198, 4s. 5d.
7. £24, 8s. 11d.
8. £751, 11s. 11d.
9. £212, 13s. 6d.
10. £39, 5s. 1d.
11. £7, 12s. 4d.
12. £544, 8s. 8d.
13. £260, 3s. 6½d.
14. £60, 7s. 7¼d.
15. £57, 0s. 9¾d.
16. £608, 19s. 7¾d.
17. £2379, 1s. 5¾d.
18. £198, 2s. 9¼d.
19. £62, 2s. 9¾d.
20. £11, 7s. 4¼d.
21. £383, 18s. 8½d.
22. £116, 19s. 2¾d.
23. £115, 17s. 10¼d.
24. £308, 18s. 9¼d.
25. £472, 5 fl. 6 m.
26. £4, 1 fl. 7 m.
27. £23, 2 fl. 3 c. 9 m.
28. £179,091, 5 fl. 5 c.
29. £957,250
30. £2,592,000
31. £1434, 7 fl. 5 c.
32. £375, 5 fl. 6 m.
33. £132, 9 fl. 4 c.
34. £17,984
35. £434, 1 fl. 5 c. 6 m.
36. £6, 1 fl. 2 c. 5 m.
37. £2, 3 fl. 4 c. 2 m.

Examples 44 (page 85).

1. 20 ; ½ ; 1½ ; 8 ; 42 ; 2880 ; £5100
2. 10 ; 6 ; 10 ; 63
3. 56
4. 5
5. 400
6. 1200
7. 16 tons 5 cwts.
8. 80
9. 275
10. (i) $\frac{100I}{RV}$; (ii) $\frac{100I}{PY}$; (iii) $\frac{100I}{PR}$; 72
11. (i) 22½ (ii) 4
12. 8
13. 3
14. 20
15. 5
16. 625
17. 14
18. 5⅓
19. 1·5708
20. 2136

Examples 45 (page 96).

1. 262·30
   2116·82
   1514·16
   18548·24
   2156·95
   1411·88
2. £35, 8 fl. 1 c. 7 m.
   £424, 6 fl. 7 c. 5 m.
   £9, 8 fl. 7 c. 7 m.
   £5, 3 fl. 4 c. 3 m.
3. £6, 14*s.* 8½*d.*
   £83, 0*s.* 11¼*d.*
   £1, 12*s.* 7¼*d.*
   £32, 10*s.* 1¼*d.*
4. 144·79
5. 438·55 francs
6. £2, 10*s.* 4¼*d.*
7. £12, 12*s.* 6½*d.*
8. 259·59 marks
9. £98, 3*s.* 8*d.*
10. 105·56 krones
11. 536·94 florins
12. 160·7 francs
13. 90 milreis
14. £825, 0*s.* 11*d.*
15. 396·79 lire
16. Gain 64·6 marks on £100
17. £234, 7*s.* 6*d.*
18. 91 marks
19. 5873 frs. 75 centimes
20. (*a*) 76·651 = £5, 2*s.*
    (*b*) 456·5698 = £30, 7*s.* 4*d.*
    (*c*) 84·974 = £2, 6*s.* 6*d.*
    (*d*) 145·4531 = £9, 13*s.* 7*d.*
    (*e*) 6·3594 = 8*s.* 5¾*d.*
    (*f*) 10·53125 = 14*s.* 0½*d.*
    (*g*) 25·2656 = £1, 13*s.* 8¼*d.*
    (*h*) 30·7[illegible]9 = £2, 1*s.* 0¾*d.*
21. £210, 16*s.* 7*d.*
22. £1 = 25·49 francs
23. 3*s.* 11*d.*
24. 4·23 milreis
25. 13*s.* loss

Examples 46 (page 97).

1. 1·380
2. 12·695
3. 4·591
4. 0·3595
5. 0·371
6. 9·864
7. 0·0447
8. 0·0480
9. 7·843
10. 1131·44
11. 78·356
12. 3·601
13. 0·0374

Examples 47 (page 97).

1. 15·432 grains
2. 4s. 10d. per yard
3. 3069·44 ft.; 935·42 metres
4. 160·9308 kilometres
5. 28·31 litres
6. 8707·23 sq. yds.
7. 2·78 oz., 28·41 litres, 28409 grammes
8. £2, 0s. 2d. per acre
9. 5s. 4·33d.
10. 1226 grammes
11. £22, 15s. 9d.
12. 70 centimes
13. 6812·3 grammes
14. £14,801 per mile
15. 170,400 tons

Examples 48 (page 100).

1. £1, 4s.
2. £2, 3s. $5\frac{1}{4}d$.
3. 12 days
4. £4, 15s. $9\frac{3}{4}d$.
5. £6, 9s. 11d.
6. £33, 6s. 8d.
7. 2s. 6d. per hour
8. 4 francs, 16 centimes
9. 1011 grammes
10. 70 kilogrammes

Examples 49 (page 103).

1. 42 days
2. 20 miles a day
3. £1, 12s. 9d.
4. £276, 17s. 3d.
5. 102·4 ft.
6. $22\frac{1}{2}$ lbs.
7. £34, 4s. 3d.
8. 24 hrs. $52\frac{1}{2}$ min.
9. $15\frac{5}{21}$ days
10. $9\frac{1}{7}d$.
11. $35\frac{1}{3}$ days in all
12. £262, 10s.
13. 26 days

Examples 49a (page 104).

1. 70; 99
2. $\frac{27}{88}$; $\frac{285}{184}$
3. (1) 155 rings; (2) £1, 18s. 6d.
4. 20 miles per hour
5. 256 shots
6. £13, 0s. $7\frac{1}{2}d$.
7. £15
8. 3s. 4d. per yard; 120 yards
9. 25014 and 16676
10. 4 tons 15 cwts. 39 lbs.
11. £2, 15s. $5\frac{1}{2}d$.
12. £3, 9s. 4d.
13. £33
14. 2 cwts. 98 lbs.
15. 34 days
16. £61, 18s.

Examples 50 (page 105).

1. £142, 10*s*.
2. Half-an-hour per day
3. 32 men
4. £10, 8*s*. 4*d*.
5. 45 men
6. 1965⅝ yds.
7. 16 days
8. 80 men
9. 180 men
10. 1250 men
11. 100 men
12. 45 men
13. £39, 4*s*. 7*d*.
14. 80 slabs
15. 7 days
16. 40·253¾ pints
17. 27 days
18. 15 men
19. £160, 16*s*. 1¾*d*.
20. 4 loaves
21. £[illegible], 10*s*. 6*d*.
22. £22
23. 8 ft. 1½ in.
24. £25, 5*s*.
25. £403, 4*s*.
26. 50 boys
27. 9½ miles

Examples 51 (page 109).

1. £25, 4*s*. 5*d*.
2. £141, 16*s*. 3*d*.
3. 9580·48 cwts.
4. 29543 people
5. £60, 8*s*. ¼*d*.
6. £832, 10*s*. 8*d*.
7. 8291 cwts. 2 qrs.
8. £9, 1*s*. 7*d*.
9. 3762 gallons
10. 34249·6 tons
11. £59,925
12. 70·0416 gallons
13. 2538
14. £60, 11*s*. 9*d*.
15. 3 hours per day
16. £3, 6*s*. 8*d*.
17. 4,737,128
18. 6270 francs
19. 211·6 litres

Examples 52 (page 111).

1. 4% ; 3% ; $\frac{1}{14}$% ; $16\frac{2}{3}$% ; $\frac{8}{9}$%
2. (*a*) $7\frac{1}{2}$% ; (*b*) $1\frac{61}{64}$% ; (*c*) $\frac{1}{8}$%
3. $3\frac{1}{3}$ per cent.
4. $33\frac{1}{3}$ per cent.
5. $14\frac{2}{7}$ per cent.
6. 40·8 per cent.
7. 99·3% ; 97·8% ; 96·8%
8. 17*s*. 6*d*. ; £56, 2*s*. 4*d*.
9. 27·5 per cent.
10. 75 per cent.
11. 25·125 per cent.
12. 95·29 per cent.
13. 1·85 per cent.
14. 1·73 per cent.
15. 0·06 per cent.
16. 0·36 and 0·18 per cent.
17. 2·55 per cent.
18. 0·58 per cent.
19. (*b*) by 0·29 per cent.
20. 0·13 ; 0·13 ; 0·26 per cent.

Examples 53 (page 113).

1. £2, 10*s*.
2. £429
3. £1260
4. 600
5. £8,000,000
6. 380 boys
7. 250
8. 23,333 c. cms.
9. 186 francs
10. 2073·6 dollars

Examples 54 (page 116).

1. 165,200
2. 6560
3. £3275
4. 3,000,000
5. £7594
6. £15 per acre
7. 80 lbs.
8. £1220, 14*s.*
9. 16 fr. 50 centimes
10. $15854
11. £800
12. £500
13. 2$\frac{3}{4}$ gallons
14. 10 gallons
15. 2 gallons
16. 40·77 kilogrammes
17. £2
18. 17 lbs.
19. Taking fatty matter as standard, nothing further has been added ; 1*s.* 3$\frac{1}{2}$*d.*

Examples 55 (page 119).

1. 461·8
2. 16·26
3. 0·105
4. 13·75375
5. 83,410
6. 7·3 hours ; 7·5 hours
7. 0·20 inches ; 0·08 inches
8. £[illegible]07, 2*s.* 7*d.*
9. £27,870,000
10. England 0·957 per person
    Wales, 2·34 „
    Scotland, 4·01 „
    Ireland, 4·64 „
    United Kingdom, 1·70 „
11. England and Wales, Population, 24458354
    Scotland, „ 3504642
    Ireland, „ 5522469
    United Kingdom, average number of letters = 30

Examples 56 (page 121).

1. Revenue, £37,845 ; Expenditure, £39,260.
2.

	Total.	United Kingdom.	Canada.	Other British Possessions.	United States of America.	Other Countries.	Percentage.
Total	1,771,307	[illegible],440	140,654	38,789	1,082,042	10,442	28·2
Yearly Average	354,273	99,888	28,131	7,758	216,408	2,088	28·2

Year.	Percentage.
1896	29·8
1897	29·5
1898	29·9
1899	26·5
1900	26·2

3. £30, 10*s.*
4. Average increase per decade 311797
   Percentage increase since 1851 . . . . 61·8
   1891 . . . . 18·2

5.

	Average No. &c.	Number of Members.	Excess Number.
England . . .	74017	498	−38
Wales	67587	30	0
Scotland . . .	67993	69	+1
Ireland	43386	64	+37
45,216,751	68407	661	0

6. 1·44 miles per hour
7. Average number of marks = 150
   Average percentage = 68
8. 4 per cent.

9.

Year.	Numbers.		Total.	% Steam	Tonnage.		Totals.	% Steam.
1896			645	47·6			296,746	92·0
1897			732	42·2			278,787	88·4
1898			1004	41·3			415,230	91·7
1899			844	46·7			435,471	93·0
1900			823	48·0			408,310	92·6
Totals	2228	1820	4048	45·0	151,342	1,683,202	1,834,544	91·8
Averages	446	364	810	44·9	30,268	336,640	366,909	91·5

10.

Year.	Numbers.		Total.	% Steam.	Tonnage.		Totals.	% Steam.
1896			270	82·6			180,663	82·7
1897			290	72·4			156,108	78·2
1898			336	81 2			212,244	96·7
1899			373	71·0			235,001	94·0
1900			319	78·7			233,569	96·5
Totals	366	1222	1588	77·0	94,663	922,922	1,017,585	90·7
Averages	73	244	318	76·8	18,933	184,584	203,517	89·6

11. 3*s.* 9¾*d.*

12.

				16*s.* 11*d.* 8*s.* 2*d.* 3*s.* 5·3*d.*	79·9 miles 51·2 „ 36·5 „
Totals .	386,577	16,231,929	100,030	5*s.* 2·1*d.*	42·0 miles

13. 3·87 inches

14. 8 stones 8 lbs.

15. 7·56 hours

16.

Income.	Dutiable Income.	Income Tax.			Rate per £ on Total Income.
£	£	£	*s.*	*d.*	*d.*
200	40	1	10	0	1·80
250	90	3	7	6	3·24
300	140	5	5	0	4·20
350	190	7	2	6	4·89
400	240	9	0	0	5·40
450	300	11	5	0	6·00
500	350	13	2	6	6·30
550	430	16	2	6	7·04
600	480	18	0	0	7·20
650	580	21	15	0	8·03
700	630	23	12	6	8·10
750	750	28	2	6	9·00

17. 10·79, 11·91, 14·08, 14·58, 13·95

18.

	1891	1901	% 1891	% 1901
			74·97 25·03 0·00	77·03 22·97 0·00
Totals . .	29,002,525	32,526,075	100·00	100·00
			72·66 24·22 3·12	75·30 21·99 2·71
Totals . .	4,025,647	4,472,000	100·00	100·00

19. 4 months
20. 24th February
21. $6\frac{1}{4}$ months
22. 5·6 months
23. 2·6 months
24. $4\frac{1}{2}$ months
25. $10\frac{1}{2}$ months
26. $17\frac{1}{2}$ months

Examples 57 (page 129).

1. (*a*) £1000 ; (*b*) £960
2. £1200
3. £7[illegible]2, 10*s.*
4. £840, 12*s.* 6*d.*
5. £486, 15*s.*
6. £64
7. £650
8. £571, 13*s.* 9*d.*
9. £1207
10. £280
11. £1200
12. £900
13. 8·84*d.*; £2, 18*s.* 9*d.*
14. £740 ; £760
15. £499 by 2*s.*
16. £263
17. £187, 0*s.* 10*d.*
18. 1*s.* in £
19. £13, 1*s.* 9*d.*
20. £388, 10*s.*
21. £250 ; £60

Examples 58 (page 131).

1. 17*s.* $8\frac{1}{4}$*d.*
2. $\frac{1}{2}$*d.* on the shilling is the better ; $8\frac{1}{2}$*d.*
3. £21, 7*s.* 2*d.*
4. 19*s.* 6*d.* ; $2\frac{2}{3}$ per cent.
5. (*a*) £5, 11*s.* $7\frac{1}{2}$*d* ; (*b*) £5, 12*s.* 10*d.* (*c*) £5, 13*s.* $11\frac{1}{4}$*d.* ; (*d*) £5, 14*s.* $3\frac{1}{4}$*d.* (*e*) £5, 16*s.* 4*d.*
6. £10, 17*s.* 6*d.*
7. $14\frac{2}{7}$ per cent.
8. 1[illegible] per cent.
9. £2, 17*s.* 4*d.*
10. £1, 11*s.* $10\frac{1}{2}$*d.*
11. The firm B, gaining 3*d.*
12. Lose 8*s.* $0\frac{3}{4}$*d.* by buying in town

Examples 59 (page 136).

1.

£	s.	d.
	16	4
	5	3
	11	8
	7	0
	8	9
2	9	0

2.

£	s.	d.
7	5	0
3	18	8
1	9	3
2	10	8
	8	9
15	12	4

3.

£	s.	d.
5	12	0
2	16	0
1	14	0
2	11	0
12	13	0

4.

£	s.	d.
2	2	0
3	10	0
6	13	0
7	10	0
6	17	6
26	12	6

5.

£	s.	d.
3	3	0
1	8	6
2	5	$4\frac{1}{2}$
8	17	6
15	14	$4\frac{1}{2}$
	7	10
15	6	$6\frac{1}{2}$

6.

£	s.	d.
	10	10
	10	0
	3	6
	11	9
	7	0
	3	0
	10	0
2	16	1

7.

	£	s.	d.
		7	6
	4	10	0
		7	0
		7	0
		1	0
		5	0
	5	17	6

8.

	£	s.	d.
	3	0	0
	2	3	7
	3	4	0
	2	13	3
	5	12	6
	16	13	4
$1\frac{1}{4}$%		4	2
	16	9	2

9.

	£	s.	d.
	1	18	6
	8	13	4
		16	$10\frac{1}{2}$
			$6\frac{1}{2}$
		5	$7\frac{1}{2}$
	11	14	$10\frac{1}{2}$

10.

	£	s.	d.
	38	9	6
	23	12	0
	2	6	$10\frac{1}{2}$
	4	5	3
	6	13	0
	1	0	10
	76	7	$5\frac{1}{2}$
5%	3	16	$4\frac{1}{2}$
	72	11	1

11.

	£	s.	d.
	3	14	8
		12	$4\frac{3}{4}$
	13	6	0
	12	12	0
	1	2	6
	31	7	$6\frac{3}{4}$
+ $2\frac{1}{2}$%		15	$8\frac{1}{4}$
	32	3	3

12.

	£	s.	d.
	1	6	3
		13	$3\frac{3}{4}$
		16	4
	1	4	6
		3	10
	4	4	$2\frac{3}{4}$
		4	$2\frac{3}{4}$
	4	0	0

13.

	£	s.	d.
	2	9	1
	4	5	0
	6	4	3
	4	11	11
	3	11	1
	21	1	4

14.

	£	s.	d.
	40	0	0
	14	0	0
		7	6
	3	17	0
	58	4	6
less	15	2	6
	43	2	0
5%	2	3	1
	40	18	11

15.

	£	s.	d.
	4	12	$11\frac{1}{2}$
		10	$3\frac{3}{4}$
	13	17	9
		6	$4\frac{1}{2}$
	2	16	$7\frac{1}{2}$
	1	5	$11\frac{3}{4}$
	23	10	0
$3\frac{1}{3}$%		15	8
	22	14	4

16.

	£	s.	d.
	107	3	9
	78	12	1
	183	15	0
	65	0	0
	79	0	0
	513	10	10

17.

	£	s.	d.
		13	$10\frac{1}{4}$
	2	15	$9\frac{1}{2}$
	4	2	5
	2	9	$10\frac{1}{2}$
		14	$4\frac{1}{2}$
		13	9
	11	10	$0\frac{3}{4}$
$2\frac{1}{2}$%		5	6
	11	4	$6\frac{3}{4}$

18.

	£	s.	d.
	4	6	$7\frac{1}{2}$
		8	$0\frac{1}{4}$
		13	$1\frac{1}{4}$
		4	$10\frac{3}{4}$
	5	12	$7\frac{3}{4}$
5%		5	$7\frac{1}{2}$
	5	7	0

19.

£	s.	d.	£	s.	d.
12	0	0			
4	10	0			
2	12	0			
5	17	6			
11	5	9			
22	19	4			
2	14	0			
1	15	0			
			63	13	7
			2	13	0
16	19	10			
	1	0			
	3	11			
			17	7	9
			83	14	4

20.

£	s.	d.	£	s.	d.
80	3	11			
97	0	7			
91	11	9			
41	19	6			
104	13	9			
39	10	10			
41	19	2			
62	0	3			
52	0	8			
			611	0	5
53	4	10			
53	1	11			
1	12	8			
	19	5			
2	4	3			
	3	3			
6	2	2			
9	3	4			
			126	11	10
			484	8	7

21.

	£	s.	d.	£	s.	d.
	4	2	6			
	17	7	6			
	3	8	0			
	1	12	0			
	2	2	0			
	14	0	0			
	1	8	4			
	18	2	8			
		9	6			
		6	0			
		1	6			
	4	10	0			
		11	7			
		12	6			
		4	6			
	7	8	9			
		3	0			
		3	6			
	1	1	0			
	1	1	0			
	10	0	0			
	20	14	7			
	113	10	5			
less 2½%	2	16	9			
				110	13	8
	6	10	2			
		3	3			
	1	7	6			
	2	19	4			
				11	0	3
				121	13	11

22.

	£	s.	d.
	17	3	5
	3	5	4
	20	8	9
less 5%	1	[illegible]	5
	23	9	0
less 5%	1	3	5
	13	7	6
	4	10	0
	2	12	0
	5	17	6
	10	10	0
	21	0	0
	4	11	0

£	s.	d.
19	8	4
22	5	7
61	8	0
2	9	1
105	11	0

23.

	£	fl.	c.	m.
	0	5	6	0
	1	2	8	2.5
	15	0	0	0
	1	7	4	0
	18	5	8	2·5
5% less		9	2	9
	17	6	5	3

24.

£	fl.	c.	m.
28	7	0	0
44	5	6	2.5
444	6	0	0
	2	5	6·75
518	1	1	9

Examples 60 (page 142).

1. £23, 7*s.* 6*d.*
2. £4874, 4*s.* 2*d.*
3. £91, 0*s.* 10*d.*
4. 1*s.* 9*d.* per cent.
5. The latter; £15, 6*s.* 9*d.*
6. The previous offer by 3*s.* 7*d.*
7. £750
8. £140
9. 6⅔ per cent.
10. £1553, 6*s.* 10½*d.*
11. 12½ per cent.
12. £4800 (approx.)

Examples 61 (page 144).

1. £423, 10*s.*
2. £7, 13*s.* 5*d.*
3. £581, 17*s.* 6*d.*
4. £9968
5. £9, 5*s.* 3*d.*
6. A, £90; B, £180; C, £450
7. £9500; £390
8. £3000
9. £66, 13*s.* 4*d.*
10. £360; £12371
11. £5258, 6*s.* 8*d.*
12. £1626; £126

Examples 62 (page 148).

1. £1, 2*s.* 6*d.*
2. 354⅙ ac.; 313⅓ ac.; 404⅙ ac.; 243⅓ ac.
3. £6, 4*s.* 10½*d.*; £3, 2*s.* 5¼*d.*
4. £1, 13*s.* 4*d.* to each boy; £3, 15*s.* 0*d.* to each girl
5. A, £42; B, £48; C, £60
6. A man receives, £1, 16*s.* 9*d.*; a woman, £1, 15*s.*; a child, 8*s.* 9*d.*
7. £11, 0*s.* 10*d.*; £8, 16*s.* 8
8. 113, 339, 678, 791
9. £14, 11*s.* 8*d.*; £5, 8*s.* 4*d.*
10. £80, 2*s.* 10*d.*; £50, 1*s.* 9*d.*; £36, 14*s.* 8*d.*
11. £11, 2*s.*; £23, 18*s.*
12. C puts in £265; A gets £113, 15*s.*; B gets £136, 10*s.*

13. A, £1, 7s.; B, £1, 4s.; C, £3
14. £903; £770; £637
15. £445, 12s. $3\frac{1}{2}d.$; £280, 16s. $10\frac{1}{2}d.$
16. £1155
17. A, £500; B, £300
18. A, £451, 16s. 8d.; B, £505, 1s.
19. 88 grains alloy; 2 grains silver
20. 57 crowns; 171 half-crowns; 285 florins; 684 shillings; 1026 sixpences
21. 187 of each
22. 80 sixpences
72 half-crowns
24 half-sovereigns
23. 90 half-sovereigns
120 half-crowns
240 sixpences
24. A, £300; B, £390; no difference

Examples 63 (page 152).

1. 17s. 6d.; 15s. $3\frac{3}{4}d.$; 2s. $2\frac{1}{4}d.$
2. 12 tons 5 cwt. 1·6 qrs.
6 tons 2 cwt. 2·8 qrs.
4 tons 1 cwt. 3·2 qrs.
3 tons 1 cwt. 1·4 qrs.
3. 431 ac. 1 rd.
143 ac. 3 rd.
575 ac.
4. £1, 19s. $4\frac{1}{2}d.$; £2, 12s. 6d.; 13s. $1\frac{1}{2}d.$
5. 16s. 6d.; £1, 4s. 9d.; £1, 7s. 6d.
6. 780 ac.; 468 ac.; 520 ac.
7. 8d. per child; 1s. $6\frac{2}{3}d.$ per woman; 2s. $0\frac{8}{9}d.$ per man
8. A, 11s. 4d.; B, 1s. 4d.; C, 7s. 4d.
9. £1, 6s. 8d.; £1, 3s. 4d.; £1, 8s. 4d.
10. £32, 5s.; £43; £15, 1s.
11. £122, 16s; £127, 18s. 4d.
12. £10, 3s. $1\frac{1}{2}d.$; £8, 15s. 5d.; £7, 0s. 4d.
13. A, £6, 12s.; B, £16, 10s.; C, £11
14. A, £176, 4s.; B, £66, 1s. 6d.; C, £146, 16s. 8d.

Examples 64 (page 154).

1. £182, 9s. $10\frac{3}{4}d.$
2. £300
3. £2295, 5s. 10d.
4. £84,454, 6s.
5. 12s. 6d. in the £
6. £2012, 18s. $7\frac{1}{2}d.$
7. £3960
8. 1s. 8d. in the £; £137, 10s.
9. £74, 6s. 1d.
10. 6s. 8d. in the £
11. 8s. 4d. in the £; £455, 5s. 10d.
12. £1333, 6s. 8d.
13. £945, 19s. 2d.
14. £1521, 17s. 5d.
15. 4s. in the £
16. 14s. 2d. in the £
17. £89, 14s.
18. 8s. in the £
19. £1000
20. Debts, £15,000; assets, £5000
21. 6s. 3d. in the £; £294, 11s. $10\frac{1}{2}d.$
22. £1049, 14s. 11d.
23. £250
24. £27
25. £105

Examples 65 (page 157).

1. 28s.
2. 7d. per lb.
3. 48s. per quarter
4. £3, 4s. $7\frac{1}{2}d.$
5. £1, 15s.
6. 10d.
7. 2·32d.
8. 6s. 6d.
9. 18s. 4d.
10. 3s. and 4s. 10d. per lb.
11. £2, 6s. 8d.
12. 1080
13. £128, 18s. 4d.
£18, 19s. 2d.
14. 168 sold
15. 45 doz. at 5 for 6d.
66 doz. at 11 for 9d.
16. $1\frac{1}{3}$ cwt.
17. 6s. per lb.
18. Loss, 8s. 2d.
19. He gained 8s. 6d.
20. $5\frac{1}{4}d.$ per lb.

Examples 66 (page 162)

1. 7*s*. $2\frac{1}{4}$*d*. per quart
2. 4·8 per cent.
3. £13, 15*s*.; $17\frac{3}{16}$ per cent.
4. 10*s*. 1*d*. per bag of 12 lbs.
5. $5\frac{3}{8}$ per cent.
6. £1, 16*s*. $10\frac{1}{2}$*d*. per gallon
7. 480 oranges
8. £30, 7*s*. 9*d*.
9. £875; $7\frac{1}{27}$ per cent.
10. £6, 13*s*. 4*d*.
11. 8 per cent.
12. $\frac{13}{33}$.
13. £2, 4*s*. 3*d*.
14. $3\frac{3}{4}$ per cent. loss.
15. Weekly sale must increase 8 times
16. 450 at 2*s*. each; 270 at 3*s*. each; $5\frac{5}{19}$ per cent. gain
17. $7\frac{1}{7}$ per cent. gain
18. 20 per cent.
19. 72 per cent. loss
20. A, £170; B, £145.
21. 10 per cent.
22. £1, 10*s*. 11*d*. per doz.
23. $42\frac{1}{2}$ per cent.
24. £1, 17*s*. 4*d*. per cent.
25. 10 per cent.
26. 2*s*. 8*d*. and 4*s*. 4*d*. per lb.
27. 60 per cent.
28. 6·94 per cent.; 11 dozen
29. He gains $6\frac{2}{3}$ per cent.
30. $36\frac{2}{3}$ per cent.
31. £1400
32. The first bookseller gives the better terms by 1*s*.
33. £25
34. $12\frac{1}{2}$ per cent.
35. 11·55 per cent. gain
36. 0·46 per cent. loss
37. 5*d*.
38. 6·15 per cent.
39. He sells 13 lbs. as a stone; original cost £15, 12*s*.
40. 8·4 per cent.
41. 1·05 per cent.
42. 6·48 per cent.
43. 1·06 per cent. gain
44. 2*s*. 5*d*. per lb.
45. $12\frac{1}{2}$% gain; $11\frac{1}{9}$% loss
46. 1*s*. $0\frac{1}{2}$*d*.
47. £7 per cwt.
48. $6\frac{2}{33}$ per cent. gain
49. 1*s*. $4\frac{1}{8}$*d*. per lb.
50. 2*s*. per lb.

Examples 67 (page 167).

1. 17 for 1*s*.
2. 66 at 2*s*. 6*d*.; 30 at 3*s*. 4*d*.; 3*s*. 6*d*.
3. 4*s*.
4. 2*s*. 9*d*.
5. 8*s*. 2*d*. per yard
6. $17\frac{11}{17}$ per cent.; 10*s*.
7. 2*s*. 6*d*. per lb.
8. ·43
9. $9\frac{3}{4}$*d*. per lb.
10. 4*s*. 6*d*. per yard
11. 2*s*. $11\frac{1}{2}$*d*.
12. £2, 4*s*. 2*d*.

Examples 68 (page 172).

1. 12*s*. per yard
2. £1 per cwt.
3. £62, 10*s*.
4. £1, 2*s*. 2*d*. per cwt.
5. 14*s*. 6*d*. per yard
6. $6\frac{2}{3}$ per cent.
7. £2, 6*s*. $10\frac{1}{2}$*d*.
8. £7, 6*s*.
9. 11*s*. 3*d*. per stone
10. £3, 15*s*. 10*d*.
11. 6*s*. 3*d*.
12. 4*s*. 2*d*.
13. $2\frac{3}{4}$ per cent.
14. 9 per cent.
15. 5·26 per cent.
16. 5*s*. 7*d*.
17. 20 per cent.

Examples 69 (page 174).

1. $7\frac{1}{2}$ per cent. gain
2. $2\frac{1}{2}$ per cent. loss
3. $12\frac{1}{2}$ per cent.
4. £14, 8*s*.
5. £1287
6. $25\frac{5}{8}$ per cent. gain
7. 50 per cent.
8. £40
9. $2\frac{1}{7}$ per cent. gain
10. $12\frac{1}{2}$ per cent. loss ; £1

Examples 70 (page 177).

1. 1 lb. at 3*s*. with 2 lbs. at 1*s*. 6*d*.
2. 2 lbs. at 1*s*. 9*d*. with 1 lb. at 1*s*. 6*d*.
3. 2 lbs. of best with 9 lbs. of second quality
4. 1 gall. at 36*s*., 5 at 37*s*., 1 at 39*s*., and 1 at 40*s*.
5. 27 galls. of water
6. 72 lbs. of coffee
7. 50 at 16*s*. 6*d*. with 40 at 15*s*.
8. 40 pairs gloves with 50 pairs stockings.
9. $33\frac{3}{4}$ galls.
10. 10 lbs. at 6*d*. with 60 lbs. at 9*d*.
11. 4 lbs. at 4*s*. 3*d*., 1 at 4*s*. 6d., 9 at 5*s*. 4*d*., and one at 5*s*. 6*d*.
12. 22 galls. at 19*s*., 2 galls. at 16*s*., 2 galls. of water

Examples 71 (page 181).

1. £50, 14*s*. 7*d*.
2. £107, 14*s*. 8*d*.
3. £62, 10*s*. 9*d*.
4. £876, 5*s*.
5. £720, 10*s*.
6. £1010, 8*s*.
7. £280, 12*s*.
8. £650
9. £550
10. $2\frac{1}{2}$ per cent.
11. 3 per cent.
12. 5 per cent.
13. $3\frac{1}{2}$ per cent.
14. $2\frac{7}{9}$ years
15. 4 years
16. 3 years
17. 6 years
18. £88, 15*s*. 7*d*
19. 6th Nov 1886
20. $8\frac{1}{3}$ per cent.
21. £563, 1*s*. 2*d*.
22. $5\frac{2}{3}$ years
23. 3 per cent. ; 5 yrs.
24. £25
25. $5\frac{2}{3}$ years
26. 6 per cent.
27. $4\frac{1}{5}$ per cent.
28. $3\frac{1}{4}$ years
29. 3 per cent.
30. $4\frac{8}{9}$ per cent.
31. 5 per cent.
32. $4\frac{1}{4}$ per cent.
33. 2·9 per cent.
34. A, £165 ; B, £150
35. £23, 6*s*. 9*d*.
36. £667, 10*s*.
37. £1500 at 3 p. cent. £1800 at $2\frac{1}{4}$ p. cent
38. $3\frac{1}{3}$ per cent.

Examples 72 (page 183).

1. £850
2. £42, 3*s*. 9*d*.
3. £850, 10*s*.
4. £227, 12*s*.
5. £660, 14*s*.
6. £978
7. £960
8. 5633, 6*s*. 8*d*.
9. £1600
10. £384, 10*s*.

Examples 73 (page 187).

1. (a) £12, 3s. 3d.
   (b) £74, 10s. 5d.
   (c) £31, 3s. 3d.
   (d) 1s. 5d.
2. (a) £1010, 15s. 1d.
   (b) £862, 8s. 10d.
   (c) £577, 5s. 2d.
3. (a) £8, 7s. 6d.
   (b) £41, 12s. 2d.
   (c) £23, 6s. 5d.
   (d) £19, 9s. 5d.
4. (a) £510, 1s. 6d.
   (b) £758, 18s. 9d.
   (c) £79, 2s. 3d.
5. £60
6. £13, 9s. 4d.
7. £5, 16s. 11d.
8. 3·14 per cent.
9. £13, 16s. 1d.
10. £8523, 10s. 2d
11. £17, 18s. 7d.
12. 2·73 per cent.; £22, 18s. 10d.
13. £337, 16s. 2d.
14. £48, 18s. 9d.
15. £670,230,307

Examples 74 (page 194).

1. £7920
2. 5426, 8s.
3. £5250; £6000 Stock
4. £4031, 5s.
5. £502
6. £16,625
7. £277, 16s. 3d.
8. £10,500 Stock
9. £1305 Stock
10. 117
11. £1055
12. 2154
13. 760
14. 6¼
15. 87½
16. 102½
17. Gained £12, 10s.
18. Gained £106

Examples 75 (page 196).

1. £180
2. £24, 7s. 6d.
3. £249, 15s.
4. £125, 10s.
5. £245
6. £460
7. £312, 10s.
8. £11,760
9. £11,000
10. 101
11. £2175, 16s. 8d.
12. £2812, 10s.
13. £2412, 10s.; 96½
14. £140
15. 1⅞ per cent.
16. £1326

Examples 76 (page 199).

1. £8400 Stock; income increased by £28
2. £800 Stock; income increased by £5, 12s.
3. Increased by £25, 17s. 6d.
4. Increased by £109, 10s.
5. Decreased by £14, 10s.
6. Decreased by £10, 15s.
7. 90
8. Increased by £24
9. 166⅔
10. Increased by £18
11. Par
12. Decreased by £5, 12s. 6d.
13. Par
14. Increased by £22, 2s. 6d.
15. £700 Stock; £1600 Stock; no change in income
16. 8 per cent.
17. Increased by £31, 10s.
18. Increased by £5
19. 83⅓
20. £4000
21. Increased by £8, 9s.
22. £190
23. £7680 Stock; £211, 4s.
24. £97,600
25. £388, 15s.

Examples 77 (page 203).

1. £20
2. Increased by £164, 5s.
3. £1010 Railway Stock
4. £1470
5. No alteration
6. Decreased by £23
7. £2063
8. 86
9. Increased by £18, 15s.
10. Increased by £18
11. Increased by £15
12. Increased by £26, 13s. 4d.
13. $77\frac{5}{8}$
14. £12087, 10s.
15. £8, 0s. $6\frac{1}{4}$d.
16. £5362, 10s.
17. £45,000 Stock; 80
18. $6\frac{1}{4}$ per cent.
19. £10,054
20. £500

Examples 78 (page 206).

1. $3\frac{1}{5}$ per cent.; £48
2. $153\frac{13}{24}$
3. £50; 189
4. £46, 13s. 4d.; $4\frac{8}{33}$ per cent.
5. The $3\frac{1}{4}$ is the better stock; £80, 6s. 11d.
6. $72\frac{13}{16}$
7. Great Western Stock at $136\frac{1}{2}$; 6s. 4d.
8. $82\frac{1}{7}$
9. £9000
10. $3\frac{1}{33}$ per cent.
11. The Railway debentures at £4, 1s. 11d.
12. $87\frac{1}{2}$; £1600 Stock
13. 3·26 per cent.
14. 75
15. Decreased by 0·92 per cent.
16. £1500 in the 3 per cents.; £500 in the 5 per cents.
17. £1155
18. £4725
19. £6175 in the [illegible] per cents.
20. 4 per cent.
21. £200; £12,500
22. $5\frac{1}{6}$ per cent.
23. The £25 shares
24. $2\frac{1}{2}$ per cent.

Examples 79 (page 209).

1. £112, 11s. 7d.
2. 4·62 per cent.
3. 108
4. £8; 99
5. £400 in $4\frac{1}{2}$ per cents. £600 in 4 per cents.
6. £1200 in 3 per cents £[illegible]60 in 5 per cents.
7. £231, 4s. 7d.
8. £3248 in Railway Debentures £2352 in Colonial Stock
9. £4640
10. $59\frac{3}{4}$
11. [illegible]; 4·386 per cent.
12. Gain, £5[illegible]
13. Buying "cum. div." he gains £29, 7s. 6d.
14. £1[illegible],847, 10s.
16. £3505
17. £4500
18. £12, 12s. 6d. gain
19. £91, 12s. 9d.
20. £[illegible]142, 17s. 9d.
21. 3·3575 per cent.
22. English Railway Stock is the better; £8, 2s. 3d.

Examples 80 (page 220).

1. £5, 2s.
2. £869, 11s. 4d.
3. £450
4. £235
5. £4000.
6. 23rd July
7. £812, 10s.
8. $4\frac{1}{2}$ per cent.
9. £635
10. 73 days
11. £1, 2s. 6d.
12. £440, 6s. 6d.
13. £927, 5s.
14. 1s. 7d.
15. 73 days
16. 9 months
17. 7s.
19. £2, 5s.
20. 13s. 4d.
21. 3d.
22. £10,400
23. £1275
    £1304, 7s.
24. $4\frac{1}{8}$ per cent.
25. £608, 6s. 7d.
26. £200, 8s. 4d.
27. £2, 11s. 5d.
28. 2s. 11d.
29. £71, 5s.
30. £620
31. $6\frac{2}{13}$ per cent.
32. £746, 5s.
    £746, 5s. $4\frac{1}{2}$d.
33. £330, 10s.
34. £505
35. £40,400
36. $3\frac{2}{13}$ per cent.
    3 per cent.
37. £230, 8s.
38. 4·8 per cent.
39. £224

Examples 81 (page 222).

1. £2, 13s. 3d.
2. £345, 8s. 3d.
3. £29, 18s. 8d.
4. £495, 3s.
5. 1s. 5d.
6. Gains, £1, 4s.
7. £57, 15s. $4\frac{1}{2}$d.
8. £12; £12, 1s. 7d.
9. £861, 7s. 6d.
10. £842, 4s. 8d.
11. £306, 2s. 5d.
12. 3·63 per cent.
13. £2, 18s. 6d. gain
14. 24997·5 francs.
15. £302.
16. £618, 11s. 2d.

Examples 82 (page 227).

1. 128, 64, 243, 125
2. 4, 2, 8, 9
3. $\frac{1}{3}$, $\frac{1}{27}$, $\frac{1}{16}$, 1
4. $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{64}$, $\frac{1}{25}$
5. 4, 9, 12, 7
6. -2, -5, -4
7. $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{3}$
8. 4, 2, 3
9. 4, 8, 36
10. 27, 16, 32
11. 216, 243, 2, $\frac{1}{4}$
12. 64, 729, 4

Examples 83 (page 230).

1. 2·6385
2. 1·2227
3. 3·6087
4. 1·9115
5. $\bar{1}$·1482
6. 2·9018
7. 0·9157
8. 5·9528
9. $\bar{2}$·9703
10. 5·6304
11. 4·7998
12. $\bar{4}$·8225
13. 70·31
14. 4·823
15. 2154
16. 0·2818
17. 5·463
18. 0·06033
19. 1·043
20. 1·204
21. 68160
22. 0·0005084
23. 685300000
24. 0·0001016

Examples 84 (page 234).

1. 150800
2. 2041
3. 0·2085
4. 0·4102
5. 25940
6. 47·02
7. 0·02919
8. 0·4667
9. 6·330
10. 566·3
11. 33480
12. 0·00004437
13. 267·7
14. 21·28
15. 3·877
16. 9·399
17. 27·64
18. 2·287
19. 1·437
20. 80·76
21. 0·5223
22. 0·1312
23. 0·0002247
24. 0·0004191

Examples 85 (page 235).

1. 3193×10^{8}
2. 3314×10^{3}
3. 2831×10^{10}
4. 1·276
5. 1·166
6. 1·090
7. 1·152
8. 1132
9. 0·1030
10. 4914×10^{-20}
11. 7461×10^{-13}
12. 2267×10^{-8}
13. 1·479
14. 8·678
15. 1·205
16. 1·543
17. 2·525; 0·9486
18. 3·324; 2·042
19. 3·371; 1·837
20. 22·56; 74330
21. 9541
22. 11·450
23. 15240
24. [illegible]488
25. 9
26. 1·085
27. 1·116
28. 0·6767
29. 0·7096
30. 0·4150

Examples 86 (page 241).

1. £41, 13*s.* 1*d.*
2. £98, 11*s.* 7*d.*
3. £3144, 8*s.*
4. £4232, 14*s.* 1*d.*
5. £384, 9*s.* 1*d.*
6. £96, 5*s.* 2*d.*
7. £46, 6*s.* 10*d.*
8. £1073, 2*s.* 1*d.*
9. £856, 6*s.* 5*d.*
10. £37, 19*s.*
11. £215, 9*s.* 6*d.*
12. £375
13. £666, 13*s.* 4*d.*
14. £3257, 13*s.* 7*d.*
15. £8, 3*s.* 5*d.*
16. £2, 1*s.* 11*d.*
17. £1, 10*s.* 11*d.*
18. £4, 3*s.* 10*d.*
19. £80
20. £1760, 5*s.* 11*d.*
21. £1341, 19*s.* 3*d.*
22. £600
23. £19, 7*s.* 8*d.*
24. £8000
25. £541, 4*s.* 4*d.*
26. £1140, 9*s.* 8*d.*
27. £46, 2*s.* 8*d.*
28. 5·0625 per cent.
29. 3·034 per cent.
30. £6250
31. 8 prizes
32. £440, 15*s.* 1*d.*
33. £7, 10*s.* 11*d.*
34. £27,454
35. £17,779, 18*s.* 6*d.*
36. £22, 9*s.* 11*d.*
37. In 9 years
38. In 10 years
39. £833, 6*s.* 8*d.*
40. £800

Examples 87 (page 245).

Where two answers are given the first is obtained by using 4 figure logarithm tables, the second by using 7 figures.

1. £610; £620, 9*s.* 9*d.*
2. £1351; £1350, 17*s.* 6*d.*
3. £820, 16*s.*; £821, 2*s.* 10*d.*
4. £36; £36, 6*s.* 7*d.*
5. £125, 4*s.*; £125, 12*s.* 6*d.*
6. £197; £195, 17*s.* 1*d.*
7. 5 years
8. 5 years; 14*s.*; 16*s.*
9. £12,000
10. £10·4; £10·052
11. 4 per cent.
12. £500
13. £341, 13*s.* 4*d.*
14. 8 years; £2144: £2143, 11*s.* 9*d.*
15. 4 per cent.
16. 3 years
17. $2\frac{1}{2}$ years
18. $3\frac{1}{4}$ years
19. 2 years
20. 79 years; 78 years
21. 3 per cent.
22. 4 per cent.
23. $1\frac{3}{4}$ per cent.
24. $3\frac{1}{2}$ per cent.

Examples 88 (page 257).

1. £1454, 10*s.* 11*d.*
2. £4800
3. £1400
4. £65
5. £52, 10*s.*
6. £1500; £56, 5*s.*
7. £292, 12*s.* 10*d.*
8. $5\frac{3}{4}$ per cent.
9. £210, 10*s.*
10. £38, 17*s.* 10*d.*

Examples 89 (page 257).

1. £2626
2. £752, 14s.
3. £60
4. £2667, 14s.
5. £1058, 5s. 6d.
6. £50
7. £45
8. £41, 9s. 6d.
9. £451, 10s.
10. £485, 12s.
11. 6 years
12. £6, 2s. 6d.
13. £924, 18s. 8d.
14. At Simple Interest, £201, 14s. At Compound Interest, £220, 6s. 5d.
15. £268, 17s. 3d.

Examples 89a (page 258).

1.

1	2	3	4	5	6	7
1	1·04000	·96154	1·00000	0·96154	1·04000	1 00000
2	1·08160	·92456	2·04000	1·88620	·53017	·49020
3	1·12486	·88900	3·12160	2·77519	·36034	·32036
4	1·16986	·85480	4·24646	3·63000	·27548	·23549
5	1·21665	·82193	5·41632	4·45192	·22462	·18463
6	1·26532	·79032	6·63297	5·24224	·19076	·15076

2. £259
3. £110, 14s. 10d.
4. £13, 7s.
5. £49, 5s. 11d.
6. £103
7. £369, 12s. 2d.
8. £220, 7s. 8d.
9. £178, 1s. 6d.
10. £379, 4s. 9d.
11. £568, 12s. 6d.
12. £220, 7s. 8d.
13. £188, 15s. 2d.
14. £133, 15s. 10d.
15. £307, 11s. 4d.
16. £248, 5s. 6d.
17. £155, 6s. 3d.
18. £298, 9s. 8d.
19. £165
20. £68, 13s. 5d.

Examples 90 (page 260).

1. 70
2. 45
3. 63
4. 88
5. 196
6. 256
7. 324
8. 625
9. 729
10. 1296
11. 42
12. 84
13. 90
14. 1008
15. 504
16. 900
17. 385
18. 150
19. 819
20. 3315

Examples 91 (page 264).

1. 34
2. 41
3. 73
4. 9·3
5. 253
6. 287
7. 999
8. 1296
9. 2307
10. 3789
11. 6809
12. 7905
13. 80·27
14. 6·907
15. 15337
16. 27091
17. 0·0253
18. 108000
19. 490·304
20. 11111·1111

Examples 92 (page 265).

1. 0·31623
2. 2·4495
3. 3·6056
4. 0·09849
5. 58·932
6. 7·9897
7. 0·00632
8. 0·09487
9. 8·1140
10. 1·9594
11. 11·446
12. 9361·1
13. 2·646
14. 3·317
15. 0·126
16. 0·045
17. 8·110
18. 94·340
19. 19·893
20. 9·478
21. 3·162
22. 9·210
23. 17·321
24. 876·356

Examples 93 (page 266).

1. $\frac{4}{5}$
2. $\frac{17}{23}$
3. $1\frac{5}{31}$
4. $1\frac{30}{53}$
5. $\frac{93}{97}$
6. $1\frac{65}{674}$
7. 0·600
8. 0·249
9. 0·614
10. 0·345
11. 0·296
12. 0·066

Examples 94 (page 267).

1. 15	4. 64	7. 126	10. 42
2. 24	5. 88	8. 132	11. 6
3. 33	6. 96	9. 8	12. 35

Examples 95 (page 272).

1. 13	6. 51	11. 468	16. 0·026
2. 19	7. 76	12. 631	17. 0·087
3. 27	8. 97	13. 10·15	18. 0·134
4. 31	9. 41·1	14. 4174	19. 0·0365
5. 44	10. 4·46	15. 60·31	20. 0·3002

Examples 96 (page 272).

1. 1·587	6. 1·804	11. 0·4595	16. 85·96
2. 0·928	7. 0·086	12. 1·898	17. 9·147
3. 2·466	8. 1·954	13. 77·02	18. 0·01912
4. 0·406	9. 4·362	14. 1·579	19. 9619
5. 14·642	10. 5·441	15. 0·09557	20. 7796

Examples 97 (page 273).

1. $\frac{3}{4}$	6. $\frac{7}{17}$	11. 0·783	16. 2·554
2. $\frac{2}{3}$	7. $\frac{11}{39}$	12. 1·893	17. 0·733
3. $\frac{17}{21}$	8. $\frac{67}{93}$	13. 0·645	18. 0·868
4. $1\frac{6}{7}$	9. $2\frac{9}{37}$	14. 3·580	19. 0·831
5. $2\frac{16}{23}$	10. $23\frac{11}{12}$	15. 0·745	20. 0·807

Examples 98 (page 275).

1. 2 ft. 7^{i}	11. 4 ft. 5 in.
2. 3 ft. 5^{i} 3^{ii}	12. 6 ft. $2\frac{11}{12}$ in.
3. 3 ft. 4^{i} 7^{ii}	13. 1 ft. $8\frac{5}{12}$ in.
4. 26 sq. ft. 1^{i}	14. 15 sq. ft. 47 sq. in.
5. 39 sq. ft. 5^{i} 7^{ii}	15. 9 sq. ft. 56 sq. in.
6. 45 sq. ft. 7^{i} 0^{ii} 9^{iii}	16. 20 sq. ft. $128\frac{1}{2}$ sq. in.
7. 10 sq. ft. 6^{i} 3^{ii} 11^{iii} 3^{iv}	17. 75 sq. ft. $119\frac{1}{3}$ sq. in.
8. 39 cub. ft. 3^{i} 3^{ii} 11^{iii}	18. 147 cub. ft. 183 cub. in.
9. 46 cub. ft. 2^{i} 0^{ii} 10^{iii} 6^{iv} 3^{v}	19. 28 cub. ft. $502\frac{1}{3}$ cub. in.
10. 291 cub. ft. 7^{i} 7^{ii} 7^{iii} 6^{iv} 8^{v}	20. 89 cub. ft. $618\frac{17}{24}$ cub. in.

Examples 99 (page 276).

1. 27 sq. ft. 7^{i} 6^{ii}	6. 115 sq. ft. 11^{i} 5^{ii} 3^{iii} 6^{iv}
2. 171 sq. ft. 11^{i} 8^{ii}	7. 54 sq. ft. 9^{i} 1^{ii} 2^{iii} 7^{iv}
3. 583 cub. ft. 4^{i}	8. 204 sq. ft. 2^{i} 1^{ii} 11^{iii} 9^{iv}
4. 176 sq. ft. 6^{i} 8^{ii}	9. 500 cub. ft. 8^{i} 9^{ii}
5. 46 sq. ft. 0^{i} 5^{ii} 4^{iii}	10. 813 cub. ft. 7^{i} 4^{ii}

Examples 100 (page 279).

1. 212 sq. ft. 48 sq. in.
2. 329 sq. ft. 48 sq. in.
3. 1484·1 sq. cms.
4. 2752·04 sq. cms.
5. 752 sq. yds. 4 sq. ft. 105 sq. in.
6. 312·308 sq. chains
7. 851·9715 sq. chains
8. 3944 sq. chains
9. 5056·32 sq. metres
10. 13707·175 sq. metres
11. 693·4 sq. yds.
12. 6076225 sq. yds.
13. 390·00025 acres
14. 120 acres 10 sq. poles.
15. 890 ac. 4 sq. ch. 7532·25 sq. lks.
16. 34 sq. yds.
17. 8 acres
18. 73 yds.
19. 1042·8 links
20. 528 yds.
21. 32 hurdles
22. 5 yds.
23. 40·166 yds.
24. 2 fur. 17 poles $3\frac{1}{2}$ yds.
25. 1 fur. 2 poles 3 yds.
26. 390 metres
27. 20 ft.
28. 220 yds.
29. 39 yds.
30. ·66 in.
31. 17 ft. 4 in.
32. 81 ft. 8 in.
33. 2·8284 metres
34. 2 furs. 24 poles 5 yds. 1 ft. 6 in.
35. 8640 blocks

Examples 101 (page 282).

1. 32·25 feet square
2. £2, 16*s.* $8\frac{3}{4}$*d.*
3. £1, 17*s.* 5*d.*
4. 1*s.* 3*d.*
5. £23, 0*s.* 4*d.*
6. $46\frac{2}{3}$ yds.
7. 128 yds. 1 ft.
8. £4, 1*s.* 1*d.*
9. 513 tiles
10. £9, 7*s.* 6*d.*
11. 72 yds.
12. £151, 4*s.*
13. £162
14. 7500 planks
15. £129, 17*s.*
16. £6, 9*s.* 10*d.*
17. 108 yds. ; £6, 15*s.*
18. 92 yds. ; £3, 16*s.* 8*d.*
19. £15, 7*s.* 1*d.*
20. 4800 meshes
21. 34 yds. ; £9, 1*s.* 4*d.*
22. £5, 11*s.* 6*d.*
23. 15 metres square
24. 57 marks 38 pfennige
25. 768·75 krones
26. 137·25 francs
27. 91803 francs
28. £16
29. 18105 planks
30. $103\frac{1}{8}$ yds.
31. £53, 8*s.*
32. 126036 stamps
33. £1, 8*s.*
34. 429 ft.
35. $\frac{154}{343}$
36. 3177·5 sq. metres.
37. Ceiling £1, 3*s.* 11*d.* ; walls 19*s.* 7*d.*
38. £9, 19*s.* 9*d.*
39. Walls £17, 0 f. 1 c. ; ceiling £6, 4 fl. 4 c.
40. 89709 stamps

Examples 102 (page 287).

1. 2 acres 400 sq. yds.
2. 12 yds.
3. 7·92 acres
4. 5108·74 sq. metres
5. 10[illegible] ft.
6. 1·8 ft.
7. 25·2 acres
8. 84 sq. yds.
9. 34·6 ft.
10. 189 chains 6 links
11. 2725·6 sq. metres ; 408 fr. 84 c.
12. 174 francs 5[illegible] centimes

Examples 103 (page 289).

1. 6·158 yds.
2. 89·03[illegible] miles
3. 51·33 cms.
4. 15·96 ft.
5. 192·27 metres
6. 82 shrubs
7. 6366 kilos. ; 3956 miles
8. 7 yds. $1\frac{1}{2}$ ft.
9. 5·1 sq. cms.
10. £1381, 10*s.* 3*d.*
11. 4·9 acres
12. 78·5 yds.
13. 126·45 sq. ft.
14. 351·8 sq. ft.
15. 98·48 sq. metres
16. 23·86 sq. metres
17. 54·76 sq. cms.
18. 490·875 sq. ft.
19. 84[illegible]·96 sq. in.
20. £25, 3*s.* 2*d.*
21. 1*s.* $3\frac{1}{2}$*d.* per sq. foot
22. 29·51 ft.
23. 7 cms.
24. 1·34 lbs. (Troy)

Examples 104 (page 292).

1. $168\frac{3}{4}$ cub. ft.
2. $688\frac{1}{2}$ cub. ft.
3. 174 cub. yds. 6 cub. ft.
4. 60·306
5. 406·5075 cub. cms.
6. 11 cub. ft. 675 cub. in.
7. 9 cub. yds. 16 cub. ft. 145 cub. in.
8. 9 cub. yds. 1 cub. ft. 243 cub. in.
9. 262·144 cub. cms.
10. 256·047875 cub. metres
11. 40 cub. ft. 660 cub. in.
12. 9 cub. yds. 15 cub. ft. 1684 cub. in.
13. 125·258 cub. cms.
14. 200·8125 cub. in.
15. 4 cub. ft. 54 cub. in.
16. 274·717 cub. cms.
17. 381·704 cub. in.
18. 371·102 cub. cms.
19. 180 cub. ft. 1512 cub. in.
20. 612·9 cub. cms.
21. 625 cub. ft.
22. 4 ft. ; 2 ft. 4 in. ; 1 ft. 6 in.
23. 2703360 bricks
24. [illegible]04 persons
25. 28·8 sq. yds.
26. $12\frac{3}{4}$ ft.
27. 38·4 in.
28. $2179\frac{1}{6}$ cub. ft.
29. 11·5 ft.
30. 8 ft.
31. 16·383.
32. 34 yds. 2 ft. 2 in.
33. 21·644 kilos.
34. £20, 16*s.* 4*d.*
35. 3333·3 grammes
36. 0·22 gallon

Examples 105 (page 294).

1. 11 ft. 2 in.
2. 509·2 cub. in.
3. 2·88 oz.
4. 83 tons 3 cwts. 2 qrs. 26 lbs. 2 oz.
5. 1 ft. 4 in.
6. 4486·5 galls.
7. 19·75 in.
8. 4000 galls.
9. 8 ft.
10. 3·08 ft.
11. 5625000 galls.
12. 14*s.* 6*d.*; 170 lbs.
13. 1391 cub. ft. 270 cub. in.; £23, 3*s.* 9*d.*
14. 1350 sq. in.
15. £13, 15*s.* 7½*d.*
16. 11 ft. 11 in.
17. £2, 0*s.* 10*d.*
18. 5 in.; 137½ cub. ft.
19. £12, 10*s.* 4*d.*
20. 8·801 in.
21. 2 hrs. 13 mins. 41 secs.
22. 70 ft. 11·25 in.
23. 3·9 in.
24. 3·04 cms.
25. 120 blocks

Examples 106 (page 297).

1. 1 cub. ft. 222 cub. in.
2. 57·27 lbs.
3. 2412·749 cub. ft.
4. 1536000
5. radius, 1·75 in.; difference of surface, 18·0136 less
6. 1 cub. ft. 1472 cub. in.
7. £10, 15*s.* 6*d.*
8. 1800 cub. ft.
9. 1157 cub. ft.
10. $2{\cdot}642 \times 10^{11}$ cub. miles
11. 116·6 cub. in.
12. 2·5 cms.

Examples 107 (page 297).

1. 817 sq. ft.
2. 763·4 sq. ft.; £14, 6*s.* 3*d.*
3. 43·6
4. 6157·536 cub. ft.
5. 2592 sq. yds.
6. £23, 6*s.* 10*d.*; 3 tons 16 cwts.
7. 70·71 sq. ft.
8. 9369 times
9. 87·5 in.
10. 10296 sq. ft.; 125 ft.; 82·368 ft.
11. 83·139 sq. ft.
12. 10 yds.; £348, 6*s.* 8*d.*
13. £15
14. 82 sq. ft.
15. 8·64 in.

Examples 108 (page 299).

£	s.	d.
24	15	0
20	5	0
108	0	0
45	0	0
63	0	0
45	0	0
63	0	0
9	0	0
6	7	6
4	10	0
23	8	0
5	8	0
11	5	0
1	16	0
47	11	0
6	5	0
21	0	0
3	4	0
0	10	6
26	5	0
3	0	0
20	5	0
31	10	0
27	0	0
22	10	0
31	10	0
22	10	0
18	0	0

Forward

	£	s.	d.
Forward			
	12	15	0
	12	15	0
	9	0	0
	18	0	0
	11	5	0
	9	15	0
	7	10	0
	6	15	0
	12	0	0
	16	10	0
	5	10	0
	1	8	0
	15	6	0
	4	8	0
	30	15	0
	7	13	0
	21	12	0
	6	12	0
	£921	4	0

£	s.	d.
£264	0	11
101	10	8
168	6	4
118	12	3
653	10	2
£267	13	10
6	13	10
£261		

MISCELLANEOUS EXAMPLES (page 301).

1. 123000
2. $3^2 \times .^2 \times 19^2$
3. £101, 4*s.* 11*d.*
4. 10022·15
5. 9 cwts. 1 qr. 18 lbs.
6. £4, 19*s.* 3*d.*
7. The $3\frac{1}{2}$ per cents. at $92\frac{1}{8}$; £7, 3*s.* 2*d.*
8. $4\frac{2}{13}$*d.* ; £12, 15*s.* $0\frac{3}{4}$*d.* loss
9. $2\frac{1}{8}$ per cent.
10. 162100 tons
11. ·82397
12. 5.919
13. 1026420 in.
14. £10, 0*s.* 7*d.*
15. 2·1*s.* ; 2·45*s.* ; 2·8*s.* ; 3·15*s.*
16. 280 sacks
17. 2*s.* 6*d.* in the £
18. £5, 3*s.* 6*d.*
19. 0·1608
20. £376, 13*s.* 4*d.*

21. £235, 3*s.* 5*d.*
22. 3 tons 9 cwts. 2 qrs. $12\frac{1}{4}$ lbs.
23. 8·026
24. Gain 25 p. cent. ; loss 20 p. cent.
25. 247·08
26. 269 yds. 1 ft.
27. 18·6
28. £13788; 1*s.* $11\frac{3}{4}$*d.*
29. $12\frac{3}{4}$ ft. ; 12954·199 galls.
30. £50, 8*s.*
31. ·9564 marks per kilogramme.
32. $3519\frac{1}{2}$ hours
33. £33, 19*s.*
34. 53 lbs. 1 oz. (Troy)
35. £136, 12*s.* ; £163, 18*s.* 4*d.* £229, 9*s.* 8*d.*
36. 4 per cent.
37. £13, 15*s.*
38. 387,420,489
39. Total £34 ; B, £5 ; C, £15 ; D, £5, 12*s.*
40. 87·5 shillings
41. £245, 10*s.*
42. £6, 8*s.* 2*d.*
43. 2·56 per cent.
44. 5 in. ; $137\frac{1}{2}$ cub. ft.
45. 4*s.*
46. 16 years
47. £3750
48. 2809 cub. ft. 1′ 4″ 6‴ = 2809 cub. ft. 198 cub. in.
49. 0·010
50. 20·06 oz.
51. $36\frac{5}{9}$ miles
52. 91·87 sq. in.
53. 17·334 kilogrammes
54. $\frac{321726}{4995}$
55. 50 per cent.
56. Apples 3*d.* per doz. ; oranges 4*d.* per doz.
57. £1317, 6*s.* 8*d.*
58. £3, 2*s.* 6*d.*
59. 11·03
60. 8*s.* $4\frac{3}{4}$*d.*
61. 158000
62. 1860
63. 441
64. £2, 13*s.* $5\frac{1}{4}$*d.*
65. 80 tons
66. 18·8 runs
67. £6241, 4*s.*
68. £1, 15*s.* 3*d.*
69. £2, 6 c. 4 m.
70. A, £250 ; B, £150
71. £201, 5*s.*
72. £2, 6*s.* 10*d.*
73. $12\frac{9}{13}$
74. $3{\cdot}277 \times 10^{-11}$
75. 162 dollars 3·5 cents
76. £82, 7*s.* 11*d.*
77. 25*s.* ; A, 7*s.* 6*d.* ; B, 10*s.* 6*d.*
78. £10
79. 6 per cent.
80. £4, 18*s.* 6*d.*
81. 6*s.* $8\frac{1}{2}$*d.* per yard
82. 699 tons
83. 7·6893
84. 29 francs 40 centimes
85. 930
86. £815, 10*s.*
87. £173, 0*s.* 5*d.* ; £107, 11*s.* 11*d.* ; £47, 14*s.* 4*d.*
88. 1·86*d.*
89. £130, 8*s.* 4*d.*
90. £3, 1*s.* 8*d.* gain
91. $4\frac{2}{7}$ per cent.
92. £6, 15*s.*
93. 176 yds. ; 45 miles per hour
94. 12·071 minutes
95. 2268 ; 1512 ; 4253
96. 19·856 sq. in.
97. 9*s.*
98. £5479, 7*s.* 7*d.*
99. 0·0125
100. £16, 15*s.* $3\frac{3}{4}$*d.*
101. 29
102. £285, 8*s.* 4*d.*
103. 7 lbs.

104. 15s. 1d.
105. £396, 10s. 7d.
106. 1244073·6 galls.
107. £766, 12s. 5d.
108. £25·58 francs

109. $\frac{7}{12}$; $\frac{43}{104}$
110. £1; £1, 4s.; £1, 2s. 6d.; £2, 10s.
111. £158, 15s. 1d.
112. £3, 6s. 11¼d.
113. £58, 0s. 3d.
114. £2, 15s.
115. 22½ poles; 6 poles
116. 116 hrs. 31 mins. 29 secs.
117. £328, 4s.
118. 215 yds. 3 in.
119. £444, 2s.
120. 150 lbs.

121. 268·446
122. $2^7 \times 11 \times 13$; $3^2 \times 5^2 \times 7 \times 13$; $3^2 \times 5 \times 7^2 \times 11$; 5148
123. 43·24375
124. £118, 4s. 9d.
125. 8⅓ per cent.
126. £41, 4s. 4d.
127. £526, 10s. 8d.
128. 509·2 cub. in.
129. £20, 18s.
130. 11·2 per cent. per annum
131. 2s. 8d. per lb.; 4s. 4d. per lb.
132. £1370, 14s.

133. 12s. 3¼d.
134. 4·512
135. £28, 2s. 6d.
136. £945, 17s.; £953, 7s. 4d.
137. 14 yds. 2 ft. 5⅓ in.
138. 0·46 per cent. loss
139. 11·46 sq. cms.
140. 6473 francs
141. 453·02 grammes; 4530·23 c. cms.
142. £1020
143. ⅔ of a second
144. £7, 10s. 8d.
145. 7478·860
146. 0·1463
147. 887640
148. 64 at 4 for 3d.; 36 at 3 for 4d.
149. A, £467, 6s. 8d.; B, £350, 10s.; C, £175, 5s.; D, £408, 18s. 4d.
150. 316 yds.
151. 100·28 dollars
152. £1, 16s. 3d.
153. £7, 3s. 10½d.; £4, 15s. 11d.; £2, 7s. 11½d.
154. $\frac{3}{13}$
155. £112, 10s. Stock.
156. 10s. 10d.

157. 0·252847 of a day
158. G.C.M., 5544; L.C.M., 1790712
159. 43·8 per cent.
160. £6232, 10s.
161. 3⅜ per cent.
162. £391, 16s. 4d.
163. £126
164. 64 yds.
165. £1 = 15·15 krones
166. 5⅙ per cent.
167. £9, 7s. 6d.
168. £31, 12s. 11d.

169. 0·017453
170. 0·22 of a yard
171. 10s. 4·875d.
172. £1, 13s. 7d.
173. 1·04060401; 2706
174. B gains £1, 11s. 8d.
175. £2, 14s. 2d.
176. 6¾ per cent.
177. £1, 6s. 9¼d.
178. £25, 2s. 6d.
179. £721, 8s.
180. 3258 cub. ft. 9'; 774 sq. ft. 4' 6''

181. 68·423
182. $\frac{586}{1155}$
183. £262, 10s.; £583, 6s. 8d.
184. 2½d. per lb.
185. £3, 15s. 10d.
186. 9 years

187. £65, 7*s.* 11*d.*
188. £606, 13*s.* 4*d.*
189. B, £1363, 6*s.* 8*d.* ; T.D., £30 ; B.D., £30, 13*s.* 6*d.*
190. 60·172 lbs.
191. 825 francs 60 centimes
192. 7 in.
193. $\frac{5}{91}$
194. 0·190
195. £16,235, 8*s.* 9*d.*
196. $2\frac{3}{4}$ per cent.
197. 8*d.* per pint
198. £1957, 4*s.* 5*d.*
199. Captain, £48 ; 1st mate, £21 ; 2nd mate, £14, 8*s.*; seamen, £32, 8*s.*
200. 6 yds. 2 ft. 5 in.
201. 108
202. 6·21 in.
203. £989, 1*s.* 9*d.*; £988, 19*s.* 2*d.*
204. 11*s.*
205. 22·237
206. 107·25
207. 45 doz. at 5 for 6*d.* ; 66 doz. at 11 for 9*d.*
208. 3*s.* per cwt. gain
209. 8 destroyed
210. A, £800 ; B, £590 ; C, £1110
211. 82164 bricks
212. £56, 17*s.* 9*d.*
213. 13179660
214. 3*s.* 6*d.* per cent.
215. 17*s.* 5*d.*
216. $3\frac{1}{2}$ per cent.
217. 14833·177
218. 246,483,000,000,000
219. 37
220. $9\frac{1}{2}$ months
221. 3504 marks 66 pfennige
222. 22·5 cents.
223. 12·7 chains
224. $6\frac{1}{4}$ per cent.
225. £643, 9*s.* 11*d.*
226. $10\frac{2}{3}$ per cent.
227. £3, 16*s.* 6*d.*
228. £137, 14*s.* 8*d.*
229. 141 : 158
230. 5
231. 272·417 yds.
232. £248, 8*s.* $11\frac{1}{2}$*d.*
233. $43\frac{3}{4}$ yds. ; £8, 5*s.*
234. Assets, £1000 ; liabilities, £3000
235. £1836, 14*s.*
236. 16923 francs 52 centimes
237. 100776960
238. £144, 7 f. 1 c. 3 m. ; £144, 14*s.* 3*d.*
239. 1550 sq. ft. 1^{i} 4^{ii}
240. £640, 7*s.* 4*d.* ; £643, 2*s.* 11*d.*
241. 192 and 150
242. 2·58989 sq. kilos.
243. £22,666, 13*s.* 4*d.*
244. 14 centimes
245. £14,475
246. £11, 7*s.* 9*d.*
247. 4·3589
248. £275
249. 15443 tiles ; 1003 fr. 80 c.
250. £93, 8*s.* 8*d.* ; 2335 fr. 84 c.
251. 16·50 miles per hour
252. $5·866 \times 10^{12}$
253. 4·129 civil years ; 4129 civil years ; 4000
254. 1·21 per cent. ; 31020
255. 12·92 per cent. ; 3·36 per cent.
256. 17 months
257. $5·04 \times 10^{6}$
258. 5878 sq. yds.
259. 3·307 percent. ; 86·772 percent.
260. 90 miles
261. 201 sq. yds. 7 sq. ft.
262. 153·16 lbs. (with lid)
263. 16·16 miles
264. 9*s.* 5*d.*
265. £2000
266. 28,000 bullets

267. £516, 19s.
268. £745, 4s. 3d.
269. 14 per cent.
270. £265
271. 9·47d.
272. 3·80 per cent.
273. £15,000
274. 5·1 per cent.

275. £204, 18s. 4d.
276. $35\frac{1}{3}$ days in all
277. £25, 5s.
278. £922, 16s. 8d.
279. 100 gallons
280. £1030 ; $4\frac{1}{2}$ per cent.
281. Cow, £14 ; lamb, 17s. 6d.
282. £8000
283. £929 ; £927, 3s.
284. £329, 5s. 3d.
285. £200, 16s. 5d.
286. 27 years

287. £4952 ; £4951, 16s. 5d.
288. £1288 ; £1289
289. £10, 13s. 4d.
290. £2500 ; 5 per cent.

EXAMINATION PAPERS.

SCOTCH LEAVING CERTIFICATE.

Lower Grade.

1909 (page 319).

1. 2 cwts. 1 qr. 9 lbs. 7 oz.
2. £122, 5s. $2\frac{1}{4}d.$
3. $3^2 \times 5^3 \times 7$; $2 \times 3^3 \times 5^2 \times 7$; G.C.F. $= 3^2 \times 5^2 \times 7$; L.C.M. $= 2 \times 3^3 \times 5^3 \times 7$
4. 226·8 kilogrammes
5. £106, 5s. ; $10\frac{18}{39}$ per cent.
6. 37·5 mils , 118·75 mils ; 829·16 mils ; £0·0375 ; £0·1188 ; £0·8292

1910 (page 319).

1. 75
2. $\frac{17}{20}$; 196·292
3. £3, 13s. 11d.
4. 614,400 bricks
5. £26, 16s. 3d.
6. $\frac{1}{36}$ sq. mile

1911 (page 320).

1. 4s. 9d.
2. £77, 13s. 9d.
3. ·7, ·71, ·73, ·74, ·78, ·83, ·86 ; 3 decimal places
4. 32 men
5. 10·758 sq. ft. and 10·765 sq. 10·76 sq. ft.

1912 (page 320).

1. 1 ft. 4 in.
2. £1600 ; £1813, 6s. 8d. ; £2186, 13s. 4d.
3. 2·00312 cub. metres ; 2003·12 litres
4. 1s. $5\frac{3}{4}d.$; $8\frac{1}{4}d.$
5. £4550

COMMERCIAL ARITHMETIC.

SECOND PAPER.

1909 (page 321).

1. 60 planks
2. £114, 6s. 2d.
3. 1s. 5d.
4. 20 per cent.
5. 100 sq. metres; 0·001 cub. metres. See page 67
6. £424, 12s. 11d.
8. £1, 14s. 1d.

1910 (page 322).

1. 2s. $8\frac{1}{2}$d. per lb.
2. $\frac{1}{4}$ lb.
3. £3, 18s. 9d. rise
4. £1260, 14s. 7d.
5. £59, 10s.
6. 1s. $4\frac{1}{16}$d.
7. £400, 3s. 2d.

1911 (page 323).

1. $6\frac{3}{4}$ inches
2. £36,754, 11s. 11d.
4. £6, 13s. 4d.
5. £2670
6. £21
7. £8500: $81\frac{9}{40}$

1912 (page 323).

1. 1 gallon
2. 16,000 blocks; £323, 19s. 2d.
3. £1478
4. £54, 13s. 9d.
5. See pages 235, 140, 215, 213
6. £1086, 3s. 7d.
7. 20 marks, 48 pf.

HIGHER LEAVING CERTIFICATE.

1. £15,000
2. 3339; 3340 kilogrammes
3. £10,550
4. £53, 6s. 8d. See pages 178, 235, 239
5. 4 per cent.
6. $3\frac{3}{8}$ inches

THE INSTITUTE OF CHARTERED ACCOUNTANTS IN ENGLAND AND WALES.

PRELIMINARY EXAMINATION.

December 1910 (page 325).

1. Total, 79,001,810
   Value, £18,247,482
2. Average reduction of $4\frac{1}{4}$ hrs.
3. 1905 7·38s.
   1906 7·15s.
   1907 7·97s.
   1908 8·21s.
   1909 9·15s.
4. $\frac{1}{\pi}$ = 0·31831; (a) 0·318$\dot{3}$, (b) 0·3183083
   7292·3 cms.; 7292·9 cms.
5. 90·68 secs.; 39·7 miles per hour; 58·2 ft. per sec.; during 9th mile
6. £2, 12s.; $18\frac{3}{4}$ per cent.
7. £2, 15s. 3d.
8. £45, 2s.; £45, 5s. 3d.
9. £4000; £200; 5·04 per cent.

June 1911 (page 327).

1. *Lusitania*, 5 days 22 7 hrs
   Mauretania, 6 days 1 hr.
2. 365
3. (i) 43·31 ; (ii) 0 6 . . .
   (iii) 298800 ; (iv) 834·7 sq. ft.
   (v) Slightly greater than 1000 ft.
4. 3·338 miles ; 44936 tsun.
5. 1 cwt. 1 qr. 14·07 lbs.
7. 1913
8. 6 26 P.M. : 45·5 miles per hr.
9. £1, 0s. 0d. ; $4\frac{1}{4}d$.
10. £91, 3s.
11. 3 lbs. at 2s. to 5 lbs. at 1s. 4d.
    11 cwts. 3 qrs. $10\frac{1}{2}$ lbs.
12. (a) Simple Interest, £188, 2s. 1d.
    (b) Principal, £287, 11s. 5d.
    (c) Years, 18

December 1911 (page 329).

1.

		Increase or Decrease	Incr. or Decr. per Cent.
		£	
		− 1,000,000	− ·20
		+ 2,500,000	+ ·53
		+ 2,600,000	+ ·74
1,314,400,000	1,318,500,000	+ 4,100,000	+ ·31
		+ 136,000	+ 4·2
		− 276,000	− 11·0
		+ 1 277,000	+ 4 0
		+ 160,000	+ 3·5
		+ 250,000	+ ·2 8
51,205,000	52,758,000	+ 1,553 000	+ 3 0
		+ 2,000,000	+ 3·4
110,705,000	114,258,000	+ 3,553,000	+ 3·2

2. 1 6 per cent.
   0·641 ; ·929
3. (i) 10d ; (ii) 9d. ; (iii) 300 per cent. ; (iv) 150
4. (i) 300 (nearer) ; (ii) 681 lbs. ; (iii) £2 5s. (nearer) ; (iv) 0·14
5. $1\frac{7}{8}$ mins. ; $2\frac{1}{2}$ mins.
7. £135, 8s. ; £327 ; 7·1 per cent. ;
   13·1 per cent.
8. 5 lbs ; $3\frac{3}{4}$ per cent.
9. 0 3 ; 0 6 ; 1·5

June 1912 (page 331).

1. 24 295 inches ; 24 inches
2. Average output, 265·8 ; 662·2 ;
   238 7
   Cash value, £106,333,893, 15s. ;
   £124,152,750 ; £74,723,468, 15s.
   32·62 per cent. ; 4·65 per cent.
3. (i) 50 ; (ii) 1·04 ; 2360·4
   £9, 2s. $2\frac{1}{2}d$.
4. £2, 0s. 5d. ; 14·76 per cent. ;
   2s. 7d.
5. £20
6. £12, 14s. 6d. loss
7. (i) [illegible] ; (ii) [illegible] ; (iii) 63 yards
   in 275 minutes
8. £4, 6s. 9d.

SCOTTISH CHARTERED ACCOUNTANTS.

INTERMEDIATE ARITHMETIC.

December 1910 (page 332).

1. 12680 R.
2. 15352·3
3. 0·31831
4. £737, 10*s.*
5. £2367, 15*s.* 9*d.*
6. £49, 12*s.* 2*d.*
7. 3·7 per cent.
8. $3\frac{1}{5}$ per cent.
9. £57, 17*s.* 8*d.*
10. 14·98 kilometres

June 1911 (page 333).

1. 22,400 acres; 9,600 acres (arable); 9,800 acres (pasture) 1,200 acres (woodland)
2. 19599·6
3. 0·0000000318
4. £848, 12*s.* 6*d.*

179 tons 4 cwt. 1 qr. 14 lbs.

5. 5 cwts.
6. £3, 1*s.* 5*d.*
7. £51, 5*s.* 2*d.*
8. £4, 9*s.* 3*d.* increase
9. A, £384; B, £336; C, £280
10. 71 miles per hour

December 1911 (page 334).

1. $\frac{7}{27}$; £1166, 13*s.* 4*d.*
2. 5584·4
3. 0 0150
4. £150, 10*s.* 6*d.*
5. Tea, 1*s.* 8*d.*; sugar, $2\frac{1}{2}$*d.*
6. £2087, 1*s.* 10*d.*
7. £4, 17*s.* 8*d.*
8. £283, 10*s.*
9. £2[illegible]80
10. 0·45

June 1912 (page 334).

1. $\frac{85}{144}$
2. 171·1
3. 0·0125
4. £608, 1*s.*
5. £1, 6*s.* 3*d.*
6. 17*s.* $10\frac{1}{2}$*d.*; $48\frac{23}{24}$ per cent.
7. 8*d.*
8. £5[illegible]3, 10*s.*
9. $256\frac{1}{4}$
10. 72·6 centimes; 7*d.*

December 1912 (page 335).

1. 95·4
2. 4975
3. 0·297
4. £141, 11*s.* 11*d.*
5. 200
6. 4 per cent.; £1627, 12*s.* 1*d.*
7. £5, 2*s.* 1*d.*; $10\frac{1}{2}$*d.*
8. 1 ton $14\frac{2}{7}$ cwt.; 2 tons $5\frac{5}{7}$ cwt.; 1 ton $8\frac{4}{7}$ cwt.; 2 tons $11\frac{3}{7}$ cwt.
9. £172, 10*s.*
10. 43·9 pence per yard

INTERMEDIATE, ANNUITIES.

page 336).

1. See page 248
2. £840
3. See page 250
4. £376 ; £381, 19*s.* 3½*d.*
5. £688, 15*s.* 7*d.*
6. £258, 10*s.*
7. See page 248

$$\log n = \log \frac{(Si+1)}{(1+i)}$$

FINAL, ACTUARIAL SCIENCE.

(page 336).

1. £927, 10*s.*
2. 53 years
3. £230
4. 5·0945
5. See page 215
7. (*a*) £480 ; (*b*) £23, 6*s.* 3*d.*
8. See page 254
10. £573 ; £427
11. 4·060401
12. £59·744
13. See page 252
14. See page 254
15. £210
16. $\frac{69}{\text{Rate}}$
17. 8*s.* per cent.
18. 0·111 ; 1·553
19. £110, 6*s* ; see page 252
20. £861
21. £329, 5*s.*
22. £154, 14*s.*

INSTITUTE OF BANKERS

1909 (page 338).

1. (i) 2·155 ; (ii) 1206·356 ; (iii) 15·194
2. £39.445, 8*s.* 5*d.*
3. 44,457 francs 45 centimes

4.

£	Marks.	pf	s	Marks.	pf	d.	pf.
1	20	52	1	1	02·6	1	8·55
2	41	04	2	2	05·2	2	17·10
3	61	56	3	3	07·8	3	25·65
4	82	08	4	4	10·4	4	34·20
5	102	60	5	5	13·0	5	42·75
6	123	12	6	6	15·6	6	51·30
7	143	64	7	7	18·2	7	59·85
8	164	16	8	8	20·8	8	68·40
9	184	68	9	9	23·4	9	76·95

1431 marks 80 pf. ; 1742 marks 90 pf.

5. 3·125 centimes ; $7\frac{1}{2}$ per cent.
6. 73 per cent.
7. B loses £4, 10s. ; 15 per cent.
8. £16, 0s. 7d.
9. 15057.58 francs
10. $83\frac{5}{24}$; £1200
11. £21, 5s ; $17\frac{1}{7}$ per cent.
12. 2450 shares
13. 20 per cent ; 50 per cent.
14. $23\frac{1}{3}$ per cent. ; 8 98 per cent.

1910 (page 339).

1. (i) 67·519
(ii) 0 044
(iii) 2 158925
(iv) 3·18
2.

Dollars.	Fr.	c.
1	5	18
2	10	36
3	15	54
4	20	72
5	25	91
6	31	09
7	36	27
8	41	45
9	46	63

5112 fr. 48 c.
4068 fr. 71 c.

3. 2596 for 2956
6439 for 1639
4. £64, 9s. ; £64, 9s. 8d.
5. German, 3 561 per cent
French, 3 077 per cent.
Consols, 3 051 per cent.
6. 122 907 sovereigns
7. £305, 5s.
8. $6\frac{1}{3}$ per cent , £12,000
9. 2,000,000
Increase of £1666, 13s 4d
10. £146, 11s. 1d. ; £144, 8s 4d
11. £64,273, 7s. 9d. ; $6\frac{1}{4}$ per cent.
12. £5, 6s.
13. $12\frac{1}{2}$ per cent
14. £272, 8s. 4d. ; 7 9 per cent.

1911 (page 341).

1. (i) 1036·169
(ii) 0·0666
(iii) 3·442
(iv) 2 012200
2. £1282, 13s. 6d.
3. 2·41 per cent. ; 3·12 per cent.
4. 50·55666
176·95 marks
£2, 9s. $5\frac{1}{2}$d.
5. £29, 16s. 6d.
6. £10, 13s. ; $11\frac{1}{9}$ per cent. ;
$7\frac{1}{27}$ per cent.
7. £782, 2s. 10d.
8. 62·7 marks
9. £62, 12s.
10.

Marks	Francs
1	1 2340
2	2·4680
3	3·7020
4	4 9360
5	6·1700
6	7·4040
7	8 6380
8	9 8720
9	11·1060

7120 francs 85 centimes
4972 francs 9 centimes
11. 16·6 centimes ; 10 per cent.
12. A ; 15 per cent.
13. 2·38 dollars better *via* London
14. 19·1 per cent. ; 17·8 per cent.

1912 (page 342).

1. (i) 373·660
   (ii) 2·865
   (iii) 2·9522
2. $\frac{7}{12}$; $14\frac{7}{12}$; $30\frac{6}{7}$
3. £1 = 25·22 francs
   1 franc = 0·8099 marks
4. £4000
5. £13, 2*s*. 6*d*. gain ; $4\frac{3}{4}$ per cent.
6. £5336
7. £1000
8. 15·68 per cent.
9. 3082
10. 10 per cent. ; $6\frac{2}{3}$ per cent. ; £378
11. 11·72 pence
12. £137, 5*s*. 3*d*.

BANKERS' INSTITUTE OF SCOTLAND.

1909 (page 344).

1. $0\dot{3}$
2. 86·3037
3. £900
4. £220 ; £40
5. 1·47 per cent. ; £7, 8*s*. 11*d*.
6. 9*s*. 11*d*.
7. ·105

1910 (page 345).

1. £6747, 8*s*. 9*d*.
2. £2, 17*s*. $6\frac{1}{2}d$.
3. $7\frac{3}{4}d$.
4. £5520 ; £1, 4*s*. per cent.
5. A, £2100 ; B, £800
6. Stock 0·18 per cent. better
7. £153, 15*s*. $10\frac{1}{2}d$.

1911 (page 345).

1. (i) $\frac{28}{111}$
   (ii) 0·00552
2. 8*s*. 4*d*.
3. £5580 ; 5 per cent.
4. 5·6 per cent.
5. £249, 11*s*. 5*d*.
6. 4·17 per cent.
7. 3560·5 florins ; 15·92 florins less

1912 (page 346).

1. 12*s*. $2\frac{1}{4}d$. ; 0·07
2. 85470
3. £204, 18*s*. 4*d*.
4. 6·4 per cent.
5. $3\frac{3}{4}$ per cent.
6. 3 per cent.
7. 2665·21 dollars

THE LONDON CHAMBER OF COMMERCE.

Junior Arithmetic.

1909 (page 347).

1. 1213
2. 67 ; 2
3. 13 ; 10 ; 2
4. 27 grains
5. 42 per cent.
6. 4 per cent.
7. 228 acres ; 30·9 sq. in.
8. $7\frac{1}{2}$ minutes on slow train ; $7\frac{3}{4}$ minutes on fast train
9. (i) 17 mins. 44 secs. past 12
   (ii) 19 mins. 40 secs. to 6

1910 (page 348).

1. 3^3 ; $2^7 \times 3^4 \times 13 \times 67$
2. England and Wales, 1·2 per cent.
   Scotland, 1·5 „
   Ireland, 4·3 „
   Total, 1·5 „
3. 24,137 times ; 2·76 miles
4. 6250 galls. ; 1[illegible]0,000 litres
5. 86,100 sq. miles ; 467 miles
6. £1, 16*s.* 2*d.* ; 213 cub. ft.
7. £6, 5*s.*
8. (*a*) 4 hr. 6 min. 41 sec. ;
   4 hr. 4 min. 30 sec. ;
   3 hr. 42 min
   (*b*) 5 hr. 27 min. 35 sec. P.M.
   5 hr. 22 min. 35 sec P.M.

1911 (page 349).

1. H.C.F. $13 \times 29 = 377$
   L.C.M. $2^3 \times 3 \times 5 \times 7 \times 13 \times 29 = 316,680$
2. £91 ; £35 ; £17 ; £20
3. 0·78 per cent.
4. 56 miles ; 9*s.* 4*d.*
5. £3, 1*s.* 4½*d.*
6. 31·84 , 28·98 ; 39·18 ; 391·8 grammes
7. 50 min. 21¾ secs. ; 671,500 litres
8. 0·675 ; 0·667

1912 (page 350)

1. 1078
2. 660.
3. 291
4. 28·8 sq. yds.; 1⅜ cub. yds.
5. 11,900 blocks
6. 1*s.* 10¼d. per lb.
7. 12*s.* 1*d.*
8. 10 per cent.
9. (i) £1, 16*s.* 6*d.*
   (ii) 4*s.*

Commercial Arithmetic.

1909 (page 351).

1. 9 for a shilling ; 10*d.*
2. 13 per cent.
3. £15.
4. 8 : 7 ; 1*d.*
5. £7, 5*s.* 10½*d.*
6. £335 ; £207, 14*s.* ; £77, 6*s.*
7. See pages 88–90
8. A pays B £94, 14*s* 8*d.*
9. £92, 5*s.* 8*d.*
   5·3 per cent. nearly

1910 (page 352).

1. (*a*) 6 sq. yds. 1 sq. ft. 59 sq. in.
   (*b*) 3 cub. yds. 3 cub. ft. 704 cub. in.
2. 73⅝ yards ; £2, 3*s.* 8*d.*
3. 336±2 sq. ins. ; 0·006 ; 2 sq. in.
4. 7*s.* 1*d.*
5. £5338, 6*s.*
6. 7*s.* ; 3 1/9 per cent.
7. (*a*) £441, 10*s.* 6*d.*
   (*b*) £4[illegible]1, 7*s.* 9*d.*
   (*c*) £316, 1*s.*
8. See pages 88–90
9.

	Beetroot	Cane	Total
Largest	2·29	0·38	2 6
Smallest	0·07	0 1	0·06

1911 (page 354).

1. (a) 25,465
   (b) 0·012324
2. 1s. 4·78d.; £2, 13s. 2d.
3. 5s. 9d.
4. 1600; 2400
5. Diminished; 12 to 11 nearly
6. £1600; £1464
7. (1) 4·65d.
   (2) 7·35d.
   (3) 8·25d.
8. £262, 4s. 6d.
9. 3 years 1·124864
   6 years 1·265318
   £119, 7s. 10d.

1912 (page 355).

1. (a) 9,979
   (b) 0·066554
2. £291, 13s. 4d.
3. 35·4 lbs.
4. 776 francs
5. Wheat, 14·4 per cent.
   Sugar, 39·9 per cent.
   Coffee, 21·6 per cent.
6. (a) £455, 10s. 3d. (b) £308
7. £103$\frac{9}{16}$
8. £10, 12s. 5d.

INDEX

Printed at THE BALLANTYNE PRESS
SPOTTISWOODE, BALLANTYNE & CO. LTD.
Colchester, London & Eton, England